

## NAG Library Function Document

### nag\_ode\_bvp\_coll\_nlin\_contin (d02txc)

#### 1 Purpose

nag\_ode\_bvp\_coll\_nlin\_contin (d02txc) allows a solution to a nonlinear two-point boundary value problem computed by nag\_ode\_bvp\_coll\_nlin\_solve (d02tlc) to be used as an initial approximation in the solution of a related nonlinear two-point boundary value problem in a continuation call to nag\_ode\_bvp\_coll\_nlin\_solve (d02tlc).

#### 2 Specification

```
#include <nag.h>
#include <nagd02.h>

void nag_ode_bvp_coll_nlin_contin (Integer mxmesh, Integer nmesh,
    const double mesh[], const Integer ipmesh[], double rcomm[],
    Integer icomm[], NagError *fail)
```

#### 3 Description

nag\_ode\_bvp\_coll\_nlin\_contin (d02txc) and its associated functions (nag\_ode\_bvp\_coll\_nlin\_solve (d02tlc), nag\_ode\_bvp\_coll\_nlin\_setup (d02tvc), nag\_ode\_bvp\_coll\_nlin\_interp (d02tyc) and nag\_ode\_bvp\_coll\_nlin\_diag (d02tzc)) solve the two-point boundary value problem for a nonlinear system of ordinary differential equations

$$\begin{aligned} y_1^{(m_1)}(x) &= f_1 \left( x, y_1, y_1^{(1)}, \dots, y_1^{(m_1-1)}, y_2, \dots, y_n^{(m_n-1)} \right) \\ y_2^{(m_2)}(x) &= f_2 \left( x, y_1, y_1^{(1)}, \dots, y_1^{(m_1-1)}, y_2, \dots, y_n^{(m_n-1)} \right) \\ &\vdots \\ y_n^{(m_n)}(x) &= f_n \left( x, y_1, y_1^{(1)}, \dots, y_1^{(m_1-1)}, y_2, \dots, y_n^{(m_n-1)} \right) \end{aligned}$$

over an interval  $[a, b]$  subject to  $p$  ( $> 0$ ) nonlinear boundary conditions at  $a$  and  $q$  ( $> 0$ ) nonlinear boundary conditions at  $b$ , where  $p + q = \sum_{i=1}^n m_i$ . Note that  $y_i^{(m)}(x)$  is the  $m$ th derivative of the  $i$ th solution component. Hence  $y_i^{(0)}(x) = y_i(x)$ . The left boundary conditions at  $a$  are defined as

$$g_i(z(y(a))) = 0, \quad i = 1, 2, \dots, p,$$

and the right boundary conditions at  $b$  as

$$\bar{g}_j(z(y(b))) = 0, \quad j = 1, 2, \dots, q,$$

where  $y = (y_1, y_2, \dots, y_n)$  and

$$z(y(x)) = \left( y_1(x), y_1^{(1)}(x), \dots, y_1^{(m_1-1)}(x), y_2(x), \dots, y_n^{(m_n-1)}(x) \right).$$

First, nag\_ode\_bvp\_coll\_nlin\_setup (d02tvc) must be called to specify the initial mesh, error requirements and other details. Then, nag\_ode\_bvp\_coll\_nlin\_solve (d02tlc) can be used to solve the boundary value problem. After successful computation, nag\_ode\_bvp\_coll\_nlin\_diag (d02tzc) can be used to ascertain details about the final mesh. nag\_ode\_bvp\_coll\_nlin\_interp (d02tyc) can be used to compute the approximate solution anywhere on the interval  $[a, b]$  using interpolation.

If the boundary value problem being solved is one of a sequence of related problems, for example as part of some continuation process, then nag\_ode\_bvp\_coll\_nlin\_contin (d02txc) should be used between calls to nag\_ode\_bvp\_coll\_nlin\_solve (d02tlc). This avoids the overhead of a complete initialization when the setup function nag\_ode\_bvp\_coll\_nlin\_setup (d02tvc) is used. nag\_ode\_bvp\_coll\_nlin\_contin

(d02txc) allows the solution values computed in the previous call to `nag_ode_bvp_coll_nlin_solve` (d02tlc) to be used as an initial approximation for the solution in the next call to `nag_ode_bvp_coll_nlin_solve` (d02tlc).

You must specify the new initial mesh. The previous mesh can be obtained by a call to `nag_ode_bvp_coll_nlin_diag` (d02tzc). It may be used unchanged as the new mesh, in which case any fixed points in the previous mesh remain as fixed points in the new mesh. Fixed and other points may be added or subtracted from the mesh by manipulation of the contents of the array argument **ipmesh**. Initial values for the solution components on the new mesh are computed by interpolation on the values for the solution components on the previous mesh.

The functions are based on modified versions of the codes COLSYS and COLNEW (see Ascher *et al.* (1979) and Ascher and Bader (1987)). A comprehensive treatment of the numerical solution of boundary value problems can be found in Ascher *et al.* (1988) and Keller (1992).

## 4 References

Ascher U M and Bader G (1987) A new basis implementation for a mixed order boundary value ODE solver *SIAM J. Sci. Stat. Comput.* **8** 483–500

Ascher U M, Christiansen J and Russell R D (1979) A collocation solver for mixed order systems of boundary value problems *Math. Comput.* **33** 659–679

Ascher U M, Mattheij R M M and Russell R D (1988) *Numerical Solution of Boundary Value Problems for Ordinary Differential Equations* Prentice–Hall

Keller H B (1992) *Numerical Methods for Two-point Boundary-value Problems* Dover, New York

## 5 Arguments

- 1: **mxmesh** – Integer *Input*  
*On entry:* the maximum number of points allowed in the mesh.  
*Constraint:* this must be identical to the value supplied for the argument **mxmesh** in the prior call to `nag_ode_bvp_coll_nlin_setup` (d02tvc).
  
- 2: **nmesh** – Integer *Input*  
*On entry:* the number of points to be used in the new initial mesh. It is strongly recommended that if this function is called that the suggested value (see below) for **nmesh** is used. In this case the arrays **mesh** and **ipmesh** returned by `nag_ode_bvp_coll_nlin_diag` (d02tzc) can be passed to this function without any modification.  
*Suggested value:*  $(n^* + 1)/2$ , where  $n^*$  is the number of mesh points used in the previous mesh as returned in the argument **nmesh** of `nag_ode_bvp_coll_nlin_diag` (d02tzc).  
*Constraint:*  $6 \leq \mathbf{nmesh} \leq (\mathbf{mxmesh} + 1)/2$ .
  
- 3: **mesh**[**mxmesh**] – const double *Input*  
*On entry:* the **nmesh** points to be used in the new initial mesh as specified by **ipmesh**.  
*Suggested value:* the argument **mesh** returned from a call to `nag_ode_bvp_coll_nlin_diag` (d02tzc).  
*Constraint:*  $\mathbf{mesh}[i_j - 1] < \mathbf{mesh}[i_{j+1} - 1]$ , for  $j = 1, 2, \dots, \mathbf{nmesh} - 1$ , the values of  $i_1, i_2, \dots, i_{\mathbf{nmesh}}$  are defined in **ipmesh**.  
**mesh**[ $i_1 - 1$ ] must contain the left boundary point,  $a$ , and **mesh**[ $i_{\mathbf{nmesh}} - 1$ ] must contain the right boundary point,  $b$ , as specified in the previous call to `nag_ode_bvp_coll_nlin_setup` (d02tvc).

- 4: **ipmesh**[**mxmesh**] – const Integer *Input*
- On entry:* specifies the points in **mesh** to be used as the new initial mesh. Let  $\{i_j : j = 1, 2, \dots, \mathbf{nmesh}\}$  be the set of array indices of **ipmesh** such that **ipmesh**[ $i_j - 1$ ] = 1 or 2 and  $1 = i_1 < i_2 < \dots < i_{\mathbf{nmesh}}$ . Then **mesh**[ $i_j - 1$ ] will be included in the new initial mesh.
- If **ipmesh**[ $i_j - 1$ ] = 1, **mesh**[ $i_j - 1$ ] will be a fixed point in the new initial mesh.
- If **ipmesh**[ $k - 1$ ] = 3 for any  $k$ , then **mesh**[ $k - 1$ ] will not be included in the new mesh.
- Suggested value:* the argument **ipmesh** returned in a call to nag\_ode\_bvp\_coll\_nlin\_diag (d02tzc).
- Constraints:*
- $$\mathbf{ipmesh}[k - 1] = 1, 2 \text{ or } 3, \text{ for } k = 1, 2, \dots, i_{\mathbf{nmesh}};$$
- $$\mathbf{ipmesh}[0] = \mathbf{ipmesh}[i_{\mathbf{nmesh}} - 1] = 1.$$
- 5: **rcomm**[*dim*] – double *Communication Array*
- Note:** the dimension, *dim*, of this array is dictated by the requirements of associated functions that must have been previously called. This array **MUST** be the same array passed as argument **rcomm** in the previous call to nag\_ode\_bvp\_coll\_nlin\_solve (d02tlc).
- On entry:* this must be the same array as supplied to nag\_ode\_bvp\_coll\_nlin\_solve (d02tlc) and **must** remain unchanged between calls.
- On exit:* contains information about the solution for use on subsequent calls to associated functions.
- 6: **icomm**[*dim*] – Integer *Communication Array*
- Note:** the dimension, *dim*, of this array is dictated by the requirements of associated functions that must have been previously called. This array **MUST** be the same array passed as argument **icomm** in the previous call to nag\_ode\_bvp\_coll\_nlin\_solve (d02tlc).
- On entry:* this must be the same array as supplied to nag\_ode\_bvp\_coll\_nlin\_solve (d02tlc) and **must** remain unchanged between calls.
- On exit:* contains information about the solution for use on subsequent calls to associated functions.
- 7: **fail** – NagError \* *Input/Output*
- The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument *<value>* had an illegal value.

### NE\_CONVERGENCE\_SOL

The solver function did not produce any results suitable for remeshing.

**NE\_INT**

An element of **ipmesh** was set to  $-1$  before **nmesh** elements containing 1 or 2 were detected.

**ipmesh**[ $i$ ]  $\neq -1, 1, 2$  or 3 for some  $i$ .

On entry, **ipmesh**[0] =  $\langle value \rangle$ .

Constraint: **ipmesh**[0] = 1.

On entry, **nmesh** =  $\langle value \rangle$ .

Constraint: **nmesh**  $\geq 6$ .

You have set the element of **ipmesh** corresponding to the last element of **mesh** to be included in the new mesh as  $\langle value \rangle$ , which is not 1.

**NE\_INT\_2**

On entry, **nmesh** =  $\langle value \rangle$  and **mxmesh** =  $\langle value \rangle$ .

Constraint: **nmesh**  $\leq (\mathbf{mxmesh} + 1)/2$ .

**NE\_INT\_CHANGED**

On entry, **mxmesh** =  $\langle value \rangle$  and **mxmesh** =  $\langle value \rangle$  in nag\_ode\_bvp\_coll\_nlin\_setup (d02tvc).

Constraint: **mxmesh** = **mxmesh** in nag\_ode\_bvp\_coll\_nlin\_setup (d02tvc).

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

**NE\_MESH\_ERROR**

The first element of array **mesh** does not coincide with the left hand end of the range previously specified.

First element of **mesh**:  $\langle value \rangle$ ; left hand of the range:  $\langle value \rangle$ .

The last point of the new mesh does not coincide with the right hand end of the range previously specified.

Last point of the new mesh:  $\langle value \rangle$ ; right hand end of the range:  $\langle value \rangle$ .

**NE\_MISSING\_CALL**

The solver function does not appear to have been called.

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.

See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

**NE\_NOT\_STRICTLY\_INCREASING**

The entries in **mesh** are not strictly increasing.

**7 Accuracy**

Not applicable.

**8 Parallelism and Performance**

nag\_ode\_bvp\_coll\_nlin\_contin (d02txc) is not threaded in any implementation.

## 9 Further Comments

For problems where sharp changes of behaviour are expected over short intervals it may be advisable to:

- cluster the mesh points where sharp changes in behaviour are expected;
- maintain fixed points in the mesh using the argument **ipmesh** to ensure that the remeshing process does not inadvertently remove mesh points from areas of known interest.

In the absence of any other information about the expected behaviour of the solution, using the values suggested in Section 5 for **nmesh**, **ipmesh** and **mesh** is strongly recommended.

## 10 Example

This example illustrates the use of continuation, solution on an infinite range, and solution of a system of two differential equations of orders 3 and 2. See also `nag_ode_bvp_coll_nlin_solve` (d02tlc), `nag_ode_bvp_coll_nlin_setup` (d02tvc), `nag_ode_bvp_coll_nlin_interp` (d02tyc) and `nag_ode_bvp_coll_nlin_diag` (d02tzc), for the illustration of other facilities.

Consider the problem of swirling flow over an infinite stationary disk with a magnetic field along the axis of rotation. See Ascher *et al.* (1988) and the references therein. After transforming from a cylindrical coordinate system  $(r, \theta, z)$ , in which the  $\theta$  component of the corresponding velocity field behaves like  $r^{-n}$ , the governing equations are

$$\begin{aligned} f''' + \frac{1}{2}(3-n)ff'' + n(f')^2 + g^2 - sf' &= \gamma^2 \\ g'' + \frac{1}{2}(3-n)fg' + (n-1)gf' - s(g-1) &= 0 \end{aligned}$$

with boundary conditions

$$f(0) = f'(0) = g(0) = 0, \quad f'(\infty) = 0, \quad g(\infty) = \gamma,$$

where  $s$  is the magnetic field strength, and  $\gamma$  is the Rossby number.

Some solutions of interest are for  $\gamma = 1$ , small  $n$  and  $s \rightarrow 0$ . An added complication is the infinite range, which we approximate by  $[0, L]$ . We choose  $n = 0.2$  and first solve for  $L = 60.0, s = 0.24$  using the initial approximations  $f(x) = -x^2e^{-x}$  and  $g(x) = 1.0 - e^{-x}$ , which satisfy the boundary conditions, on a uniform mesh of 21 points. Simple continuation on the parameters  $L$  and  $s$  using the values  $L = 120.0, s = 0.144$  and then  $L = 240.0, s = 0.0864$  is used to compute further solutions. We use the suggested values for **nmesh**, **ipmesh** and **mesh** in the call to `nag_ode_bvp_coll_nlin_contin` (d02txc) prior to a continuation call, that is only every second point of the preceding mesh is used.

The equations are first mapped onto  $[0, 1]$  to yield

$$\begin{aligned} f''' &= L^3(\gamma^2 - g^2) + L^2sf' - L\left(\frac{1}{2}(3-n)ff'' + n(f')^2\right) \\ g'' &= L^2s(g-1) - L\left(\frac{1}{2}(3-n)fg' + (n-1)f'g\right). \end{aligned}$$

### 10.1 Program Text

```
/* nag_ode_bvp_coll_nlin_contin (d02txc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nag02.h>

#define Y(I, J)    y[J*neq + I-1]
```

```

typedef struct
{
  double el, en, s;
} func_data;

#ifdef __cplusplus
extern "C"
{
#endif
  static void NAG_CALL ffun(double x, const double y[], Integer neq,
                           const Integer m[], double f[], Nag_Comm *comm);
  static void NAG_CALL fjac(double x, const double y[], Integer neq,
                           const Integer m[], double dfdy[], Nag_Comm *comm);
  static void NAG_CALL gafun(const double ya[], Integer neq,
                            const Integer m[], Integer nlbc, double ga[],
                            Nag_Comm *comm);
  static void NAG_CALL gbfun(const double yb[], Integer neq,
                            const Integer m[], Integer nrbc, double gb[],
                            Nag_Comm *comm);
  static void NAG_CALL gajac(const double ya[], Integer neq,
                            const Integer m[], Integer nlbc, double dgady[],
                            Nag_Comm *comm);
  static void NAG_CALL gbjac(const double yb[], Integer neq,
                            const Integer m[], Integer nrbc, double dgbdy[],
                            Nag_Comm *comm);
  static void NAG_CALL guess(double x, Integer neq, const Integer m[],
                            double y[], double dym[], Nag_Comm *comm);
#ifdef __cplusplus
}
#endif

int main(void)
{
  static double ruser[7] = { -1.0, -1.0, -1.0, -1.0, -1.0, -1.0, -1.0 };
  Integer exit_status = 0;
  Integer neq, mmax, nlbc, nrbc, nleft, nright;
  Integer i, iermx, ijermx, j, licomm, lrcomm, mxmesh, ncol, ncont, nmesh;
  double xsplit = 30.0;
  double dx, el, el_init, en, erm, s, s_init, xx;
  double *mesh = 0, *rcomm = 0;
  double *tol = 0, *y = 0;
  Integer *ipmesh = 0, *icomm = 0, *m = 0;
  func_data fd;
  Nag_Comm comm;
  NagError fail;

  INIT_FAIL(fail);

  printf("nag_ode_bvp_coll_nlin_contin (d02txc) Example Program Results\n\n");

  /* For communication with user-supplied functions: */
  comm.user = ruser;

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[\n] ");
#else
  scanf("%*[\n] ");
#endif
#ifdef _WIN32
  scanf_s("%" NAG_IFMT "%" NAG_IFMT "", &neq, &mmax);
#else
  scanf("%" NAG_IFMT "%" NAG_IFMT "", &neq, &mmax);
#endif
#ifdef _WIN32
  scanf_s("%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &nlbc, &nrbc);
#else
  scanf("%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &nlbc, &nrbc);
#endif
#ifdef _WIN32

```

```

    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &nleft, &nright);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &nleft, &nright);
#endif
    /* Read method parameters */
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &ncol, &mesh,
            &mxmesh);
#else
    scanf("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "%*[\n] ", &ncol, &mesh,
            &mxmesh);
#endif
    licomm = mxmesh * (11 * neq + 6);
    lrcomm = mxmesh * (109 * pow((double) neq, 2) + 78 * neq + 7);
    /* Allocate memory */
    if (!(tol = NAG_ALLOC(neq, double)) ||
        !(y = NAG_ALLOC(neq * mmax, double)) ||
        !(m = NAG_ALLOC(neq, Integer)) ||
        !(mesh = NAG_ALLOC(mxmesh, double)) ||
        !(rcomm = NAG_ALLOC(lrcomm, double)) ||
        !(ipmesh = NAG_ALLOC(mxmesh, Integer)) ||
        !(icomm = NAG_ALLOC(licomm, Integer))
        )
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
    for (i = 0; i < neq; i++) {
#ifdef _WIN32
        scanf_s("%" NAG_IFMT "", &m[i]);
#else
        scanf("%" NAG_IFMT "", &m[i]);
#endif
    }
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
    for (i = 0; i < neq; i++) {
#ifdef _WIN32
        scanf_s("%lf", &tol[i]);
#else
        scanf("%lf", &tol[i]);
#endif
    }
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
    /* Read problem (initial) parameters */
#ifdef _WIN32
    scanf_s("%lf%lf%lf%*[\n] ", &en, &el_init, &s_init);
#else
    scanf("%lf%lf%lf%*[\n] ", &en, &el_init, &s_init);
#endif
    /* Initialize data */
    el = el_init;
    s = s_init;

    /* Set data required for the user-supplied functions */
    fd.el = el;
    fd.en = en;
    fd.s = s;
    /* Associate the data structure with comm.p */
    comm.p = (Pointer) &fd;

    dx = 1.0 / (double) (nmesh - 1);
    mesh[0] = 0.0;

```

```

for (i = 1; i < nmesh - 1; i++) {
    mesh[i] = mesh[i - 1] + dx;
}
mesh[nmesh - 1] = 1.0;
ipmesh[0] = 1;
for (i = 1; i < nmesh - 1; i++) {
    ipmesh[i] = 2;
}
ipmesh[nmesh - 1] = 1;

/* nag_ode_bvp_coll_nlin_setup (d02tvc).
 * Ordinary differential equations, general nonlinear boundary value problem,
 * setup for nag_ode_bvp_coll_nlin_solve (d02tlc).
 */
nag_ode_bvp_coll_nlin_setup(neq, m, nlbc, nrbc, ncol, tol, mxmesh, nmesh,
                           mesh, ipmesh, rcomm, lrcomm, icomm, licomm,
                           &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_ode_bvp_coll_nlin_setup (d02tvc).\n%s\n",
          fail.message);
    exit_status = 1;
    goto END;
}

/* Initialize number of continuation steps in el and s */
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%*[\n] ", &ncont);
#else
    scanf("%" NAG_IFMT "%*[\n] ", &ncont);
#endif

for (j = 0; j < ncont; j++) {
    printf("\n Tolerance = %8.1e", tol[0]);
    printf("  l = %8.3f  s = %7.4f\n", el, s);
    /* Solve */

    /* nag_ode_bvp_coll_nlin_solve (d02tlc).
     * Ordinary differential equations, general nonlinear boundary value
     * problem, collocation technique.
     */
    nag_ode_bvp_coll_nlin_solve(ffun, fjac, gafun, gbfun, gajac, gbjac, guess,
                               rcomm, icomm, &comm, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_ode_bvp_coll_nlin_solve (d02tlc).\n%s\n",
              fail.message);
        exit_status = 2;
        goto END;
    }

    /* Extract mesh */

    /* nag_ode_bvp_coll_nlin_diag (d02tzc).
     * Ordinary differential equations, general nonlinear boundary value
     * problem, diagnostics for nag_ode_bvp_coll_nlin_solve (d02tlc).
     */
    nag_ode_bvp_coll_nlin_diag(mxmesh, &nmesh, mesh, ipmesh, &ermx, &iermx,
                               &ijermx, rcomm, icomm, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_ode_bvp_coll_nlin_diag (d02tzc).\n%s\n",
              fail.message);
        exit_status = 3;
        goto END;
    }

    printf("\n Used a mesh of %4" NAG_IFMT " points\n", nmesh);
    printf(" Maximum error = %10.2e", ermx);
    printf(" in interval %4" NAG_IFMT " ", iermx);
    printf(" for component %4" NAG_IFMT " \n", ijermx);
    /* Print solution components on mesh */
    printf("\n\n Solution on original interval:\n      x          f          g\n");
    /* Left side domain [0,xsplit], evaluate at nleft+1 uniform grid points. */

```



```

dx = xsplit / (double) (nleft) / el;
xx = 0.0;
for (i = 0; i <= nleft; i++) {

    /* nag_ode_bvp_coll_nlin_interp (d02tyc).
    * Ordinary differential equations, general nonlinear boundary value
    * problem, interpolation for nag_ode_bvp_coll_nlin_solve (d02t1c).
    */
    nag_ode_bvp_coll_nlin_interp(xx, y, neq, mmax, rcomm, icomm, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_ode_bvp_coll_nlin_interp (d02tyc).\n%s\n",
            fail.message);
        exit_status = 5;
        goto END;
    }

    printf("%8.2f %10.4f %10.4f \n", xx * el, Y(1, 0), Y(2, 0));
    xx = xx + dx;
}
/* Right side domain (xsplit,L], evaluate at nright uniform grid points. */
dx = (el - xsplit) / (double) (nright) / el;
xx = xsplit / el;
for (i = 0; i < nright; i++) {
    xx = MIN(1.0, xx + dx);

    /* nag_ode_bvp_coll_nlin_interp (d02tyc).
    * Ordinary differential equations, general nonlinear boundary value
    * problem, interpolation for nag_ode_bvp_coll_nlin_solve (d02t1c).
    */
    nag_ode_bvp_coll_nlin_interp(xx, y, neq, mmax, rcomm, icomm, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_ode_bvp_coll_nlin_interp (d02tyc).\n%s\n",
            fail.message);
        exit_status = 6;
        goto END;
    }

    printf("%8.2f %10.4f %10.4f \n", xx * el, Y(1, 0), Y(2, 0));
}
/* Select mesh for continuation and update continuation parameters. */
if (j < ncont - 1) {
    el = 2.0 * el;
    s = 0.6 * s;
    fd.el = el;
    fd.s = s;
    nmesh = (nmesh + 1) / 2;

    /* nag_ode_bvp_coll_nlin_contin (d02txc).
    * Ordinary differential equations, general nonlinear boundary value
    * problem, continuation facility for
    * nag_ode_bvp_coll_nlin_solve (d02t1c).
    */
    nag_ode_bvp_coll_nlin_contin(mymesh, nmesh, mesh, ipmesh, rcomm, icomm,
        &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_ode_bvp_coll_nlin_contin (d02txc).\n%s\n",
            fail.message);
        exit_status = 7;
        goto END;
    }
}
}
}
END:
NAG_FREE(mesh);
NAG_FREE(m);
NAG_FREE(tol);
NAG_FREE(rcomm);
NAG_FREE(y);
NAG_FREE(ipmesh);
NAG_FREE(icomm);

```

```

    return exit_status;
}

static void NAG_CALL ffun(double x, const double y[], Integer neq,
                          const Integer m[], double f[], Nag_Comm *comm)
{
    func_data *fd = (func_data *) comm->p;
    double el, en, s, t1, y11, y20;
    double half = 0.5;
    double one = 1.0;
    double three = 3.0;

    if (comm->user[0] == -1.0) {
        printf("(User-supplied callback ffun, first invocation.)\n");
        comm->user[0] = 0.0;
    }
    el = fd->el;
    en = fd->en;
    s = fd->s;
    t1 = half * (three - en) * Y(1, 0);
    y11 = Y(1, 1);
    y20 = Y(2, 0);
    f[0] = (pow(el, 3)) * (one - pow(y20, 2)) + (pow(el, 2)) * s * y11 -
            el * (t1 * Y(1, 2) + en * pow(y11, 2));
    f[1] = (pow(el, 2)) * s * (y20 - one) - el *
            (t1 * Y(2, 1) + (en - one) * y11 * y20);
}

static void NAG_CALL fjac(double x, const double y[], Integer neq,
                          const Integer m[], double dfdy[], Nag_Comm *comm)
{
#define DFDY(I, J, K) dfdy[I-1 + (J-1)* neq + K * neq * neq]
    func_data *fd = (func_data *) comm->p;
    double el, en, s;
    double half = 0.5;
    double one = 1.0;
    double two = 2.0;
    double three = 3.0;

    if (comm->user[1] == -1.0) {
        printf("(User-supplied callback fjac, first invocation.)\n");
        comm->user[1] = 0.0;
    }
    el = fd->el;
    en = fd->en;
    s = fd->s;
    DFDY(1, 2, 0) = -two * pow(el, 3) * Y(2, 0);
    DFDY(1, 1, 0) = -el * half * (three - en) * Y(1, 2);
    DFDY(1, 1, 1) = pow(el, 2) * s - el * two * en * Y(1, 1);
    DFDY(1, 1, 2) = -el * half * (three - en) * Y(1, 0);
    DFDY(2, 2, 0) = pow(el, 2) * s - el * (en - one) * Y(1, 1);
    DFDY(2, 2, 1) = -el * half * (three - en) * Y(1, 0);
    DFDY(2, 1, 0) = -el * half * (three - en) * Y(2, 1);
    DFDY(2, 1, 1) = -el * (en - one) * Y(2, 0);
}

static void NAG_CALL gafun(const double ya[], Integer neq, const Integer m[],
                           Integer nlbc, double ga[], Nag_Comm *comm)
{
#define YA(I, J) ya[J * neq + I-1]

    if (comm->user[2] == -1.0) {
        printf("(User-supplied callback gafun, first invocation.)\n");
        comm->user[2] = 0.0;
    }
    ga[0] = YA(1, 0);
    ga[1] = YA(1, 1);
    ga[2] = YA(2, 0);
}

static void NAG_CALL gbfun(const double yb[], Integer neq, const Integer m[],

```

```

                                Integer nrbc, double gb[], Nag_Comm *comm)
{
#define YB(I, J) yb[J * neq + I-1]

    if (comm->user[3] == -1.0) {
        printf("(User-supplied callback gbfun, first invocation.)\n");
        comm->user[3] = 0.0;
    }
    gb[0] = YB(1, 1);
    gb[1] = YB(2, 0) - 1.0;
}

static void NAG_CALL gajac(const double ya[], Integer neq, const Integer m[],
                           Integer nlbc, double dgady[], Nag_Comm *comm)
{
#define DGADY(I, J, K) dgady[I-1 + (J-1)* nlbc + K * nlbc * neq]
    double one = 1.0;

    if (comm->user[4] == -1.0) {
        printf("(User-supplied callback gajac, first invocation.)\n");
        comm->user[4] = 0.0;
    }
    DGADY(1, 1, 0) = one;
    DGADY(2, 1, 1) = one;
    DGADY(3, 2, 0) = one;
}

static void NAG_CALL gbjac(const double yb[], Integer neq, const Integer m[],
                           Integer nrbc, double dgbdy[], Nag_Comm *comm)
{
#define DGBDY(I, J, K) dgbdy[I-1 + (J-1)* nrbc + K * nrbc * neq]
    double one = 1.0;

    if (comm->user[5] == -1.0) {
        printf("(User-supplied callback gbjac, first invocation.)\n");
        comm->user[5] = 0.0;
    }
    DGBDY(1, 1, 1) = one;
    DGBDY(2, 2, 0) = one;
}

static void NAG_CALL guess(double x, Integer neq, const Integer m[],
                           double y[], double dym[], Nag_Comm *comm)
{
    func_data *fd = (func_data *) comm->p;
    double ex, expmx;
    double one = 1.0;
    double two = 2.0;

    if (comm->user[6] == -1.0) {
        printf("(User-supplied callback guess, first invocation.)\n");
        comm->user[6] = 0.0;
    }
    ex = x * fd->e1;
    expmx = exp(-ex);
    Y(1, 0) = -pow(ex, 2) * expmx;
    Y(1, 1) = (-two * ex + pow(ex, 2)) * expmx;
    Y(1, 2) = (-two + 4.0 * ex - pow(ex, 2)) * expmx;
    Y(2, 0) = one - expmx;
    Y(2, 1) = expmx;
    dym[0] = (6.0 - 6.0 * ex + pow(ex, 2)) * expmx;
    dym[1] = -expmx;
}

```

## 10.2 Program Data

```
nag_ode_bvp_coll_nlin_contin (d02txc) Example Program Data
  2  3  3  2      : neq, mmax, nlbc, nrbc
 15 10          : nleft, nright
  6 21 250      : (method parameters) ncol, nmesh, mxmesh
  3  2          : m
1.0e-5 1.0e-5   : tolerances
0.2 60.0 0.24   : (problem parameters) en, el_init, s_init
  3            : ncont
```

## 10.3 Program Results

```
nag_ode_bvp_coll_nlin_contin (d02txc) Example Program Results
```

```
Tolerance = 1.0e-05  l = 60.000  s = 0.2400
(User-supplied callback guess, first invocation.)
(User-supplied callback gafun, first invocation.)
(User-supplied callback gajac, first invocation.)
(User-supplied callback gbfun, first invocation.)
(User-supplied callback gbjac, first invocation.)
(User-supplied callback ffun, first invocation.)
(User-supplied callback fjac, first invocation.)
```

```
Used a mesh of 21 points
Maximum error = 2.66e-08 in interval 7 for component 1
```

Solution on original interval:

x	f	g
0.00	0.0000	0.0000
2.00	-0.9769	0.8011
4.00	-2.0900	1.1459
6.00	-2.6093	1.2389
8.00	-2.5498	1.1794
10.00	-2.1397	1.0478
12.00	-1.7176	0.9395
14.00	-1.5465	0.9206
16.00	-1.6127	0.9630
18.00	-1.7466	1.0068
20.00	-1.8286	1.0244
22.00	-1.8338	1.0185
24.00	-1.7956	1.0041
26.00	-1.7582	0.9940
28.00	-1.7445	0.9926
30.00	-1.7515	0.9965
33.00	-1.7695	1.0019
36.00	-1.7730	1.0018
39.00	-1.7673	0.9998
42.00	-1.7645	0.9993
45.00	-1.7659	0.9999
48.00	-1.7672	1.0002
51.00	-1.7671	1.0001
54.00	-1.7666	0.9999
57.00	-1.7665	0.9999
60.00	-1.7666	1.0000

```
Tolerance = 1.0e-05  l = 120.000  s = 0.1440
```

```
Used a mesh of 21 points
Maximum error = 6.88e-06 in interval 7 for component 2
```

Solution on original interval:

x	f	g
0.00	0.0000	0.0000
2.00	-1.1406	0.7317
4.00	-2.6531	1.1315
6.00	-3.6721	1.3250

8.00	-4.0539	1.3707
10.00	-3.8285	1.3003
12.00	-3.1339	1.1407
14.00	-2.2469	0.9424
16.00	-1.6146	0.8201
18.00	-1.5472	0.8549
20.00	-1.8483	0.9623
22.00	-2.1761	1.0471
24.00	-2.3451	1.0778
26.00	-2.3236	1.0600
28.00	-2.1784	1.0165
30.00	-2.0214	0.9775
39.00	-2.1109	1.0155
48.00	-2.0362	0.9931
57.00	-2.0709	1.0023
66.00	-2.0588	0.9995
75.00	-2.0616	1.0000
84.00	-2.0615	1.0001
93.00	-2.0611	0.9999
102.00	-2.0614	1.0000
111.00	-2.0613	1.0000
120.00	-2.0613	1.0000

Tolerance = 1.0e-05 l = 240.000 s = 0.0864

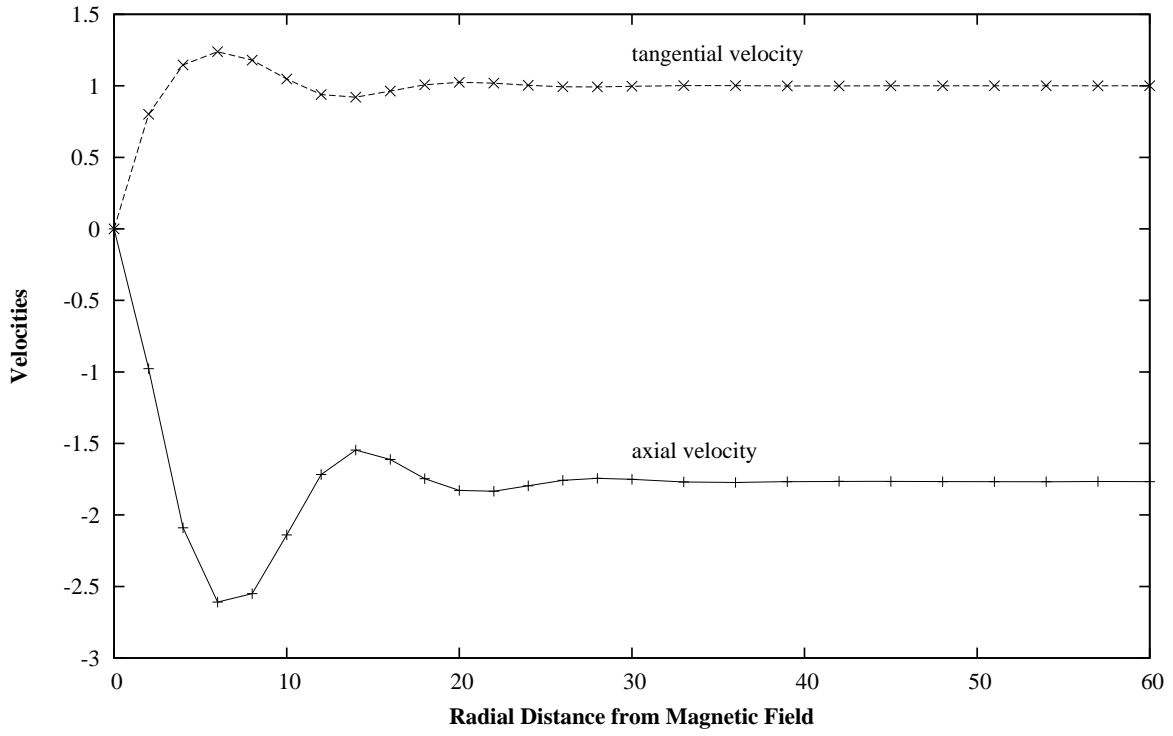
Used a mesh of 81 points

Maximum error = 3.30e-07 in interval 19 for component 2

Solution on original interval:

x	f	g
0.00	0.0000	0.0000
2.00	-1.2756	0.6404
4.00	-3.1604	1.0463
6.00	-4.7459	1.3011
8.00	-5.8265	1.4467
10.00	-6.3412	1.5036
12.00	-6.2862	1.4824
14.00	-5.6976	1.3886
16.00	-4.6568	1.2263
18.00	-3.3226	1.0042
20.00	-2.0328	0.7718
22.00	-1.4035	0.6943
24.00	-1.6603	0.8218
26.00	-2.2975	0.9928
28.00	-2.8661	1.1139
30.00	-3.1641	1.1641
51.00	-2.5307	1.0279
72.00	-2.3520	0.9919
93.00	-2.3674	0.9975
114.00	-2.3799	1.0003
135.00	-2.3800	1.0002
156.00	-2.3792	1.0000
177.00	-2.3791	1.0000
198.00	-2.3792	1.0000
219.00	-2.3792	1.0000
240.00	-2.3792	1.0000

**Example Program**  
Swirling Flow over Disc under Axial Magnetic Field  
using  $L=60$  and Magnetic Field Strength,  $s=0.24$



Swirling Flow over Disc under Axial Magnetic Field  
Continued to  $L=120$  and Magnetic Field Strength,  $s=0.144$

