

# NAG Library Function Document

## nag\_nd\_shep\_eval (e01znc)

### 1 Purpose

nag\_nd\_shep\_eval (e01znc) evaluates the multidimensional interpolating function generated by nag\_nd\_shep\_interp (e01zmc) and its first partial derivatives.

### 2 Specification

```
#include <nag.h>
#include <nage01.h>

void nag_nd_shep_eval (Integer d, Integer m, const double x[],
    const double f[], const Integer iq[], const double rq[], Integer n,
    const double xe[], double q[], double qx[], NagError *fail)
```

### 3 Description

nag\_nd\_shep\_eval (e01znc) takes as input the interpolant  $Q(\mathbf{x})$ ,  $\mathbf{x} \in \mathbb{R}^d$  of a set of scattered data points  $(\mathbf{x}_r, f_r)$ , for  $r = 1, 2, \dots, m$ , as computed by nag\_nd\_shep\_interp (e01zmc), and evaluates the interpolant and its first partial derivatives at the set of points  $\mathbf{x}_i$ , for  $i = 1, 2, \dots, n$ .

nag\_nd\_shep\_eval (e01znc) must only be called after a call to nag\_nd\_shep\_interp (e01zmc).

nag\_nd\_shep\_eval (e01znc) is derived from the new implementation of QS3GRD described by Renka (1988). It uses the modification for high-dimensional interpolation described by Berry and Minser (1999).

### 4 References

Berry M W, Minser K S (1999) Algorithm 798: high-dimensional interpolation using the modified Shepard method *ACM Trans. Math. Software* **25** 353–366

Renka R J (1988) Algorithm 661: QSHEP3D: Quadratic Shepard method for trivariate interpolation of scattered data *ACM Trans. Math. Software* **14** 151–152

### 5 Arguments

- 1: **d** – Integer *Input*  
*On entry:* **must** be the same value supplied for argument **d** in the preceding call to nag\_nd\_shep\_interp (e01zmc).  
*Constraint:*  $\mathbf{d} \geq 2$ .
- 2: **m** – Integer *Input*  
*On entry:* **must** be the same value supplied for argument **m** in the preceding call to nag\_nd\_shep\_interp (e01zmc).  
*Constraint:*  $\mathbf{m} \geq (\mathbf{d} + 1) \times (\mathbf{d} + 2)/2 + 2$ .
- 3: **x**[**d** × **m**] – const double *Input*  
**Note:** the  $i$ th ordinate of the point  $x_j$  is stored in **x**[( $j - 1$ ) × **d** +  $i - 1$ ].  
*On entry:* **must** be the same array supplied as argument **x** in the preceding call to nag\_nd\_shep\_interp (e01zmc). It **must** remain unchanged between calls.

- 4: **f[m]** – const double *Input*  
*On entry:* **must** be the same array supplied as argument **f** in the preceding call to `nag_nd_shep_interp` (e01zmc). It **must** remain unchanged between calls.
- 5: **iq[2 × m + 1]** – const Integer *Input*  
*On entry:* **must** be the same array returned as argument **iq** in the preceding call to `nag_nd_shep_interp` (e01zmc). It **must** remain unchanged between calls.
- 6: **rq[dim]** – const double *Input*  
**Note:** the dimension, *dim*, of the array **rq** must be at least  $((\mathbf{d} + 1) \times (\mathbf{d} + 2)/2) \times \mathbf{m} + 2 \times \mathbf{d} + 1$ .  
*On entry:* **must** be the same array returned as argument **rq** in the preceding call to `nag_nd_shep_interp` (e01zmc). It **must** remain unchanged between calls.
- 7: **n** – Integer *Input*  
*On entry:* *n*, the number of evaluation points.  
*Constraint:*  $\mathbf{n} \geq 1$ .
- 8: **xe[d × n]** – const double *Input*  
**Note:** the *i*th ordinate of the point  $x_j$  is stored in  $\mathbf{xe}[(j - 1) \times \mathbf{d} + i - 1]$ .  
*On entry:*  $\mathbf{xe}[(j - 1) \times \mathbf{d}], \dots, \mathbf{xe}[(j - 1) \times \mathbf{d} + \mathbf{d} - 1]$  must be set to the evaluation point  $\mathbf{x}_j$ , for  $j = 1, 2, \dots, n$ .
- 9: **q[n]** – double *Output*  
*On exit:*  $\mathbf{q}[i - 1]$  contains the value of the interpolant, at  $\mathbf{x}_i$ , for  $i = 1, 2, \dots, n$ . If any of these evaluation points lie outside the region of definition of the interpolant the corresponding entries in **q** are set to an extrapolated approximation, and `nag_nd_shep_eval` (e01znc) returns with **fail.code** = NE\_BAD\_POINT.
- 10: **qx[d × n]** – double *Output*  
**Note:** the (*i, j*)th element of the matrix is stored in  $\mathbf{qx}[(j - 1) \times \mathbf{d} + i - 1]$ .  
*On exit:*  $\mathbf{qx}[(j - 1) \times \mathbf{d} + i - 1]$  contains the value of the partial derivatives with respect to the *i*th independent variable (dimension) of the interpolant  $Q(\mathbf{x})$  at  $\mathbf{x}_j$ , for  $j = 1, 2, \dots, n$ , and for each of the partial derivatives  $i = 1, 2, \dots, d$ . If any of these evaluation points lie outside the region of definition of the interpolant, the corresponding entries in **qx** are set to extrapolated approximations to the partial derivatives, and `nag_nd_shep_eval` (e01znc) returns with **fail.code** = NE\_BAD\_POINT.
- 11: **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

**NE\_BAD\_PARAM**

On entry, argument  $\langle value \rangle$  had an illegal value.

**NE\_BAD\_POINT**

On entry, at least one evaluation point lies outside the region of definition of the interpolant. At such points the corresponding values in  $\mathbf{q}$  and  $\mathbf{qx}$  contain extrapolated approximations. Points should be evaluated one by one to identify extrapolated values.

**NE\_INT**

On entry,  $\mathbf{d} = \langle value \rangle$ .

Constraint:  $\mathbf{d} \geq 2$ .

On entry,  $\mathbf{n} = \langle value \rangle$ .

Constraint:  $\mathbf{n} \geq 1$ .

**NE\_INT\_2**

On entry,  $((\mathbf{d} + 1) \times (\mathbf{d} + 2)/2) \times \mathbf{m} + 2 \times \mathbf{d} + 1$  exceeds the largest machine integer.

$\mathbf{d} = \langle value \rangle$  and  $\mathbf{m} = \langle value \rangle$ .

On entry,  $\mathbf{m} = \langle value \rangle$  and  $\mathbf{d} = \langle value \rangle$ .

Constraint:  $\mathbf{m} \geq (\mathbf{d} + 1) \times (\mathbf{d} + 2)/2 + 2$ .

**NE\_INT\_ARRAY**

On entry, values in  $\mathbf{iq}$  appear to be invalid. Check that  $\mathbf{iq}$  has not been corrupted between calls to `nag_nd_shep_interp` (e01zmc) and `nag_nd_shep_eval` (e01znc).

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

**NE\_REAL\_ARRAY**

On entry, values in  $\mathbf{rq}$  appear to be invalid. Check that  $\mathbf{rq}$  has not been corrupted between calls to `nag_nd_shep_interp` (e01zmc) and `nag_nd_shep_eval` (e01znc).

**7 Accuracy**

Computational errors should be negligible in most practical situations.

**8 Parallelism and Performance**

`nag_nd_shep_eval` (e01znc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

`nag_nd_shep_eval` (e01znc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

The time taken for a call to `nag_nd_shep_eval` (e01znc) will depend in general on the distribution of the data points. If the data points are approximately uniformly distributed, then the time taken should be only  $O(n)$ . At worst  $O(mn)$  time will be required.

## 10 Example

This program evaluates the function (in six variables)

$$f(x) = \frac{x_1 x_2 x_3}{1 + 2x_4 x_5 x_6}$$

at a set of randomly generated data points and calls `nag_nd_shep_interp` (e01zmc) to construct an interpolating function  $Q_x$ . It then calls `nag_nd_shep_eval` (e01znc) to evaluate the interpolant at a set of points on the line  $x_i = x$ , for  $i = 1, 2, \dots, 6$ . To reduce the time taken by this example, the number of data points is limited. Increasing this value to the suggested minimum of 4000 improves the interpolation accuracy at the expense of more time.

See also Section 10 in `nag_nd_shep_interp` (e01zmc).

### 10.1 Program Text

```

/* nag_nd_shep_eval (e01znc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */
#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nage01.h>
#include <nagg05.h>

static double funct(double *x);

int main(void)
{
    /* Scalars */
    Integer d = 6, exit_status = 0, lseed = 1, subid = 0;
    Integer i, j, liq, lrq, lstate, m, n, nq, nw, tmpdsm;
    double fun;
    /* Arrays */
    double *f = 0, *q = 0, *qx = 0, *rq = 0, *x = 0, *xe = 0;
    Integer *iq = 0, *state = 0;
    Integer seed[] = { 1762543 };
    /* Nag Types */
    Nag_BaseRNG genid = Nag_Basic;
    NagError fail;

    INIT_FAIL(fail);

    printf("nag_nd_shep_eval (e01znc) Example Program Results\n\n");
    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
    /* Set up state array for generating a random sample of data locations
     * using nag_rand_init_repeatable (g05kfc).
     *
     * First get the length of the state array by setting lstate = -1.
     */
    lstate = -1;

```

```

nag_rand_init_repeatabe(genid, subid, seed, lseed, state, &lstate, &fail);
if (fail.code == NE_NOERROR) {
    /* Allocate arrays */
    if (!(state = NAG_ALLOC(lstate, Integer)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    /* Then initialize the generator to a repeatable sequence */
    nag_rand_init_repeatabe(genid, subid, seed, lseed, state, &lstate,
        &fail);
}
if (fail.code != NE_NOERROR) {
    printf("Error from nag_rand_init_repeatabe (g05kfc).\n%s\n",
        fail.message);
    exit_status = 1;
    goto END;
}

/* Input the number of nodes. */
#ifdef _WIN32
scanf_s("%" NAG_IFMT "%*[\n] ", &m);
#else
scanf("%" NAG_IFMT "%*[\n] ", &m);
#endif
liq = 2 * m + 1;
lrq = (d + 1) * (d + 2) / 2 * m + 2 * d + 1;
if (!(x = NAG_ALLOC(d * m, double)) ||
    !(f = NAG_ALLOC(m, double)) ||
    !(iq = NAG_ALLOC(liq, Integer)) || !(rq = NAG_ALLOC(lrq, double))
    )
{
    printf("Allocation failure\n");
    exit_status = -2;
    goto END;
}
/* Generate d*m pseudorandom numbers in U(0,1) using
 * nag_rand_basic (g05sac).
 */
tmpdsm = d * m;
nag_rand_basic(tmpdsm, state, x, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_rand_basic (g05sac).\n%s\n", fail.message);
    exit_status = 2;
    goto END;
}

/* Evaluate f at x */
for (i = 0; i < m; i++)
    f[i] = funct(&x[i * d]);

/* Generate the interpolant using nag_nd_shep_interp (e01zmc):
   Interpolating functions, modified Shepard's method, d variables.
 */
nq = 0;
nw = 0;
nag_nd_shep_interp(d, m, x, f, nw, nq, iq, rq, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_nd_shep_interp (e01zmc).\n%s\n", fail.message);
    exit_status = 3;
    goto END;
}

/* Input the number of evaluation points and allocate arrays with lengths
   based on this.
 */
#ifdef _WIN32
scanf_s("%" NAG_IFMT "%*[\n]", &n);
#else

```

```

scanf("%" NAG_IFMT "%*[\n]", &n);
#endif
if (!(xe = NAG_ALLOC(d * n, double)) ||
    !(q = NAG_ALLOC(n, double)) || !(qx = NAG_ALLOC(d * n, double))
    )
{
printf("Allocation failure\n");
exit_status = -3;
goto END;
}

/* Generate a set of evaluation points lying on diagonal line
 * xe(1:d,i) = xe(1,i) = i/(n+1).
 */
for (i = 0; i < n; i++)
for (j = 0; j < d; j++)
    xe[i * d + j] = (double) (i + 1) / (double) (n + 1);

/* Evaluate the interpolant using nag_nd_shep_eval (e01znc), at given
interpolated values, where interpolant previously computed by e01zmc.
 */
nag_nd_shep_eval(d, m, x, f, iq, rq, n, xe, q, qx, &fail);
if (fail.code != NE_NOERROR) {
printf("Error from nag_nd_shep_eval (e01znc).\n%s\n", fail.message);
exit_status = 4;
goto END;
}

/* Print interpolated function values against actual function values
 * at the points on the diagonal line. */
/* Header */
printf("    i | f(i)          q(i)      | |f(i)-q(i)|\n");
printf("    ---|-----+-----\n");
/* Results */
for (i = 0; i < n; i++) {
fun = funct(&xe[i * d]);
printf("%5" NAG_IFMT " %10.4f%10.4f%10.4f\n", i, fun, q[i],
        fabs(fun - q[i]));
}

END:

NAG_FREE(f);
NAG_FREE(q);
NAG_FREE(qx);
NAG_FREE(rq);
NAG_FREE(x);
NAG_FREE(xe);
NAG_FREE(iq);
NAG_FREE(state);

return exit_status;
}

static double funct(double *x)
{
double funct_return;
funct_return = x[0] * x[1] * x[2] / (1.0 + 2.0 * x[3] * x[4] * x[5]);
return funct_return;
}

```

## 10.2 Program Data

```

nag_nd_shep_eval (e01znc) Example Program Data
120                : m, the number of data points
9                  : n, the number of evaluation points

```

### 10.3 Program Results

nag\_nd\_shep\_eval (e01znc) Example Program Results

i	f(i)	q(i)	f(i)-q(i)
0	0.0010	0.0050	0.0040
1	0.0079	0.0046	0.0033
2	0.0256	0.0214	0.0042
3	0.0567	0.0537	0.0030
4	0.1000	0.0986	0.0014
5	0.1508	0.1523	0.0015
6	0.2034	0.2071	0.0036
7	0.2530	0.2560	0.0030
8	0.2966	0.2940	0.0025

---