NAG Library Function Document

nag_dhsein (f08pkc)

1 Purpose

nag_dhsein (f08pkc) computes selected left and/or right eigenvectors of a real upper Hessenberg matrix corresponding to specified eigenvalues, by inverse iteration.

2 Specification

```
#include <nag.h>
#include <nagf08.h>
```

```
void nag_dhsein (Nag_OrderType order, Nag_SideType side,
     Nag_EigValsSourceType eig_source, Nag_InitVeenumtype initv,
     Nag_Boolean select[], Integer n, const double h[], Integer pdh,
     double wr[], const double wi[], double vl[], Integer pdvl, double vr[],
     Integer pdvr, Integer mm, Integer *m, Integer ifaill[],
     Integer ifailr[], NagError *fail)
```

3 Description

nag_dhsein (f08pkc) computes left and/or right eigenvectors of a real upper Hessenberg matrix H, corresponding to selected eigenvalues.

The right eigenvector x, and the left eigenvector y, corresponding to an eigenvalue λ, are defined by:

$$Hx = \lambda x \quad \text{and} \quad y^{\mathrm{H}}H = \lambda y^{\mathrm{H}} \left(\quad \text{or } H^{\mathrm{T}}y = \bar{\lambda} y \right).$$

Note that even though H is real, λ, x and y may be complex. If x is an eigenvector corresponding to a complex eigenvalue λ, then the complex conjugate vector $\bar{x}$ is the eigenvector corresponding to the complex conjugate eigenvalue $\bar{\lambda}$.

The eigenvectors are computed by inverse iteration. They are scaled so that, for a real eigenvector x, $\max(|x_i|) = 1$, and for a complex eigenvector, $\max(|\mathrm{Re}(x_i)| + |\mathrm{Im}\, x_i|) = 1$.

If H has been formed by reduction of a real general matrix A to upper Hessenberg form, then the eigenvectors of H may be transformed to eigenvectors of A by a call to nag_dormhr (f08ngc).

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **side** – Nag_SideType *Input*

On entry: indicates whether left and/or right eigenvectors are to be computed.

side = Nag_RightSide
Only right eigenvectors are computed.

side = Nag_LeftSide
Only left eigenvectors are computed.

side = Nag_BothSides
Both left and right eigenvectors are computed.

Constraint: **side** = Nag_RightSide, Nag_LeftSide or Nag_BothSides.

3: **eig_source** – Nag_EigValsSourceType *Input*

On entry: indicates whether the eigenvalues of H (stored in **wr** and **wi**) were found using nag_dhseqr (f08pec).

eig_source = Nag_HSEQRSource
The eigenvalues of H were found using nag_dhseqr (f08pec); thus if H has any zero subdiagonal elements (and so is block triangular), then the jth eigenvalue can be assumed to be an eigenvalue of the block containing the jth row/column. This property allows the function to perform inverse iteration on just one diagonal block.

eig_source = Nag_NotKnown
No such assumption is made and the function performs inverse iteration using the whole matrix.

Constraint: **eig_source** = Nag_HSEQRSource or Nag_NotKnown.

4: **initv** – Nag_InitVeenumtype *Input*

On entry: indicates whether you are supplying initial estimates for the selected eigenvectors.

initv = Nag_NoVec
No initial estimates are supplied.

initv = Nag_UserVec
Initial estimates are supplied in **vl** and/or **vr**.

Constraint: **initv** = Nag_NoVec or Nag_UserVec.

5: **select**[dim] – Nag_Boolean *Input/Output*

Note: the dimension, dim, of the array **select** must be at least $\max(1, \mathbf{n})$.

On entry: specifies which eigenvectors are to be computed. To obtain the real eigenvector corresponding to the real eigenvalue **wr**[$j-1$], **select**[$j-1$] must be set Nag_TRUE. To select the complex eigenvector corresponding to the complex eigenvalue (**wr**[$j-1$], **wi**[$j-1$]) with complex conjugate (**wr**[j], **wi**[j]), **select**[$j-1$] and/or **select**[j] must be set Nag_TRUE; the eigenvector corresponding to the **first** eigenvalue in the pair is computed.

On exit: if a complex eigenvector was selected as specified above, then **select**[$j-1$] is set to Nag_TRUE and **select**[j] to Nag_FALSE.

6: **n** – Integer *Input*

On entry: n, the order of the matrix H.

Constraint: $\mathbf{n} \geq 0$.

7: **h**[dim] – const double *Input*

Note: the dimension, dim, of the array **h** must be at least $\max(1, \mathbf{pdh} \times \mathbf{n})$.

The (i, j)th element of the matrix H is stored in

$\mathbf{h}[(j-1) \times \mathbf{pdh} + i - 1]$ when **order** = Nag_ColMajor;
$\mathbf{h}[(i-1) \times \mathbf{pdh} + j - 1]$ when **order** = Nag_RowMajor.

On entry: the n by n upper Hessenberg matrix H. If a NaN is detected in **h**, the function will return with **fail.code** = NE_BAD_PARAM.

Constraint: No element of **h** is equal to NaN.

8: **pdh** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **h**.

Constraint: $\mathbf{pdh} \geq \max(1, \mathbf{n})$.

9: **wr**[*dim*] – double *Input/Output*
10: **wi**[*dim*] – const double *Input*

Note: the dimension, *dim*, of the arrays **wr** and **wi** must be at least $\max(1, \mathbf{n})$.

On entry: the real and imaginary parts, respectively, of the eigenvalues of the matrix H. Complex conjugate pairs of values must be stored in consecutive elements of the arrays. If **eig_source** = Nag_HSEQRSource, the arrays **must** be exactly as returned by nag_dhseqr (f08pec).

On exit: some elements of **wr** may be modified, as close eigenvalues are perturbed slightly in searching for independent eigenvectors.

11: **vl**[*dim*] – double *Input/Output*

Note: the dimension, *dim*, of the array **vl** must be at least

$\max(1, \mathbf{pdvl} \times \mathbf{mm})$ when **side** = Nag_LeftSide or Nag_BothSides and **order** = Nag_ColMajor;
$\max(1, \mathbf{n} \times \mathbf{pdvl})$ when **side** = Nag_LeftSide or Nag_BothSides and **order** = Nag_RowMajor;
1 when **side** = Nag_RightSide.

The (i, j)th element of the matrix is stored in

$\mathbf{vl}[(j-1) \times \mathbf{pdvl} + i - 1]$ when **order** = Nag_ColMajor;
$\mathbf{vl}[(i-1) \times \mathbf{pdvl} + j - 1]$ when **order** = Nag_RowMajor.

On entry: if **initv** = Nag_UserVec and **side** = Nag_LeftSide or Nag_BothSides, **vl** must contain starting vectors for inverse iteration for the left eigenvectors. Each starting vector must be stored in the same rows or columns as will be used to store the corresponding eigenvector (see below).

If **initv** = Nag_NoVec, **vl** need not be set.

On exit: if **side** = Nag_LeftSide or Nag_BothSides, **vl** contains the computed left eigenvectors (as specified by **select**). The eigenvectors are stored consecutively in the rows or columns of the array (depending on the value of **order**), in the same order as their eigenvalues. Corresponding to each selected real eigenvalue is a real eigenvector, occupying one row or column. Corresponding to each selected complex eigenvalue is a complex eigenvector, occupying two rows or columns: the first row or column holds the real part and the second row or column holds the imaginary part.

If **side** = Nag_RightSide, **vl** is not referenced.

12: **pdvl** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **vl**.

Constraints:

if **order** $=$ Nag_ColMajor,

if **side** $=$ Nag_LeftSide or Nag_BothSides, **pdvl** $\geq$ **n**;
if **side** $=$ Nag_RightSide, **pdvl** ≥ 1.;

if **order** $=$ Nag_RowMajor,

if **side** $=$ Nag_LeftSide or Nag_BothSides, **pdvl** $\geq \max(1, \mathbf{mm})$;
if **side** $=$ Nag_RightSide, **pdvl** ≥ 1..

13: **vr**$[dim]$ – double *Input/Output*

Note: the dimension, *dim*, of the array **vr** must be at least

$\max(1, \mathbf{pdvr} \times \mathbf{mm})$ when **side** $=$ Nag_RightSide or Nag_BothSides and **order** $=$ Nag_ColMajor;
$\max(1, \mathbf{n} \times \mathbf{pdvr})$ when **side** $=$ Nag_RightSide or Nag_BothSides and **order** $=$ Nag_RowMajor;
1 when **side** $=$ Nag_LeftSide.

The (i, j)th element of the matrix is stored in

vr$[(j-1) \times \mathbf{pdvr} + i - 1]$ when **order** $=$ Nag_ColMajor;
vr$[(i-1) \times \mathbf{pdvr} + j - 1]$ when **order** $=$ Nag_RowMajor.

On entry: if **initv** $=$ Nag_UserVec and **side** $=$ Nag_RightSide or Nag_BothSides, **vr** must contain starting vectors for inverse iteration for the right eigenvectors. Each starting vector must be stored in the same rows or columns as will be used to store the corresponding eigenvector (see below).

If **initv** $=$ Nag_NoVec, **vr** need not be set.

On exit: if **side** $=$ Nag_RightSide or Nag_BothSides, **vr** contains the computed right eigenvectors (as specified by **select**). The eigenvectors are stored consecutively in the rows or columns of the array (depending on the **order** argument), in the same order as their eigenvalues. Corresponding to each selected real eigenvalue is a real eigenvector, occupying one row or column. Corresponding to each selected complex eigenvalue is a complex eigenvector, occupying two rows or columns: the first row or column holds the real part and the second row or column holds the imaginary part.

If **side** $=$ Nag_LeftSide, **vr** is not referenced.

14: **pdvr** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **vr**.

Constraints:

if **order** $=$ Nag_ColMajor,

if **side** $=$ Nag_RightSide or Nag_BothSides, **pdvr** $\geq$ **n**;
if **side** $=$ Nag_LeftSide, **pdvr** ≥ 1.;

if **order** $=$ Nag_RowMajor,

if **side** $=$ Nag_RightSide or Nag_BothSides, **pdvr** $\geq \max(1, \mathbf{mm})$;
if **side** $=$ Nag_LeftSide, **pdvr** ≥ 1..

15: **mm** – Integer *Input*

On entry: the number of columns in the arrays **vl** and/or **vr** if **order** $=$ Nag_ColMajor or the number of rows in the arrays if **order** $=$ Nag_RowMajor. The actual number of rows or columns required, $required_rowcol$, is obtained by counting 1 for each selected real eigenvector and 2 for each selected complex eigenvector (see **select**); $0 \leq required_rowcol \leq n$.

Constraint: **mm** $\geq required_rowcol$.

16: **m** – Integer * *Output*

On exit: $required_rowcol$, the number of rows or columns of **vl** and/or **vr** required to store the selected eigenvectors.

17: **ifaill**[dim] – Integer *Output*

Note: the dimension, *dim*, of the array **ifaill** must be at least

max(1, **mm**) when **side** = Nag_LeftSide or Nag_BothSides;
1 when **side** = Nag_RightSide.

On exit: if **side** = Nag_LeftSide or Nag_BothSides, then **ifaill**[$i-1$] $= 0$ if the selected left eigenvector converged and **ifaill**[$i-1$] $= j \geq 0$ if the eigenvector stored in the ith row or column of **vl** (corresponding to the jth eigenvalue as held in (**wr**[$j-1$], **wi**[$j-1$]) failed to converge. If the ith and $(i+1)$th rows or columns of **vl** contain a selected complex eigenvector, then **ifaill**[$i-1$] and **ifaill**[i] are set to the same value.

If **side** = Nag_RightSide, **ifaill** is not referenced.

18: **ifailr**[dim] – Integer *Output*

Note: the dimension, *dim*, of the array **ifailr** must be at least

max(1, **mm**) when **side** = Nag_RightSide or Nag_BothSides;
1 when **side** = Nag_LeftSide.

On exit: if **side** = Nag_RightSide or Nag_BothSides, then **ifailr**[$i-1$] $= 0$ if the selected right eigenvector converged and **ifailr**[$i-1$] $= j \geq 0$ if the eigenvector stored in the ith row or column of **vr** (corresponding to the jth eigenvalue as held in (**wr**[$j-1$], **wi**[$j-1$])) failed to converge. If the ith and $(i+1)$th rows or columns of **vr** contain a selected complex eigenvector, then **ifailr**[$i-1$] and **ifailr**[i] are set to the same value.

If **side** = Nag_LeftSide, **ifailr** is not referenced.

19: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

Constraint: No element of **h** is equal to NaN.

On entry, argument ⟨*value*⟩ had an illegal value.

NE_CONVERGENCE

⟨*value*⟩ eigenvectors (as indicated by arguments **ifaill** and/or **ifailr**) failed to converge. The corresponding columns of **vl** and/or **vr** contain no useful information.

NE_ENUM_INT_2

On entry, **side** = ⟨*value*⟩, **pdvl** = ⟨*value*⟩, **mm** = ⟨*value*⟩.
Constraint: if **side** = Nag_LeftSide or Nag_BothSides, **pdvl** ≥ max(1, **mm**);
if **side** = Nag_RightSide, **pdvl** ≥ 1.

On entry, **side** = $\langle value \rangle$, **pdvl** = $\langle value \rangle$ and **n** = $\langle value \rangle$.
Constraint: if **side** = Nag_LeftSide or Nag_BothSides, **pdvl** $\geq$ **n**;
if **side** = Nag_RightSide, **pdvl** ≥ 1.

On entry, **side** = $\langle value \rangle$, **pdvr** = $\langle value \rangle$, **mm** = $\langle value \rangle$.
Constraint: if **side** = Nag_RightSide or Nag_BothSides, **pdvr** $\geq \max(1, \mathbf{mm})$;
if **side** = Nag_LeftSide, **pdvr** ≥ 1.

On entry, **side** = $\langle value \rangle$, **pdvr** = $\langle value \rangle$ and **n** = $\langle value \rangle$.
Constraint: if **side** = Nag_RightSide or Nag_BothSides, **pdvr** $\geq$ **n**;
if **side** = Nag_LeftSide, **pdvr** ≥ 1.

NE_INT

On entry, **mm** = $\langle value \rangle$.
Constraint: **mm** $\geq required_rowcol$, where $required_rowcol$ is obtained by counting 1 for each selected real eigenvector and 2 for each selected complex eigenvector.

On entry, **n** = $\langle value \rangle$.
Constraint: **n** ≥ 0.

On entry, **pdh** = $\langle value \rangle$.
Constraint: **pdh** > 0.

On entry, **pdvl** = $\langle value \rangle$.
Constraint: **pdvl** > 0.

On entry, **pdvr** = $\langle value \rangle$.
Constraint: **pdvr** > 0.

NE_INT_2

On entry, **pdh** = $\langle value \rangle$ and **n** = $\langle value \rangle$.
Constraint: **pdh** $\geq \max(1, \mathbf{n})$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

Each computed right eigenvector x_i is the exact eigenvector of a nearby matrix $A + E_i$, such that $\|E_i\| = O(\epsilon)\|A\|$. Hence the residual is small:

$$\|Ax_i - \lambda_i x_i\| = O(\epsilon)\|A\|.$$

However, eigenvectors corresponding to close or coincident eigenvalues may not accurately span the relevant subspaces.

Similar remarks apply to computed left eigenvectors.

8 Parallelism and Performance

nag_dhsein (f08pkc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag_dhsein (f08pkc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The complex analogue of this function is nag_zhsein (f08pxc).

10 Example

See Section 10 in nag_dormhr (f08ngc).