

# NAG Library Function Document

## nag\_zungrq (f08cwc)

### 1 Purpose

nag\_zungrq (f08cwc) generates all or part of the complex  $n$  by  $n$  unitary matrix  $Q$  from an  $RQ$  factorization computed by nag\_zgerqf (f08cvc).

### 2 Specification

```
#include <nag.h>
#include <nagf08.h>

void nag_zungrq (Nag_OrderType order, Integer m, Integer n, Integer k,
                 Complex a[], Integer pda, const Complex tau[], NagError *fail)
```

### 3 Description

nag\_zungrq (f08cwc) is intended to be used following a call to nag\_zgerqf (f08cvc), which performs an  $RQ$  factorization of a complex matrix  $A$  and represents the unitary matrix  $Q$  as a product of  $k$  elementary reflectors of order  $n$ .

This function may be used to generate  $Q$  explicitly as a square matrix, or to form only its trailing rows.

Usually  $Q$  is determined from the  $RQ$  factorization of a  $p$  by  $n$  matrix  $A$  with  $p \leq n$ . The whole of  $Q$  may be computed by:

```
nag_zungrq(order, n, n, p, a, pda, tau, info)
```

(note that the matrix  $A$  must have at least  $n$  rows), or its trailing  $p$  rows as:

```
nag_zungrq(order, p, n, p, a, pda, tau, info)
```

The rows of  $Q$  returned by the last call form an orthonormal basis for the space spanned by the rows of  $A$ ; thus nag\_zgerqf (f08cvc) followed by nag\_zungrq (f08cwc) can be used to orthogonalize the rows of  $A$ .

The information returned by nag\_zgerqf (f08cvc) also yields the  $RQ$  factorization of the trailing  $k$  rows of  $A$ , where  $k < p$ . The unitary matrix arising from this factorization can be computed by:

```
nag_zungrq(order, n, n, k, a, pda, tau, info)
```

or its leading  $k$  columns by:

```
nag_zungrq(order, k, n, k, a, pda, tau, info)
```

### 4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

### 5 Arguments

1: **order** – Nag\_OrderType *Input*

*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by

**order** = Nag\_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.

- 2: **m** – Integer *Input*  
*On entry:*  $m$ , the number of rows of the matrix  $Q$ .  
*Constraint:*  $m \geq 0$ .
- 3: **n** – Integer *Input*  
*On entry:*  $n$ , the number of columns of the matrix  $Q$ .  
*Constraint:*  $n \geq m$ .
- 4: **k** – Integer *Input*  
*On entry:*  $k$ , the number of elementary reflectors whose product defines the matrix  $Q$ .  
*Constraint:*  $m \geq k \geq 0$ .
- 5: **a**[*dim*] – Complex *Input/Output*  
**Note:** the dimension, *dim*, of the array **a** must be at least  
 $\max(1, \mathbf{pda} \times \mathbf{n})$  when **order** = Nag\_ColMajor;  
 $\max(1, \mathbf{m} \times \mathbf{pda})$  when **order** = Nag\_RowMajor.  
*On entry:* details of the vectors which define the elementary reflectors, as returned by nag\_zgerqf (f08cvc).  
*On exit:* the  $m$  by  $n$  matrix  $Q$ .  
If **order** = Nag\_ColMajor, the  $(i, j)$ th element of the matrix is stored in  $\mathbf{a}[(j-1) \times \mathbf{pda} + i - 1]$ .  
If **order** = Nag\_RowMajor, the  $(i, j)$ th element of the matrix is stored in  $\mathbf{a}[(i-1) \times \mathbf{pda} + j - 1]$ .
- 6: **pda** – Integer *Input*  
*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **a**.  
*Constraints:*  
if **order** = Nag\_ColMajor,  $\mathbf{pda} \geq \max(1, \mathbf{m})$ ;  
if **order** = Nag\_RowMajor,  $\mathbf{pda} \geq \max(1, \mathbf{n})$ .
- 7: **tau**[*dim*] – const Complex *Input*  
**Note:** the dimension, *dim*, of the array **tau** must be at least  $\max(1, \mathbf{k})$ .  
*On entry:*  $\mathbf{tau}[i-1]$  must contain the scalar factor of the elementary reflector  $H_i$ , as returned by nag\_zgerqf (f08cvc).
- 8: **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INT

On entry,  $\mathbf{m} = \langle value \rangle$ .

Constraint:  $\mathbf{m} \geq 0$ .

On entry,  $\mathbf{pda} = \langle value \rangle$ .

Constraint:  $\mathbf{pda} > 0$ .

### NE\_INT\_2

On entry,  $\mathbf{m} = \langle value \rangle$  and  $\mathbf{k} = \langle value \rangle$ .

Constraint:  $\mathbf{m} \geq \mathbf{k} \geq 0$ .

On entry,  $\mathbf{n} = \langle value \rangle$  and  $\mathbf{m} = \langle value \rangle$ .

Constraint:  $\mathbf{n} \geq \mathbf{m}$ .

On entry,  $\mathbf{pda} = \langle value \rangle$  and  $\mathbf{m} = \langle value \rangle$ .

Constraint:  $\mathbf{pda} \geq \max(1, \mathbf{m})$ .

On entry,  $\mathbf{pda} = \langle value \rangle$  and  $\mathbf{n} = \langle value \rangle$ .

Constraint:  $\mathbf{pda} \geq \max(1, \mathbf{n})$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

### NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

## 7 Accuracy

The computed matrix  $Q$  differs from an exactly unitary matrix by a matrix  $E$  such that

$$\|E\|_2 = O\epsilon$$

and  $\epsilon$  is the *machine precision*.

## 8 Parallelism and Performance

nag\_zungrq (f08cwc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

The total number of floating-point operations is approximately  $16mnk - 8(m+n)k^2 + \frac{16}{3}k^3$ ; when  $m = k$  this becomes  $\frac{8}{3}m^2(3n - m)$ .

The real analogue of this function is nag\_dorghr (f08cjc).

## 10 Example

This example generates the first four rows of the matrix  $Q$  of the  $RQ$  factorization of  $A$  as returned by nag\_zgerqf (f08cvc), where

$$A = \begin{pmatrix} 0.96 - 0.81i & -0.98 + 1.98i & 0.62 - 0.46i & -0.37 + 0.38i & 0.83 + 0.51i & 1.08 - 0.28i \\ -0.03 + 0.96i & -1.20 + 0.19i & 1.01 + 0.02i & 0.19 - 0.54i & 0.20 + 0.01i & 0.20 - 0.12i \\ -0.91 + 2.06i & -0.66 + 0.42i & 0.63 - 0.17i & -0.98 - 0.36i & -0.17 - 0.46i & -0.07 + 1.23i \\ -0.05 + 0.41i & -0.81 + 0.56i & -1.11 + 0.60i & 0.22 - 0.20i & 1.47 + 1.59i & 0.26 + 0.26i \end{pmatrix}.$$

### 10.1 Program Text

```

/* nag_zungrq (f08cwc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf08.h>
#include <nagx04.h>

int main(void)
{
    /* Scalars */
    Integer i, j, m, n, pda;
    Integer exit_status = 0;
    /* Arrays */
    char *title = 0;
    Complex *a = 0, *tau = 0;
    /* Nag Types */
    Nag_OrderType order;
    NagError fail;

#ifdef NAG_COLUMN_MAJOR
#define A(I, J) a[(J - 1) * pda + I - 1]
    order = Nag_ColMajor;
#else
#define A(I, J) a[(I - 1) * pda + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_zungrq (f08cwc) Example Program Results\n\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%*[\n]", &m, &n);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%*[\n]", &m, &n);

```

```

#endif

#ifdef NAG_COLUMN_MAJOR
    pda = m;
#else
    pda = n;
#endif

/* Allocate memory */
if (!(title = NAG_ALLOC(27, char)) ||
    !(a = NAG_ALLOC(m * n, Complex)) || !(tau = NAG_ALLOC(n, Complex)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Read A from data file */
for (i = 1; i <= m; ++i)
    for (j = 1; j <= n; ++j)
#ifdef _WIN32
    scanf_s(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
#else
    scanf(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
#endif
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif

/* nag_zgerqf (f08cvc).
 * Compute the RQ factorization of A.
 */
nag_zgerqf(order, m, n, a, pda, tau, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_zgerqf (f08cvc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* nag_zungrq (f08cwc).
 * Form the leading m rows of Q explicitly.
 */
nag_zungrq(order, m, n, m, a, pda, tau, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_zungrq (f08cwc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

#ifdef _WIN32
    sprintf_s(title, 27, "The leading %4" NAG_IFMT " rows of Q", m);
#else
    sprintf(title, "The leading %4" NAG_IFMT " rows of Q", m);
#endif

/* nag_gen_complx_mat_print_comp (x04dbc).
 * Print the leading m rows of Q.
 */
fflush(stdout);
nag_gen_complx_mat_print_comp(order, Nag_GeneralMatrix, Nag_NonUnitDiag, m,
                              n, a, pda, Nag_BracketForm, "%7.4f", title,
                              Nag_IntegerLabels, 0, Nag_IntegerLabels, 0,
                              80, 0, 0, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_complx_mat_print_comp (x04dbc).\n%s\n",
          fail.message);
    exit_status = 1;
}

```

```

END:
  NAG_FREE(title);
  NAG_FREE(a);
  NAG_FREE(tau);

  return exit_status;
}

#undef A

```

## 10.2 Program Data

nag\_zungrq (f08cwc) Example Program Data

```

      4              6                      :Values of M and N

( 0.96,-0.81) (-0.98, 1.98) ( 0.62,-0.46) (-0.37, 0.38) ( 0.83, 0.51)
( 1.08,-0.28)
(-0.03, 0.96) (-1.20, 0.19) ( 1.01, 0.02) ( 0.19,-0.54) ( 0.20, 0.01)
( 0.20,-0.12)
(-0.91, 2.06) (-0.66, 0.42) ( 0.63,-0.17) (-0.98,-0.36) (-0.17,-0.46)
(-0.07, 1.23)
(-0.05, 0.41) (-0.81, 0.56) (-1.11, 0.60) ( 0.22,-0.20) ( 1.47, 1.59)
( 0.26, 0.26)                               :End of matrix A

```

## 10.3 Program Results

nag\_zungrq (f08cwc) Example Program Results

```

The leading      4 rows of Q
      1              2              3              4
1 ( 0.2810, 0.5020) ( 0.2707,-0.3296) (-0.2864,-0.0094) ( 0.2262,-0.3854)
2 (-0.2051,-0.1092) ( 0.5711, 0.0432) (-0.5416, 0.0454) (-0.3387, 0.2228)
3 ( 0.3083,-0.6874) ( 0.2251,-0.1313) (-0.2062, 0.0691) ( 0.3259, 0.1178)
4 ( 0.0181,-0.1483) ( 0.2930,-0.2025) ( 0.4015,-0.2170) (-0.0796, 0.0723)

      5              6
1 ( 0.0341,-0.0760) (-0.3936,-0.2083)
2 ( 0.0098,-0.0712) (-0.1296, 0.3691)
3 ( 0.0753, 0.1412) ( 0.0264,-0.4134)
4 (-0.5317,-0.5751) (-0.0940,-0.0940)

```

---