

NAG Library Function Document

nag_dtbcon (f07vgc)

1 Purpose

nag_dtbcon (f07vgc) estimates the condition number of a real triangular band matrix.

2 Specification

```
#include <nag.h>
#include <nagf07.h>

void nag_dtbcon (Nag_OrderType order, Nag_NormType norm, Nag_UploType uplo,
                Nag_DiagType diag, Integer n, Integer kd, const double ab[],
                Integer pdab, double *rcond, NagError *fail)
```

3 Description

nag_dtbcon (f07vgc) estimates the condition number of a real triangular band matrix A , in either the 1-norm or the ∞ -norm:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \quad \text{or} \quad \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1(A^T)$.

Because the condition number is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of the condition number.

The function computes $\|A\|_1$ or $\|A\|_\infty$ exactly, and uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\|A^{-1}\|_1$ or $\|A^{-1}\|_\infty$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **norm** – Nag_NormType *Input*

On entry: indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is estimated.

norm = Nag_OneNorm
 $\kappa_1(A)$ is estimated.

norm = Nag_InfNorm
 $\kappa_\infty(A)$ is estimated.

Constraint: **norm** = Nag_OneNorm or Nag_InfNorm.

- 3: **uplo** – Nag_UploType *Input*
On entry: specifies whether A is upper or lower triangular.
uplo = Nag_Upper
 A is upper triangular.
uplo = Nag_Lower
 A is lower triangular.
Constraint: **uplo** = Nag_Upper or Nag_Lower.
- 4: **diag** – Nag_DiagType *Input*
On entry: indicates whether A is a nonunit or unit triangular matrix.
diag = Nag_NonUnitDiag
 A is a nonunit triangular matrix.
diag = Nag_UnitDiag
 A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.
Constraint: **diag** = Nag_NonUnitDiag or Nag_UnitDiag.
- 5: **n** – Integer *Input*
On entry: n , the order of the matrix A .
Constraint: $n \geq 0$.
- 6: **kd** – Integer *Input*
On entry: k_d , the number of superdiagonals of the matrix A if **uplo** = Nag_Upper, or the number of subdiagonals if **uplo** = Nag_Lower.
Constraint: $kd \geq 0$.
- 7: **ab**[*dim*] – const double *Input*
Note: the dimension, *dim*, of the array **ab** must be at least $\max(1, \mathbf{pdab} \times \mathbf{n})$.
On entry: the n by n triangular band matrix A .
This is stored as a notional two-dimensional array with row elements or column elements stored contiguously. The storage of elements of A_{ij} , depends on the **order** and **uplo** arguments as follows:
- if **order** = Nag_ColMajor and **uplo** = Nag_Upper,
 A_{ij} is stored in **ab**[$k_d + i - j + (j - 1) \times \mathbf{pdab}$], for $j = 1, \dots, n$ and
 $i = \max(1, j - k_d), \dots, j$;
 - if **order** = Nag_ColMajor and **uplo** = Nag_Lower,
 A_{ij} is stored in **ab**[$i - j + (j - 1) \times \mathbf{pdab}$], for $j = 1, \dots, n$ and
 $i = j, \dots, \min(n, j + k_d)$;
 - if **order** = Nag_RowMajor and **uplo** = Nag_Upper,
 A_{ij} is stored in **ab**[$j - i + (i - 1) \times \mathbf{pdab}$], for $i = 1, \dots, n$ and
 $j = i, \dots, \min(n, i + k_d)$;
 - if **order** = Nag_RowMajor and **uplo** = Nag_Lower,
 A_{ij} is stored in **ab**[$k_d + j - i + (i - 1) \times \mathbf{pdab}$], for $i = 1, \dots, n$ and
 $j = \max(1, i - k_d), \dots, i$.
- If **diag** = Nag_UnitDiag, the diagonal elements of AB are assumed to be 1, and are not referenced.

- 8: **pdab** – Integer *Input*
On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix *A* in the array **ab**.
Constraint: **pdab** \geq **kd** + 1.
- 9: **rcond** – double * *Output*
On exit: an estimate of the reciprocal of the condition number of *A*. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*, *A* is singular to working precision.
- 10: **fail** – NagError * *Input/Output*
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument *<value>* had an illegal value.

NE_INT

On entry, **kd** = *<value>*.

Constraint: **kd** \geq 0.

On entry, **n** = *<value>*.

Constraint: **n** \geq 0.

On entry, **pdab** = *<value>*.

Constraint: **pdab** $>$ 0.

NE_INT_2

On entry, **pdab** = *<value>* and **kd** = *<value>*.

Constraint: **pdab** \geq **kd** + 1.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where **rcond** is much larger.

8 Parallelism and Performance

nag_dtbcon (f07vgc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

A call to nag_dtbcon (f07vgc) involves solving a number of systems of linear equations of the form $Ax = b$ or $A^T x = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $2nk$ floating-point operations (assuming $n \gg k$) but takes considerably longer than a call to nag_dtbtrs (f07vec) with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The complex analogue of this function is nag_ztbcon (f07vuc).

10 Example

This example estimates the condition number in the 1-norm of the matrix A , where

$$A = \begin{pmatrix} -4.16 & 0.00 & 0.00 & 0.00 \\ -2.25 & 4.78 & 0.00 & 0.00 \\ 0.00 & 5.86 & 6.32 & 0.00 \\ 0.00 & 0.00 & -4.82 & 0.16 \end{pmatrix}.$$

Here A is treated as a lower triangular band matrix with one subdiagonal. The true condition number in the 1-norm is 69.62.

10.1 Program Text

```

/* nag_dtbcon (f07vgc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx02.h>

int main(void)
{
    /* Scalars */
    Integer i, j, k, kd, n, pdab;
    Integer exit_status = 0;
    double rcond;
    NagError fail;
    Nag_UploType uplo;
    Nag_OrderType order;
    /* Arrays */
    char nag_enum_arg[40];
    double *ab = 0;

#ifdef NAG_COLUMN_MAJOR
#define AB_UPPER(I, J) ab[(J-1)*pdab + k + I - J - 1]
#define AB_LOWER(I, J) ab[(J-1)*pdab + I - J]
    order = Nag_ColMajor;

```

```

#else
#define AB_UPPER(I, J) ab[(I-1)*pdab + J - I]
#define AB_LOWER(I, J) ab[(I-1)*pdab + k + J - I - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_dtbcon (f07vgc) Example Program Results\n\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &n, &kd);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &n, &kd);
#endif
    pdab = kd + 1;

    /* Allocate memory */
    if (!(ab = NAG_ALLOC((kd + 1) * n, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    /* Read A from data file */
#ifdef _WIN32
    scanf_s(" %39s%*[\n] ", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf(" %39s%*[\n] ", nag_enum_arg);
#endif
    /* nag_enum_name_to_value (x04nac).
     * Converts NAG enum member name to value
     */
    uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);

    k = kd + 1;
    if (uplo == Nag_Upper) {
        for (i = 1; i <= n; ++i) {
            for (j = i; j <= MIN(i + kd, n); ++j)
#ifdef _WIN32
                scanf_s("%lf", &AB_UPPER(i, j));
#else
                scanf("%lf", &AB_UPPER(i, j));
#endif
        }
#ifdef _WIN32
        scanf_s("%*[\n] ");
#else
        scanf("%*[\n] ");
#endif
    }
    else {
        for (i = 1; i <= n; ++i) {
            for (j = MAX(1, i - kd); j <= i; ++j)
#ifdef _WIN32
                scanf_s("%lf", &AB_LOWER(i, j));
#else
                scanf("%lf", &AB_LOWER(i, j));
#endif
        }
#ifdef _WIN32
        scanf_s("%*[\n] ");
#else
        scanf("%*[\n] ");
#endif

```

```

#endif
}
/* Estimate condition number */
/* nag_dtbcon (f07vgc).
 * Estimate condition number of real band triangular matrix
 */
nag_dtbcon(order, Nag_OneNorm, uplo, Nag_NonUnitDiag, n,
           kd, ab, pdab, &rcond, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_dtbcon (f07vgc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* nag_machine_precision (x02ajc).
 * The machine precision
 */
if (rcond >= nag_machine_precision) {
    printf("Estimate of condition number =%11.2e\n\n", 1.0 / rcond);
}
else
    printf("A is singular to working precision\n");
END:
    NAG_FREE(ab);
    return exit_status;
}

```

10.2 Program Data

```

nag_dtbcon (f07vgc) Example Program Data
 4 1 :Values of n and kd
 Nag_Lower :Value of uplo
-4.16
-2.25 4.78
      5.86 6.32
      -4.82 0.16 :End of matrix A

```

10.3 Program Results

```

nag_dtbcon (f07vgc) Example Program Results
Estimate of condition number = 6.96e+01

```
