NAG Library Function Document

nag_pde_bs_1d_means (d03nec)

1 Purpose

nag_pde_bs_1d_means (d03nec) computes average values of a continuous function of time over the remaining life of an option. It is used together with nag_pde_bs_1d_analytic (d03ndc) to value options with time-dependent arguments.

2 Specification

```
#include <nag.h>
#include <nagd03.h>
void nag_pde_bs_1d_means (double t0, double tmat, Integer ntd,
     const double td[], const double phid[], double phiav[], NagError *fail)
```

3 Description

nag_pde_bs_1d_means (d03nec) computes the quantities

$$\phi(t_0), \qquad \hat{\phi} = \frac{1}{T-t_0}\int_{t_0}^{T} \phi(\zeta)\, d\zeta, \qquad \bar{\phi} = \left(\frac{1}{T-t_0}\int_{t_0}^{T} \phi^2(\zeta)\, d\zeta\right)^{1/2}$$

from a given set of values **phid** of a continuous time-dependent function $\phi(t)$ at a set of discrete points **td**, where t_0 is the current time and T is the maturity time. Thus $\hat{\phi}$ and $\bar{\phi}$ are first and second order averages of ϕ over the remaining life of an option.

The function may be used in conjunction with nag_pde_bs_1d_analytic (d03ndc) in order to value an option in the case where the risk-free interest rate r, the continuous dividend q, or the stock volatility σ is time-dependent and is described by values at a set of discrete times (see Section 9.2). This is illustrated in Section 10.

4 References

None.

5 Arguments

1: **t0** – double *Input*

On entry: the current time t_0.

Constraint: **td**$[0] \leq$ **t0** $\leq$ **td**$[$**ntd** $- 1]$.

2: **tmat** – double *Input*

On entry: the maturity time T.

Constraint: **td**$[0] \leq$ **tmat** $\leq$ **td**$[$**ntd** $- 1]$.

3: **ntd** – Integer *Input*

On entry: the number of discrete times at which ϕ is given.

Constraint: **ntd** ≥ 2.

4: **td**[**ntd**] – const double *Input*

On entry: the discrete times at which ϕ is specified.

Constraint: $\mathbf{td}[0] < \mathbf{td}[1] < \cdots < \mathbf{td}[\mathbf{ntd}-1]$.

5: **phid**[**ntd**] – const double *Input*

On entry: $\mathbf{phid}[i-1]$ must contain the value of ϕ at time $\mathbf{td}[i-1]$, for $i = 1, 2, \ldots, \mathbf{ntd}$.

6: **phiav**[**3**] – double *Output*

On exit: $\mathbf{phiav}[0]$ contains the value of ϕ interpolated to t_0, $\mathbf{phiav}[1]$ contains the first-order average $\hat{\phi}$ and $\mathbf{phiav}[2]$ contains the second-order average $\bar{\phi}$, where:

$$\hat{\phi} = \frac{1}{T-t_0}\int_{t_0}^{T} \phi(\zeta)d\zeta, \qquad \bar{\phi} = \left(\frac{1}{T-t_0}\int_{t_0}^{T} \phi^2(\zeta)\,d\zeta\right)^{1/2}.$$

7: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **ntd** = ⟨*value*⟩.
Constraint: **ntd** ≥ 2.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

Unexpected failure in internal call to spline function.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

NE_NOT_STRICTLY_INCREASING

On entry, $\mathbf{td}[I] \leq \mathbf{td}[I-1]$, for $I = \langle value \rangle$.

NE_REAL_3

On entry, **t0** lies outside the range $[\mathbf{td}[0], \mathbf{td}[\mathbf{ntd}-1]]$: **t0** = ⟨*value*⟩, $\mathbf{td}[0] = \langle value \rangle$ and $\mathbf{td}[\mathbf{ntd}-1] = \langle value \rangle$.

On entry, **tmat** lies outside the range $[\mathbf{td}[0], \mathbf{td}[\mathbf{ntd}-1]]$: $\mathbf{tmat} = \langle value \rangle$, $\mathbf{td}[0] = \langle value \rangle$ and $\mathbf{td}[\mathbf{ntd}-1] = \langle value \rangle$.

7 Accuracy

If $\phi \in C^4[t_0, T]$ then the error in the approximation of $\phi(t_0)$ and $\hat{\phi}$ is $O\left(H^4\right)$, where $H = \max_i(T(i+1) - T(i))$, for $i = 1, 2, \ldots, \mathbf{ntd}-1$. The approximation is exact for polynomials of degree up to 3.

The third quantity $\bar{\phi}$ is $O\left(H^2\right)$, and exact for linear functions.

8 Parallelism and Performance

nag_pde_bs_1d_means (d03nec) is not threaded in any implementation.

9 Further Comments

9.1 Timing

The time taken is proportional to **ntd**.

9.2 Use with nag_pde_bs_1d_analytic (d03ndc)

Suppose you wish to evaluate the analytic solution of the Black–Scholes equation in the case when the risk-free interest rate r is a known function of time, and is represented as a set of values at discrete times. A call to nag_pde_bs_1d_means (d03nec) providing these values in **phid** produces an output array **phiav** suitable for use as the argument **r** in a subsequent call to nag_pde_bs_1d_analytic (d03ndc).

Time-dependent values of the continuous dividend Q and the volatility σ may be handled in the same way.

9.3 Algorithmic Details

The **ntd** data points are fitted with a cubic B-spline using the function nag_1d_spline_interpolant (e01bac). Evaluation is then performed using nag_1d_spline_evaluate (e02bbc), and the definite integrals are computed using direct integration of the cubic splines in each interval. The special case of $T = t_o$ is handled by interpolating ϕ at that point.

10 Example

This example demonstrates the use of the function in conjunction with nag_pde_bs_1d_analytic (d03ndc) to solve the Black–Scholes equation for valuation of a 5-month American call option on a non-dividend-paying stock with an exercise price of \$50. The risk-free interest rate varies linearly with time and the stock volatility has a quadratic variation. Since these functions are integrated exactly by nag_pde_bs_1d_means (d03nec) the solution of the Black–Scholes equation by nag_pde_bs_1d_analytic (d03ndc) is also exact.

The option is valued at a range of times and stock prices.

10.1 Program Text

```
/* nag_pde_bs_1d_means (d03nec) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */
```

```
#include <stdio.h>
#include <string.h>
#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagd03.h>

#define F(I, J)      f[ns*((J) -1)+(I) -1]
#define THETA(I, J)  theta[ns*((J) -1)+(I) -1]
#define DELTA(I, J)  delta[ns*((J) -1)+(I) -1]
#define GAMMA(I, J)  gamma[ns*((J) -1)+(I) -1]
#define LAMBDA(I, J) lambda[ns*((J) -1)+(I) -1]
#define RHO(I, J)    rho[ns*((J) -1)+(I) -1]

int main(void)
{
  double ds, dt, tmat, x;
  Integer i, igreek, j, ns, nt, ntd, exit_status = 0;
  double *delta = 0, *f = 0, *gamma = 0, *lambda = 0, q[3], ra[3],
         *rho = 0, *s = 0;
  double siga[3], *t = 0, *theta = 0, *td = 0, *rd = 0, *sigd = 0,
         smin, smax, tmin, tmax;
  Nag_Boolean gprnt[5] = { Nag_TRUE, Nag_TRUE, Nag_TRUE, Nag_TRUE, Nag_TRUE };
  Nag_Boolean tdpar[3];
  const char *gname[5] = { "Theta", "Delta", "Gamma", "Lambda", "Rho" };
  NagError fail;

  INIT_FAIL(fail);

  printf("nag_pde_bs_1d_means (d03nec) Example Program Results\n\n");

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
#endif

  /* Read problem parameters */
#ifdef _WIN32
  scanf_s("%lf", &x);
#else
  scanf("%lf", &x);
#endif
#ifdef _WIN32
  scanf_s("%lf", &tmat);
#else
  scanf("%lf", &tmat);
#endif
#ifdef _WIN32
  scanf_s("%" NAG_IFMT "%" NAG_IFMT "", &ns, &nt);
#else
  scanf("%" NAG_IFMT "%" NAG_IFMT "", &ns, &nt);
#endif
#ifdef _WIN32
  scanf_s("%lf%lf", &smin, &smax);
#else
  scanf("%lf%lf", &smin, &smax);
#endif
#ifdef _WIN32
  scanf_s("%lf%lf", &tmin, &tmax);
#else
  scanf("%lf%lf", &tmin, &tmax);
#endif
#ifdef _WIN32
  scanf_s("%" NAG_IFMT "", &ntd);
#else
  scanf("%" NAG_IFMT "", &ntd);
#endif

  /* Allocate memory */
```

```
  if (!(s = NAG_ALLOC(ns, double)) ||
      !(t = NAG_ALLOC(nt, double)) ||
      !(f = NAG_ALLOC(ns * nt, double)) ||
      !(theta = NAG_ALLOC(ns * nt, double)) ||
      !(delta = NAG_ALLOC(ns * nt, double)) ||
      !(gamma = NAG_ALLOC(ns * nt, double)) ||
      !(lambda = NAG_ALLOC(ns * nt, double)) ||
      !(rho = NAG_ALLOC(ns * nt, double)) ||
      !(td = NAG_ALLOC(ntd, double)) ||
      !(rd = NAG_ALLOC(ntd, double)) || !(sigd = NAG_ALLOC(ntd, double)))
  {
    printf("Allocation failure\n");
    exit_status = 1;
    goto END;
  }

  /* Read discrete times and data values for r and sigma */

#ifdef _WIN32
  for (i = 0; i < ntd; i++)
    scanf_s("%lf", &td[i]);
#else
  for (i = 0; i < ntd; i++)
    scanf("%lf", &td[i]);
#endif
#ifdef _WIN32
  for (i = 0; i < ntd; i++)
    scanf_s("%lf", &rd[i]);
#else
  for (i = 0; i < ntd; i++)
    scanf("%lf", &rd[i]);
#endif
#ifdef _WIN32
  for (i = 0; i < ntd; i++)
    scanf_s("%lf", &sigd[i]);
#else
  for (i = 0; i < ntd; i++)
    scanf("%lf", &sigd[i]);
#endif

  /* Set up input parameters for nag_pde_bs_1d_analytic (d03ndc) */

  s[0] = smin;
  s[ns - 1] = smax;
  t[0] = tmin;
  t[nt - 1] = tmax;
  tdpar[0] = Nag_TRUE;
  tdpar[1] = Nag_FALSE;
  tdpar[2] = Nag_TRUE;
  q[0] = 0.0;

  ds = (s[ns - 1] - s[0]) / (ns - 1.0);
  dt = (t[nt - 1] - t[0]) / (nt - 1.0);

  /* Loop over times */

  for (j = 1; j <= nt; j++) {
    t[j - 1] = t[0] + (j - 1) * dt;

    /* Find average values of r and sigma */

    /* nag_pde_bs_1d_means (d03nec).
     * Compute average values for nag_pde_bs_1d_analytic
     * (d03ndc)
     */
    nag_pde_bs_1d_means(t[j - 1], tmat, ntd, td, rd, ra, &fail);

    if (fail.code != NE_NOERROR) {
      printf("Error from nag_pde_bs_1d_means (d03nec).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
```

```
    }

    /* nag_pde_bs_1d_means (d03nec), see above. */
    nag_pde_bs_1d_means(t[j - 1], tmat, ntd, td, sigd, siga, &fail);

    if (fail.code != NE_NOERROR) {
      printf("Error from nag_pde_bs_1d_means (d03nec).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

    /* Loop over stock prices */

    for (i = 1; i <= ns; i++) {
      s[i - 1] = s[0] + (i - 1) * ds;

      /* Evaluate analytic solution of Black-Scholes equation */

      /* nag_pde_bs_1d_analytic (d03ndc).
       * Analytic solution of the Black-Scholes equations
       */
      nag_pde_bs_1d_analytic(Nag_AmericanCall, x, s[i - 1], t[j - 1], tmat,
                             tdpar, ra, q, siga, &F(i, j), &THETA(i, j),
                             &DELTA(i, j), &GAMMA(i, j), &LAMBDA(i, j),
                             &RHO(i, j), &fail);

      if (fail.code != NE_NOERROR) {
        printf("Error from nag_pde_bs_1d_analytic (d03ndc).\n%s\n",
               fail.message);
        exit_status = 1;
        goto END;
      }
    }
  }

  /* Output option values */

  printf("\n");
  printf("Option Values\n");
  printf("-------------\n");
  printf("%14s  |  %s\n", "Stock Price", "Time to Maturity (months)");
  printf("%14s  | ", "");
  for (i = 0; i < nt; i++)
    printf(" %13.4e", 12.0 * (tmat - t[i]));
  printf("\n");
  for (i = 0; i < 74; i++)
    printf("-");
  printf("\n");
  for (i = 1; i <= ns; i++) {
    printf(" %13.4e  | ", s[i - 1]);
    for (j = 1; j <= nt; j++)
      printf(" %13.4e", F(i, j));
    printf("\n");
  }

  for (igreek = 0; igreek < 5; igreek++) {
    if (!gprnt[igreek])
      continue;

    printf("\n");
    printf("%s\n", gname[igreek]);
    for (i = 0; i < (Integer) strlen(gname[igreek]); i++)
      printf("-");
    printf("\n");
    printf("%14s  |  %s\n", "Stock Price", "Time to Maturity (months)");
    printf("%14s  | ", "");
    for (i = 0; i < nt; i++)
      printf(" %13.4e", 12.0 * (tmat - t[i]));
    printf("\n");
    for (i = 0; i < 74; i++)
      printf("-");
```

```
    printf("\n");

    for (i = 1; i <= ns; i++) {
      printf(" %13.4e  | ", s[i - 1]);
      switch (igreek) {
      case 0:
        for (j = 1; j <= nt; j++)
          printf(" %13.4e", THETA(i, j));
        break;
      case 1:
        for (j = 1; j <= nt; j++)
          printf(" %13.4e", DELTA(i, j));
        break;
      case 2:
        for (j = 1; j <= nt; j++)
          printf(" %13.4e", GAMMA(i, j));
        break;
      case 3:
        for (j = 1; j <= nt; j++)
          printf(" %13.4e", LAMBDA(i, j));
        break;
      case 4:
        for (j = 1; j <= nt; j++)
          printf(" %13.4e", RHO(i, j));
        break;
      default:
        break;
      }
      printf("\n");
    }
  }

END:
  NAG_FREE(s);
  NAG_FREE(t);
  NAG_FREE(f);
  NAG_FREE(theta);
  NAG_FREE(delta);
  NAG_FREE(gamma);
  NAG_FREE(lambda);
  NAG_FREE(rho);
  NAG_FREE(td);
  NAG_FREE(rd);
  NAG_FREE(sigd);

  return exit_status;
}
```

10.2 Program Data

```
nag_pde_bs_1d_means (d03nec) Example Program Data
  50.
  0.4166667
  21  4
  0.0  100.
  0.0  0.125
  6
  0.00 0.10 0.20
  0.30 0.40 0.50
  0.10 0.11 0.12
  0.13 0.14 0.15
  0.30 0.46 0.54
  0.54 0.46 0.30
```

10.3 Program Results

```
nag_pde_bs_1d_means (d03nec) Example Program Results


Option Values
-------------
   Stock Price  |  Time to Maturity (months)
                |     5.0000e+00    4.5000e+00    4.0000e+00    3.5000e+00
-------------------------------------------------------------------------
    0.0000e+00  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    5.0000e+00  |     3.3671e-13    7.7404e-14    7.3210e-15    2.0179e-16
    1.0000e+01  |     5.2088e-07    2.4281e-07    7.2216e-08    1.1540e-08
    1.5000e+01  |     2.6607e-04    1.6753e-04    8.0943e-05    2.7179e-05
    2.0000e+01  |     8.9697e-03    6.6505e-03    4.1780e-03    2.0942e-03
    2.5000e+01  |     8.3647e-02    6.8467e-02    5.0375e-02    3.2105e-02
    3.0000e+01  |     3.8221e-01    3.3331e-01    2.7117e-01    2.0119e-01
    3.5000e+01  |     1.1298e+00    1.0275e+00    8.9292e-01    7.3146e-01
    4.0000e+01  |     2.5164e+00    2.3541e+00    2.1380e+00    1.8699e+00
    4.5000e+01  |     4.6249e+00    4.4110e+00    4.1267e+00    3.7700e+00
    5.0000e+01  |     7.4287e+00    7.1797e+00    6.8531e+00    6.4449e+00
    5.5000e+01  |     1.0830e+01    1.0564e+01    1.0221e+01    9.7996e+00
    6.0000e+01  |     1.4707e+01    1.4436e+01    1.4097e+01    1.3689e+01
    6.5000e+01  |     1.8937e+01    1.8671e+01    1.8348e+01    1.7968e+01
    7.0000e+01  |     2.3421e+01    2.3164e+01    2.2860e+01    2.2514e+01
    7.5000e+01  |     2.8080e+01    2.7833e+01    2.7550e+01    2.7234e+01
    8.0000e+01  |     3.2857e+01    3.2620e+01    3.2354e+01    3.2064e+01
    8.5000e+01  |     3.7713e+01    3.7484e+01    3.7233e+01    3.6963e+01
    9.0000e+01  |     4.2620e+01    4.2398e+01    4.2158e+01    4.1904e+01
    9.5000e+01  |     4.7561e+01    4.7344e+01    4.7112e+01    4.6868e+01
    1.0000e+02  |     5.2523e+01    5.2310e+01    5.2084e+01    5.1848e+01

Theta
-----
   Stock Price  |  Time to Maturity (months)
                |     5.0000e+00    4.5000e+00    4.0000e+00    3.5000e+00
-------------------------------------------------------------------------
    0.0000e+00  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    5.0000e+00  |    -8.9082e-12   -3.4507e-12   -5.0884e-13   -2.1236e-14
    1.0000e+01  |    -7.2097e-06   -5.5915e-06   -2.5721e-06   -6.1830e-07
    1.5000e+01  |    -2.2499e-03   -2.3259e-03   -1.7227e-03   -8.6349e-04
    2.0000e+01  |    -4.9483e-02   -5.9355e-02   -5.6562e-02   -4.1921e-02
    2.5000e+01  |    -3.1200e-01   -4.0620e-01   -4.4765e-01   -4.1683e-01
    3.0000e+01  |    -9.8578e-01   -1.3408e+00   -1.6092e+00   -1.7186e+00
    3.5000e+01  |    -2.0479e+00   -2.8395e+00   -3.5745e+00   -4.1390e+00
    4.0000e+01  |    -3.2501e+00   -4.5165e+00   -5.8147e+00   -7.0323e+00
    4.5000e+01  |    -4.3144e+00   -5.9349e+00   -7.6762e+00   -9.4488e+00
    5.0000e+01  |    -5.0802e+00   -6.8543e+00   -8.7919e+00   -1.0815e+01
    5.5000e+01  |    -5.5225e+00   -7.2603e+00   -9.1500e+00   -1.1104e+01
    6.0000e+01  |    -5.7006e+00   -7.2722e+00   -8.9491e+00   -1.0625e+01
    6.5000e+01  |    -5.7014e+00   -7.0446e+00   -8.4366e+00   -9.7565e+00
    7.0000e+01  |    -5.6037e+00   -6.7093e+00   -7.8142e+00   -8.7951e+00
    7.5000e+01  |    -5.4653e+00   -6.3555e+00   -7.2107e+00   -7.9170e+00
    8.0000e+01  |    -5.3218e+00   -6.0329e+00   -6.6903e+00   -7.1974e+00
    8.5000e+01  |    -5.1920e+00   -5.7627e+00   -6.2736e+00   -6.6481e+00
    9.0000e+01  |    -5.0833e+00   -5.5487e+00   -5.9563e+00   -6.2492e+00
    9.5000e+01  |    -4.9969e+00   -5.3857e+00   -5.7234e+00   -5.9700e+00
    1.0000e+02  |    -4.9306e+00   -5.2651e+00   -5.5570e+00   -5.7797e+00

Delta
-----
   Stock Price  |  Time to Maturity (months)
                |     5.0000e+00    4.5000e+00    4.0000e+00    3.5000e+00
-------------------------------------------------------------------------
    0.0000e+00  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    5.0000e+00  |     1.6086e-12    3.8832e-13    3.9572e-14    1.2111e-15
    1.0000e+01  |     8.9933e-07    4.3972e-07    1.4063e-07    2.4884e-08
    1.5000e+01  |     2.3975e-04    1.5810e-04    8.1943e-05    3.0366e-05
    2.0000e+01  |     4.9150e-03    3.8095e-03    2.5596e-03    1.4100e-03
    2.5000e+01  |     3.0345e-02    2.5906e-02    2.0311e-02    1.4153e-02
    3.0000e+01  |     9.6991e-02    8.7980e-02    7.5946e-02    6.1231e-02
```

```
    3.5000e+01  |     2.0863e-01    1.9675e-01    1.8053e-01    1.5957e-01
    4.0000e+01  |     3.4875e-01    3.3719e-01    3.2158e-01    3.0109e-01
    4.5000e+01  |     4.9361e-01    4.8480e-01    4.7356e-01    4.5924e-01
    5.0000e+01  |     6.2450e-01    6.1931e-01    6.1363e-01    6.0735e-01
    5.5000e+01  |     7.3200e-01    7.3000e-01    7.2907e-01    7.2954e-01
    6.0000e+01  |     8.1439e-01    8.1462e-01    8.1681e-01    8.2145e-01
    6.5000e+01  |     8.7440e-01    8.7589e-01    8.7961e-01    8.8602e-01
    7.0000e+01  |     9.1650e-01    9.1850e-01    9.2260e-01    9.2911e-01
    7.5000e+01  |     9.4522e-01    9.4726e-01    9.5107e-01    9.5679e-01
    8.0000e+01  |     9.6441e-01    9.6624e-01    9.6946e-01    9.7406e-01
    8.5000e+01  |     9.7704e-01    9.7856e-01    9.8111e-01    9.8461e-01
    9.0000e+01  |     9.8526e-01    9.8646e-01    9.8839e-01    9.9094e-01
    9.5000e+01  |     9.9057e-01    9.9148e-01    9.9290e-01    9.9470e-01
    1.0000e+02  |     9.9397e-01    9.9464e-01    9.9567e-01    9.9691e-01

Gamma
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   Stock Price  |  Time to Maturity (months)
                |     5.0000e+00    4.5000e+00    4.0000e+00    3.5000e+00
-------------------------------------------------------------------------
    0.0000e+00  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    5.0000e+00  |     7.2334e-12    1.8390e-12    2.0276e-13    6.9267e-15
    1.0000e+01  |     1.4139e-06    7.2829e-07    2.5205e-07    4.9786e-08
    1.5000e+01  |     1.8932e-04    1.3153e-04    7.3756e-05    3.0494e-05
    2.0000e+01  |     2.2528e-03    1.8392e-03    1.3360e-03    8.2017e-04
    2.5000e+01  |     8.6933e-03    7.8126e-03    6.6135e-03    5.1251e-03
    3.0000e+01  |     1.8099e-02    1.7264e-02    1.6056e-02    1.4350e-02
    3.5000e+01  |     2.5953e-02    2.5691e-02    2.5315e-02    2.4683e-02
    4.0000e+01  |     2.9260e-02    2.9618e-02    3.0194e-02    3.0968e-02
    4.5000e+01  |     2.8046e-02    2.8736e-02    2.9814e-02    3.1368e-02
    5.0000e+01  |     2.4005e-02    2.4715e-02    2.5793e-02    2.7346e-02
    5.5000e+01  |     1.8950e-02    1.9500e-02    2.0296e-02    2.1401e-02
    6.0000e+01  |     1.4105e-02    1.4449e-02    1.4903e-02    1.5476e-02
    6.5000e+01  |     1.0054e-02    1.0221e-02    1.0396e-02    1.0555e-02
    7.0000e+01  |     6.9401e-03    6.9861e-03    6.9806e-03    6.8890e-03
    7.5000e+01  |     4.6779e-03    4.6538e-03    4.5552e-03    4.3505e-03
    8.0000e+01  |     3.0978e-03    3.0414e-03    2.9096e-03    2.6800e-03
    8.5000e+01  |     2.0250e-03    1.9598e-03    1.8291e-03    1.6205e-03
    9.0000e+01  |     1.3114e-03    1.2499e-03    1.1365e-03    9.6637e-04
    9.5000e+01  |     8.4362e-04    7.9138e-04    7.0024e-04    5.7052e-04
    1.0000e+02  |     5.4033e-04    4.9856e-04    4.2893e-04    3.3442e-04

Lambda
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   Stock Price  |  Time to Maturity (months)
                |     5.0000e+00    4.5000e+00    4.0000e+00    3.5000e+00
-------------------------------------------------------------------------
    0.0000e+00  |     0.0000e+00    0.0000e+00    0.0000e+00    0.0000e+00
    5.0000e+00  |     3.6558e-11    8.6441e-12    8.6672e-13    2.6259e-14
    1.0000e+01  |     2.8583e-05    1.3693e-05    4.3098e-06    7.5495e-07
    1.5000e+01  |     8.6115e-03    5.5645e-03    2.8375e-03    1.0404e-03
    2.0000e+01  |     1.8217e-01    1.3832e-01    9.1376e-02    4.9748e-02
    2.5000e+01  |     1.0984e+00    9.1808e-01    7.0676e-01    4.8574e-01
    3.0000e+01  |     3.2931e+00    2.9214e+00    2.4708e+00    1.9584e+00
    3.5000e+01  |     6.4272e+00    5.9173e+00    5.3025e+00    4.5851e+00
    4.0000e+01  |     9.4643e+00    8.9101e+00    8.2604e+00    7.5135e+00
    4.5000e+01  |     1.1481e+01    1.0941e+01    1.0323e+01    9.6323e+00
    5.0000e+01  |     1.2132e+01    1.1617e+01    1.1026e+01    1.0367e+01
    5.5000e+01  |     1.1588e+01    1.1091e+01    1.0498e+01    9.8169e+00
    6.0000e+01  |     1.0265e+01    9.7801e+00    9.1734e+00    8.4486e+00
    6.5000e+01  |     8.5872e+00    8.1198e+00    7.5104e+00    6.7621e+00
    7.0000e+01  |     6.8747e+00    6.4363e+00    5.8487e+00    5.1188e+00
    7.5000e+01  |     5.3194e+00    4.9219e+00    4.3812e+00    3.7109e+00
    8.0000e+01  |     4.0081e+00    3.6599e+00    3.1840e+00    2.6009e+00
    8.5000e+01  |     2.9578e+00    2.6623e+00    2.2597e+00    1.7754e+00
    9.0000e+01  |     2.1474e+00    1.9036e+00    1.5741e+00    1.1870e+00
    9.5000e+01  |     1.5392e+00    1.3429e+00    1.0806e+00    7.8078e-01
    1.0000e+02  |     1.0923e+00    9.3740e-01    7.3341e-01    5.0711e-01

Rho
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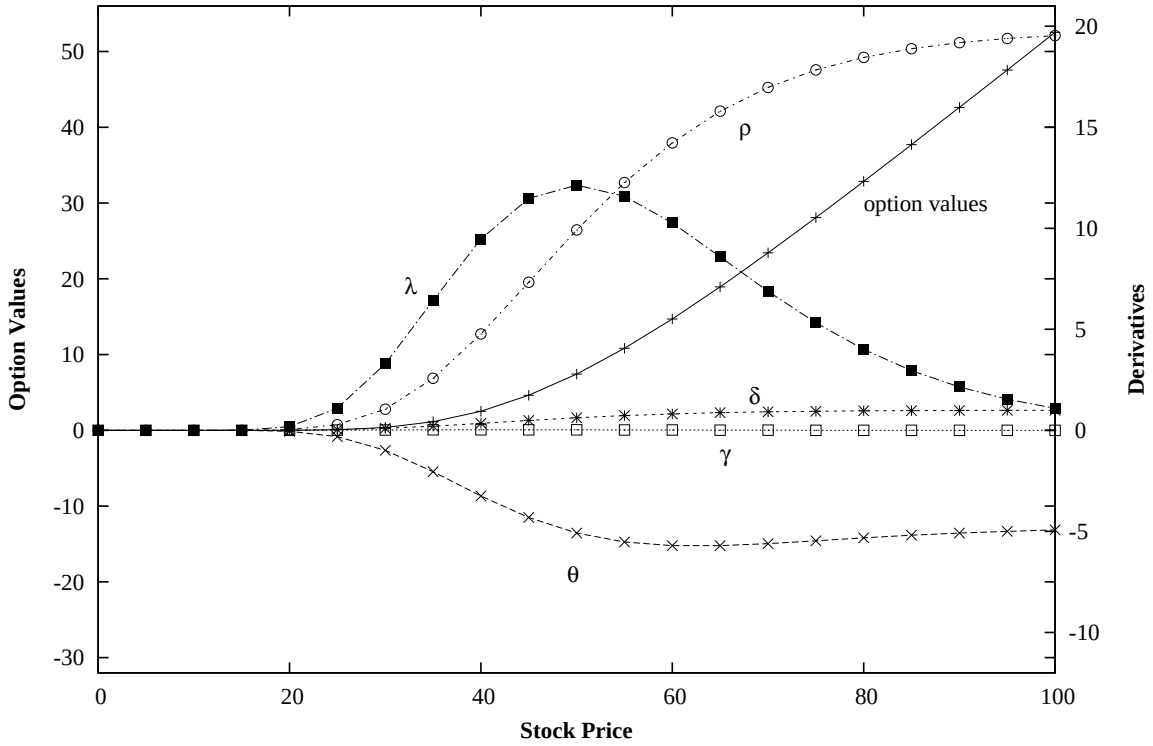
Stock Price	Time to Maturity (months)			
	5.0000e+00	4.5000e+00	4.0000e+00	3.5000e+00
0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00	0.0000e+00
5.0000e+00	3.2110e-12	6.9908e-13	6.3513e-14	1.7073e-15
1.0000e+01	3.5302e-06	1.5579e-06	4.4470e-07	6.9214e-08
1.5000e+01	1.3876e-03	8.2648e-04	3.8273e-04	1.2492e-04
2.0000e+01	3.7221e-02	2.6077e-02	1.5671e-02	7.6142e-03
2.5000e+01	2.8124e-01	2.1719e-01	1.5247e-01	9.3836e-02
3.0000e+01	1.0531e+00	8.6478e-01	6.6907e-01	4.7709e-01
3.5000e+01	2.5718e+00	2.1971e+00	1.8086e+00	1.4156e+00
4.0000e+01	4.7641e+00	4.1750e+00	3.5750e+00	2.9673e+00
4.5000e+01	7.3281e+00	6.5270e+00	5.7279e+00	4.9280e+00
5.0000e+01	9.9152e+00	8.9196e+00	7.9427e+00	6.9774e+00
5.5000e+01	1.2262e+01	1.1095e+01	9.9592e+00	8.8448e+00
6.0000e+01	1.4232e+01	1.2915e+01	1.1637e+01	1.0383e+01
6.5000e+01	1.5791e+01	1.4348e+01	1.2942e+01	1.1557e+01
7.0000e+01	1.6973e+01	1.5424e+01	1.3907e+01	1.2403e+01
7.5000e+01	1.7838e+01	1.6204e+01	1.4594e+01	1.2987e+01
8.0000e+01	1.8457e+01	1.6755e+01	1.5067e+01	1.3376e+01
8.5000e+01	1.8890e+01	1.7135e+01	1.5387e+01	1.3629e+01
9.0000e+01	1.9189e+01	1.7393e+01	1.5599e+01	1.3790e+01
9.5000e+01	1.9393e+01	1.7567e+01	1.5738e+01	1.3891e+01
1.0000e+02	1.9531e+01	1.7683e+01	1.5827e+01	1.3954e+01

Example Program 1
Option Values and Derivatives at 5 Months to Maturity



Example Program 2
Option Values and Derivatives at 3.5 Months to Maturity