NAG Library Function Document

nag_pde_parab_1d_euler_osher (d03pvc)

1 Purpose

nag_pde_parab_1d_euler_osher (d03pvc) calculates a numerical flux function using Osher's Approximate Riemann Solver for the Euler equations in conservative form. It is designed primarily for use with the upwind discretization schemes nag_pde_parab_1d_cd (d03pfc), nag_pde_parab_1d_cd_ode (d03plc) or nag_pde_parab_1d_cd_ode_remesh (d03psc), but may also be applicable to other conservative upwind schemes requiring numerical flux functions.

2 Specification

```
#include <nag.h>
#include <nagd03.h>
void nag_pde_parab_1d_euler_osher (const double uleft[],
     const double uright[], double gamma, Nag_OsherVersion path,
     double flux[], Nag_D03_Save *saved, NagError *fail)
```

3 Description

nag_pde_parab_1d_euler_osher (d03pvc) calculates a numerical flux function at a single spatial point using Osher's Approximate Riemann Solver (see Hemker and Spekreijse (1986) and Pennington and Berzins (1994)) for the Euler equations (for a perfect gas) in conservative form. You must supply the *left* and *right* solution values at the point where the numerical flux is required, i.e., the initial left and right states of the Riemann problem defined below. In the functions nag_pde_parab_1d_cd (d03pfc), nag_pde_parab_1d_cd_ode (d03plc) and nag_pde_parab_1d_cd_ode_remesh (d03psc), the left and right solution values are derived automatically from the solution values at adjacent spatial points and supplied to the function argument **numflx** from which you may call nag_pde_parab_1d_euler_osher (d03pvc).

The Euler equations for a perfect gas in conservative form are:

$$\frac{\partial U}{\partial t}+\frac{\partial F}{\partial x}=0, \tag{1}$$

with

$$U=\begin{bmatrix}\rho\\ m\\ e\end{bmatrix} \quad \text{and} \quad F=\begin{bmatrix}m\\ \frac{m^2}{\rho}+(\gamma-1)\left(e-\frac{m^2}{2\rho}\right)\\ \frac{me}{\rho}+\frac{m}{\rho}(\gamma-1)\left(e-\frac{m^2}{2\rho}\right)\end{bmatrix}, \tag{2}$$

where ρ is the density, m is the momentum, e is the specific total energy, and γ is the (constant) ratio of specific heats. The pressure p is given by

$$p=(\gamma-1)\left(e-\frac{\rho u^2}{2}\right), \tag{3}$$

where $u=m/\rho$ is the velocity.

The function calculates the Osher approximation to the numerical flux function $F(U_L,U_R)=F(U^*(U_L,U_R))$, where $U=U_L$ and $U=U_R$ are the left and right solution values, and $U^*(U_L,U_R)$ is the intermediate state $\omega(0)$ arising from the similarity solution $U(y,t)=\omega(y/t)$ of the Riemann problem defined by

$$\frac{\partial U}{\partial t}+\frac{\partial F}{\partial y}=0, \tag{4}$$

with U and F as in (2), and initial piecewise constant values $U = U_L$ for $y < 0$ and $U = U_R$ for $y > 0$. The spatial domain is $-\infty < y < \infty$, where $y = 0$ is the point at which the numerical flux is required. Osher's solver carries out an integration along a path in the phase space of U consisting of subpaths which are piecewise parallel to the eigenvectors of the Jacobian of the PDE system. There are two variants of the Osher solver termed O (original) and P (physical), which differ in the order in which the subpaths are taken. The P-variant is generally more efficient, but in some rare cases may fail (see Hemker and Spekreijse (1986) for details). The argument **path** specifies which variant is to be used. The algorithm for Osher's solver for the Euler equations is given in detail in the Appendix of Pennington and Berzins (1994).

4 References

Hemker P W and Spekreijse S P (1986) Multiple grid and Osher's scheme for the efficient solution of the steady Euler equations *Applied Numerical Mathematics* **2** 475–493

Pennington S V and Berzins M (1994) New NAG Library software for first-order partial differential equations *ACM Trans. Math. Softw.* **20** 63–99

Quirk J J (1994) A contribution to the great Riemann solver debate *Internat. J. Numer. Methods Fluids* **18** 555–574

5 Arguments

1: **uleft**[**3**] – const double *Input*

On entry: **uleft**$[i-1]$ must contain the left value of the component U_i, for $i = 1, 2, 3$. That is, **uleft**[0] must contain the left value of ρ, **uleft**[1] must contain the left value of m and **uleft**[2] must contain the left value of e.

Constraints:

uleft[0] ≥ 0.0;
Left pressure, $pl \geq 0.0$, where pl is calculated using (3).

2: **uright**[**3**] – const double *Input*

On entry: **uright**$[i-1]$ must contain the right value of the component U_i, for $i = 1, 2, 3$. That is, **uright**[0] must contain the right value of ρ, **uright**[1] must contain the right value of m and **uright**[2] must contain the right value of e.

Constraints:

uright[0] ≥ 0.0;
Right pressure, $pr \geq 0.0$, where pr is calculated using (3).

3: **gamma** – double *Input*

On entry: the ratio of specific heats, γ.

Constraint: **gamma** > 0.0.

4: **path** – Nag_OsherVersion *Input*

On entry: the variant of the Osher scheme.

path = Nag_OsherOriginal
Original.

path = Nag_OsherPhysical
Physical.

Constraint: **path** = Nag_OsherOriginal or Nag_OsherPhysical.

5: **flux**[**3**] – double *Output*

On exit: **flux**$[i-1]$ contains the numerical flux component $\hat{F}_i$, for $i = 1, 2, 3$.

6: **saved** – Nag_D03_Save * *Communication Structure*

saved may contain data concerning the computation required by nag_pde_parab_1d_euler_osher (d03pvc) as passed through to **numflx** from one of the integrator functions nag_pde_parab_1d_cd (d03pfc), nag_pde_parab_1d_cd_ode (d03plc) or nag_pde_parab_1d_cd_ode_remesh (d03psc). You should not change the components of **saved**.

7: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

NE_REAL

Left pressure value $pl < 0.0$: $pl = \langle value \rangle$.

On entry, **gamma** $= \langle value \rangle$.
Constraint: **gamma** > 0.0.

On entry, **uleft**$[0] = \langle value \rangle$.
Constraint: **uleft**$[0] \geq 0.0$.

On entry, **uright**$[0] = \langle value \rangle$.
Constraint: **uright**$[0] \geq 0.0$.

Right pressure value $pr < 0.0$: $pr = \langle value \rangle$.

7 Accuracy

nag_pde_parab_1d_euler_osher (d03pvc) performs an exact calculation of the Osher numerical flux function, and so the result will be accurate to ***machine precision***.

8 Parallelism and Performance

nag_pde_parab_1d_euler_osher (d03pvc) is not threaded in any implementation.

9 Further Comments

nag_pde_parab_1d_euler_osher (d03pvc) must only be used to calculate the numerical flux for the Euler equations in exactly the form given by (2), with **uleft**$[i-1]$ and **uright**$[i-1]$ containing the left and right values of ρ, m and e, for $i = 1, 2, 3$, respectively. It should be noted that Osher's scheme, in common with all Riemann solvers, may be unsuitable for some problems (see Quirk (1994) for examples). The time taken depends on the input argument **path** and on the left and right solution values, since inclusion of each subpath depends on the signs of the eigenvalues. In general this cannot be determined in advance.

10 Example

See Section 10 in nag_pde_parab_1d_cd_ode (d03plc).