

## NAG Library Function Document

### nag\_sparse\_sym\_precon\_ichol\_solve (f11jbc)

#### 1 Purpose

nag\_sparse\_sym\_precon\_ichol\_solve (f11jbc) solves a system of linear equations involving the incomplete Cholesky preconditioning matrix generated by nag\_sparse\_sym\_chol\_fac (f11jac).

#### 2 Specification

```
#include <nag.h>
#include <nagf11.h>

void nag_sparse_sym_precon_ichol_solve (Integer n, const double a[],
    Integer la, const Integer irow[], const Integer icol[],
    const Integer ipiv[], const Integer istr[],
    Nag_SparseSym_CheckData check, const double y[], double x[],
    NagError *fail)
```

#### 3 Description

nag\_sparse\_sym\_precon\_ichol\_solve (f11jbc) solves a system of linear equations

$$Mx = y$$

involving the preconditioning matrix  $M = PLDL^T P^T$ , corresponding to an incomplete Cholesky decomposition of a sparse symmetric matrix stored in symmetric coordinate storage (SCS) format (see Section 2.1.2 in the f11 Chapter Introduction), as generated by nag\_sparse\_sym\_chol\_fac (f11jac).

In the above decomposition  $L$  is a lower triangular sparse matrix with unit diagonal,  $D$  is a diagonal matrix and  $P$  is a permutation matrix.  $L$  and  $D$  are supplied to nag\_sparse\_sym\_precon\_ichol\_solve (f11jbc) through the matrix

$$C = L + D^{-1} - I$$

which is a lower triangular  $n$  by  $n$  sparse matrix, stored in SCS format, as returned by nag\_sparse\_sym\_chol\_fac (f11jac). The permutation matrix  $P$  is returned from nag\_sparse\_sym\_chol\_fac (f11jac) via the array **ipiv**.

It is envisaged that a common use of nag\_sparse\_sym\_precon\_ichol\_solve (f11jbc) will be to carry out the preconditioning step required in the application of nag\_sparse\_sym\_basic\_solver (f11gec) to sparse symmetric linear systems. nag\_sparse\_sym\_precon\_ichol\_solve (f11jbc) is used for this purpose by the Black Box function nag\_sparse\_sym\_chol\_sol (f11jcc).

nag\_sparse\_sym\_precon\_ichol\_solve (f11jbc) may also be used in combination with nag\_sparse\_sym\_chol\_fac (f11jac) to solve a sparse symmetric positive definite system of linear equations directly (see Section 9.4 in nag\_sparse\_sym\_chol\_fac (f11jac)). This use of nag\_sparse\_sym\_precon\_ichol\_solve (f11jbc) is demonstrated in Section 10.

#### 4 References

None.

## 5 Arguments

- 1: **n** – Integer *Input*  
*On entry:*  $n$ , the order of the matrix  $M$ . This **must** be the same value as was supplied in the preceding call to nag\_sparse\_sym\_chol\_fac (f11jac).  
*Constraint:*  $n \geq 1$ .
- 2: **a[la]** – const double *Input*  
*On entry:* the values returned in the array **a** by a previous call to nag\_sparse\_sym\_chol\_fac (f11jac).
- 3: **la** – Integer *Input*  
*On entry:* the dimension of the arrays **a**, **irow** and **icol**. This **must** be the same value returned by the preceding call to nag\_sparse\_sym\_chol\_fac (f11jac).
- 4: **irow[la]** – const Integer *Input*  
 5: **icol[la]** – const Integer *Input*  
 6: **ipiv[n]** – const Integer *Input*  
 7: **istr[n + 1]** – const Integer *Input*  
*On entry:* the values returned in arrays **irow**, **icol**, **ipiv** and **istr** by a previous call to nag\_sparse\_sym\_chol\_fac (f11jac).
- 8: **check** – Nag\_SparseSym\_CheckData *Input*  
*On entry:* specifies whether or not the input data should be checked.  
**check** = Nag\_SparseSym\_Check  
 Checks are carried out on the values of **n**, **irow**, **icol**, **ipiv** and **istr**.  
**check** = Nag\_SparseSym\_NoCheck  
 No checks are carried out.  
 See also Section 9.2.  
*Constraint:* **check** = Nag\_SparseSym\_Check or Nag\_SparseSym\_NoCheck.
- 9: **y[n]** – const double *Input*  
*On entry:* the right-hand side vector  $y$ .
- 10: **x[n]** – double *Output*  
*On exit:* the solution vector  $x$ .
- 11: **fail** – NagError \* *Input/Output*  
 The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

**NE\_INT**

On entry,  $\mathbf{n} = \langle \text{value} \rangle$ .  
 Constraint:  $\mathbf{n} \geq 1$ .

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.  
 See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

**NE\_INVALID\_ROWCOL\_PIVOT**

Check that  $\mathbf{a}$ ,  $\mathbf{irow}$ ,  $\mathbf{icol}$ ,  $\mathbf{ipiv}$  and  $\mathbf{istr}$  have not been corrupted between calls to `nag_sparse_sym_chol_fac (f11jac)` and `nag_sparse_sym_precon_ichol_solve (f11jbc)`.

**NE\_INVALID\_SCS**

Check that  $\mathbf{a}$ ,  $\mathbf{irow}$ ,  $\mathbf{icol}$ ,  $\mathbf{ipiv}$  and  $\mathbf{istr}$  have not been corrupted between calls to `nag_sparse_sym_chol_fac (f11jac)` and `nag_sparse_sym_precon_ichol_solve (f11jbc)`.

**NE\_INVALID\_SCS\_PRECOND**

Check that  $\mathbf{a}$ ,  $\mathbf{irow}$ ,  $\mathbf{icol}$ ,  $\mathbf{ipiv}$  and  $\mathbf{istr}$  have not been corrupted between calls to `nag_sparse_sym_chol_fac (f11jac)` and `nag_sparse_sym_precon_ichol_solve (f11jbc)`.

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.  
 See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

**NE\_NOT\_STRICTLY\_INCREASING**

Check that  $\mathbf{a}$ ,  $\mathbf{irow}$ ,  $\mathbf{icol}$ ,  $\mathbf{ipiv}$  and  $\mathbf{istr}$  have not been corrupted between calls to `nag_sparse_sym_chol_fac (f11jac)` and `nag_sparse_sym_precon_ichol_solve (f11jbc)`.

**7 Accuracy**

The computed solution  $x$  is the exact solution of a perturbed system of equations  $(M + \delta M)x = y$ , where

$$|\delta M| \leq c(n)\epsilon P|L||D||L^T|P^T,$$

$c(n)$  is a modest linear function of  $n$ , and  $\epsilon$  is the *machine precision*.

**8 Parallelism and Performance**

`nag_sparse_sym_precon_ichol_solve (f11jbc)` is not threaded in any implementation.

**9 Further Comments****9.1 Timing**

The time taken for a call to `nag_sparse_sym_precon_ichol_solve (f11jbc)` is proportional to the value of `nnzc` returned from `nag_sparse_sym_chol_fac (f11jac)`.

**9.2 Use of check**

It is expected that a common use of `nag_sparse_sym_precon_ichol_solve (f11jbc)` will be to carry out the preconditioning step required in the application of `nag_sparse_sym_basic_solver (f11gec)` to sparse symmetric linear systems. In this situation `nag_sparse_sym_precon_ichol_solve (f11jbc)` is likely to be

called many times with the same matrix  $M$ . In the interests of both reliability and efficiency, you are recommended to set `check = Nag_SparseSym_Check` for the first of such calls, and to set `check = Nag_SparseSym_NoCheck` for all subsequent calls.

## 10 Example

This example reads in a symmetric positive definite sparse matrix  $A$  and a vector  $y$ . It then calls `nag_sparse_sym_chol_fac` (f11jac), with `lfill = -1` and `dtol = 0.0`, to compute the **complete** Cholesky decomposition of  $A$ :

$$A = PLDL^T P^T.$$

Then it calls `nag_sparse_sym_precon_ichol_solve` (f11jbc) to solve the system

$$PLDL^T P^T x = y.$$

It then repeats the exercise for the same matrix permuted with the bandwidth-reducing Reverse Cuthill–McKee permutation, calculated with `nag_sparse_sym_rcm` (f11yec).

### 10.1 Program Text

```

/* nag_sparse_sym_precon_ichol_solve (f11jbc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <nag_stdlib.h>
#include <nagf11.h>
#include <nagm01.h>

void do_rcm(Integer n, Integer nnz, Integer *irow, Integer *icol,
            double *a, double *y, Integer *istr, Integer *perm_fwd,
            Integer *perm_inv);

int main(void)
{
    /* Scalars */
    Integer exit_status = 0;
    double dscale, dtol;
    Integer i, la, lfill, n, nnz, nnzc, npivm;
    /* Arrays */
    double *a = 0, *x = 0, *y = 0;
    Integer *icol = 0, *ipiv = 0, *irow = 0, *istr = 0,
            *perm_fwd = 0, *perm_inv = 0;
    /* NAG types */
    Nag_SparseSym_Fact mic;
    Nag_SparseSym_Piv pstrat;
    Nag_SparseSym_CheckData check;
    Nag_Sparse_Comm comm;
    NagError fail;

    INIT_FAIL(fail);

    printf("nag_sparse_sym_precon_ichol_solve (f11jbc) Example Program Results");
    printf("\n");
    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
    /* Read order of matrix and number of nonzero entries */
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%*[\n]", &n);

```

```

#else
    scanf("%" NAG_IFMT "%*[\n]", &n);
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%*[\n]", &nnz);
#else
    scanf("%" NAG_IFMT "%*[\n]", &nnz);
#endif

/* Allocate memory */
la = 3 * nnz;
if (!(a = NAG_ALLOC(la, double)) ||
    !(x = NAG_ALLOC(n, double)) ||
    !(y = NAG_ALLOC(n, double)) ||
    !(icol = NAG_ALLOC(la, Integer)) ||
    !(ipiv = NAG_ALLOC(n, Integer)) ||
    !(irow = NAG_ALLOC(la, Integer)) ||
    !(istr = NAG_ALLOC(n + 1, Integer)) ||
    !(perm_fwd = NAG_ALLOC(n, Integer)) ||
    !(perm_inv = NAG_ALLOC(n, Integer))
    )
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Read the matrix A */
for (i = 0; i < nnz; i++)
#ifdef _WIN32
    scanf_s("%lf%" NAG_IFMT "%" NAG_IFMT "%*[\n]", &a[i], &irow[i],
            &icol[i]);
#else
    scanf("%lf%" NAG_IFMT "%" NAG_IFMT "%*[\n]", &a[i], &irow[i], &icol[i]);
#endif
/* Read the vector y */
for (i = 0; i < n; i++)
#ifdef _WIN32
    scanf_s("%lf", &y[i]);
#else
    scanf("%lf", &y[i]);
#endif

lfill = -1;
dtol = 0.0;
dscale = 0.0;
mic = Nag_SparseSym_UnModFact;
pstrat = Nag_SparseSym_MarkPiv;
/* Calculate Cholesky factorization using
 * nag_sparse_sym_chol_fac (f11jac).
 */
nag_sparse_sym_chol_fac(n, nnz, &a, &la, &irow, &icol, lfill, dtol, mic,
                        dscale, pstrat, ipiv, istr, &nnzc, &npivm, &comm,
                        &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_sparse_sym_chol_fac (f11jac).\n%s\n",
           fail.message);
    exit_status = 1;
    goto END;
}
/* Check the output value of npivm */
if (npivm != 0)
    printf("Factorization is not complete \n");
else {
    /* Solve linear system involving incomplete Cholesky factorization
     *
     *           T T
     *       P L D L P x = y
     *
     * using nag_sparse_sym_precon_ichol_solve (f11jbc).
     */
}

```

```

check = Nag_SparseSym_Check;
nag_sparse_sym_precon_ichol_solve(n, a, la, irow, icol, ipiv, istr,
                                check, y, x, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_sparse_sym_precon_ichol_solve (f11jbc).\n%s\n",
          fail.message);
    exit_status = 2;
    goto END;
}
/* Output results */
printf(" Solution of linear system \n");
for (i = 0; i < n; i++)
    printf("%16.4e\n", x[i]);
printf("\n");
}

/* Repeat with Cuthill-McKee permutation
 * Compute reverse Cuthill-McKee permutation for bandwidth reduction
 */
do_rcm(n, nnz, irow, icol, a, y, istr, perm_fwd, perm_inv);

SET_FAIL(fail);
nag_sparse_sym_chol_fac(n, nnz, &a, &la, &irow, &icol, lfill, dtol, mic,
                       dscale, pstrat, ipiv, istr, &nnzc, &npivm, &comm,
                       &fail);

if (npivm != 0)
    printf("Factorization is not complete \n");
else {
    check = Nag_SparseSym_Check;
    nag_sparse_sym_precon_ichol_solve(n, a, la, irow, icol, ipiv, istr,
                                    check, y, x, &fail);
    printf(" Solution of linear system with Reverse Cuthill-McKee\n");
    for (i = 0; i < n; i++)
        printf("%16.4e\n", x[perm_inv[i] - 1]);
    printf("\n");
}
}

END:
NAG_FREE(a);
NAG_FREE(x);
NAG_FREE(y);
NAG_FREE(icol);
NAG_FREE(ipiv);
NAG_FREE(irow);
NAG_FREE(istr);
NAG_FREE(perm_fwd);
NAG_FREE(perm_inv);
return exit_status;
}

void do_rcm(Integer n, Integer nnz, Integer *irow, Integer *icol,
           double *a, double *y, Integer *istr, Integer *perm_fwd,
           Integer *perm_inv)
{
    Integer j, i, nnz_cs, nnz_scs, info[4], mask[1];
    double *yy;
    Nag_Boolean lopts[5] =
        { Nag_FALSE, Nag_FALSE, Nag_TRUE, Nag_TRUE, Nag_TRUE };
    NagError fail;

    SET_FAIL(fail);
    yy = NAG_ALLOC(n, double);
    /* SCS to CS, must add the upper triangle entries. */
    j = nnz;
    for (i = 0; i < nnz; i++) {
        if (irow[i] > icol[i]) {
            /* strictly lower triangle, add the transposed */
            a[j] = a[i];
            irow[j] = icol[i];
            icol[j] = irow[i];
            j++;
        }
    }
}

```

```

    }
  }
  nnz_cs = j;
  /* Reorder, CS to CCS, icolzp in istr */
  nag_sparse_nsym_sort(n, &nnz_cs, a, icol, irow, Nag_SparseNsym_FailDups,
                      Nag_SparseNsym_FailZeros, istr, &fail);

  /* Calculate reverse Cuthill-McKee */
  nag_sparse_sym_rcm(n, nnz_cs, istr, irow, lopts, mask, perm_fwd, info,
                    &fail);

  /* compute inverse perm, in perm_inv */
  for (i = 0; i < n; i++)
    perm_inv[perm_fwd[i] - 1] = i + 1;

  /* Apply permutation on column/row indices */
  for (i = 0; i < nnz_cs; i++) {
    icol[i] = perm_inv[icol[i] - 1];
    irow[i] = perm_inv[irow[i] - 1];
  }
  /* restrict to lower triangle, SCS format
   * copying entries upwards
   */
  j = 0;
  for (i = 0; i < nnz_cs; i++) {
    if (irow[i] >= icol[i]) {
      /* non-upper triangle, bubble up */
      a[j] = a[i];
      icol[j] = icol[i];
      irow[j] = irow[i];
      j++;
    }
  }
  nnz_scs = j;
  /* sort */
  nag_sparse_sym_sort(n, &nnz_scs, a, irow, icol, Nag_SparseSym_SumDups,
                     Nag_SparseSym_KeepZeros, istr, &fail);

  /* permute rhs vector */
  for (i = 0; i < n; i++)
    yy[i] = y[perm_fwd[i] - 1];
  for (i = 0; i < n; i++)
    y[i] = yy[i];
  NAG_FREE(yy);
}

```

## 10.2 Program Data

nag\_sparse\_sym\_precon\_ichol\_solve (f11jbc) Example Program Data

```

9          : n
23         : nnz
4.   1    1
-1.  2    1
6.   2    2
1.   3    2
2.   3    3
3.   4    4
2.   5    1
4.   5    5
1.   6    3
2.   6    4
6.   6    6
-4.  7    2
1.   7    5
-1.  7    6
6.   7    7
-1.  8    4
-1.  8    6
3.   8    8
1.   9    1

```

```
1. 9 5
-1. 9 6
1. 9 8
4. 9 9 : a[i], irow[i], icol[i], i=0,...,nnz-1
4.10 -2.94 1.41
2.53 4.35 1.29
5.01 0.52 4.57 : y[i], i=0,...,n-1
```

### 10.3 Program Results

nag\_sparse\_sym\_precon\_ichol\_solve (f11jbc) Example Program Results

Solution of linear system

```
7.0000e-01
1.6000e-01
5.2000e-01
7.7000e-01
2.8000e-01
2.1000e-01
9.3000e-01
2.0000e-01
9.0000e-01
```

Solution of linear system with Reverse Cuthill-McKee

```
7.0000e-01
1.6000e-01
5.2000e-01
7.7000e-01
2.8000e-01
2.1000e-01
9.3000e-01
2.0000e-01
9.0000e-01
```

---