NAG Library Function Document

nag_zero_cont_func_brent_binsrch (c05auc)

1 Purpose

nag_zero_cont_func_brent_binsrch (c05auc) locates a simple zero of a continuous function from a given starting value. It uses a binary search to locate an interval containing a zero of the function, then Brent's method, which is a combination of nonlinear interpolation, linear extrapolation and bisection, to locate the zero precisely.

2 Specification

```
#include <nag.h>
#include <nagc05.h>
void nag_zero_cont_func_brent_binsrch (double *x, double h, double eps,
     double eta,
     double (*f)(double x, Nag_Comm *comm),
     double *a, double *b, Nag_Comm *comm, NagError *fail)
```

3 Description

nag_zero_cont_func_brent_binsrch (c05auc) attempts to locate an interval $[a, b]$ containing a simple zero of the function $f(x)$ by a binary search starting from the initial point $x = \mathbf{x}$ and using repeated calls to nag_interval_zero_cont_func (c05avc). If this search succeeds, then the zero is determined to a user-specified accuracy by a call to nag_zero_cont_func_brent (c05ayc). The specifications of functions nag_interval_zero_cont_func (c05avc) and nag_zero_cont_func_brent (c05ayc) should be consulted for details of the methods used.

The approximation x to the zero α is determined so that at least one of the following criteria is satisfied:

(i) $|x - \alpha| \leq \mathbf{eps}$,

(ii) $|f(x)| \leq \mathbf{eta}$.

4 References

Brent R P (1973) *Algorithms for Minimization Without Derivatives* Prentice–Hall

5 Arguments

1: **x** – double * *Input/Output*

On entry: an initial approximation to the zero.

On exit: if **fail.code** = NE_NOERROR or NW_TOO_MUCH_ACC_REQUESTED, **x** is the final approximation to the zero.

If **fail.code** = NE_PROBABLE_POLE, **x** is likely to be a pole of $f(x)$.

Otherwise, **x** contains no useful information.

2: **h** – double *Input*

On entry: a step length for use in the binary search for an interval containing the zero. The maximum interval searched is $[\mathbf{x} - 256.0 \times \mathbf{h}, \mathbf{x} + 256.0 \times \mathbf{h}]$.

Constraint: **h** must be sufficiently large that $\mathbf{x} + \mathbf{h} \neq \mathbf{x}$ on the computer.

3: **eps** – double *Input*

On entry: the termination tolerance on x (see Section 3).

Constraint: **eps** > 0.0.

4: **eta** – double *Input*

On entry: a value such that if $|f(x)| \leq$ **eta**, x is accepted as the zero. **eta** may be specified as 0.0 (see Section 7).

5: **f** – function, supplied by the user *External Function*

f must evaluate the function f whose zero is to be determined.

The specification of **f** is:

```
double f (double x, Nag_Comm *comm)
```

1: **x** – double *Input*

On entry: the point at which the function must be evaluated.

2: **comm** – Nag_Comm *

Pointer to structure of type Nag_Comm; the following members are relevant to **f**.

user – double *
iuser – Integer *
p – Pointer

The type Pointer will be `void *`. Before calling nag_zero_cont_func_brent_binsrch (c05auc) you may allocate memory and initialize these pointers with various quantities for use by **f** when called from nag_zero_cont_func_brent_binsrch (c05auc) (see Section 2.3.1.1 in How to Use the NAG Library and its Documentation).

6: **a** – double * *Output*
7: **b** – double * *Output*

On exit: the lower and upper bounds respectively of the interval resulting from the binary search. If the zero is determined exactly such that $f(x) = 0.0$ or is determined so that $|f(x)| \leq$ **eta** at any stage in the calculation, then on exit **a** $=$ **b** $= x$.

8: **comm** – Nag_Comm *

The NAG communication argument (see Section 2.3.1.1 in How to Use the NAG Library and its Documentation).

9: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 2.3.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 2.7.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 2.7.5 in How to Use the NAG Library and its Documentation for further information.

NE_PROBABLE_POLE

Solution may be a pole rather than a zero.

NE_REAL

On entry, **eps** $= \langle value \rangle$.
Constraint: **eps** > 0.0.

NE_REAL_2

On entry, **x** $= \langle value \rangle$ and **h** $= \langle value \rangle$.
Constraint: $\mathbf{x} + \mathbf{h} \neq \mathbf{x}$ (to machine accuracy).

NE_ZERO_NOT_FOUND

An interval containing the zero could not be found. Increasing **h** and calling nag_zero_cont_func_brent_binsrch (c05auc) again will increase the range searched for the zero. Decreasing **h** and calling nag_zero_cont_func_brent_binsrch (c05auc) again will refine the mesh used in the search for the zero.

NW_TOO_MUCH_ACC_REQUESTED

The tolerance **eps** has been set too small for the problem being solved. However, the value **x** returned is a good approximation to the zero. **eps** $= \langle value \rangle$.

7 Accuracy

The levels of accuracy depend on the values of **eps** and **eta**. If full machine accuracy is required, they may be set very small, resulting in an exit with **fail.code** = NW_TOO_MUCH_ACC_REQUESTED, although this may involve many more iterations than a lesser accuracy. You are recommended to set **eta** $= 0.0$ and to use **eps** to control the accuracy, unless you have considerable knowledge of the size of $f(x)$ for values of x near the zero.

8 Parallelism and Performance

nag_zero_cont_func_brent_binsrch (c05auc) is not threaded in any implementation.

9 Further Comments

The time taken by nag_zero_cont_func_brent_binsrch (c05auc) depends primarily on the time spent evaluating **f** (see Section 5). The accuracy of the initial approximation **x** and the value of **h** will have a somewhat unpredictable effect on the timing.

If it is important to determine an interval of relative length less than $2 \times$ **eps** containing the zero, or if **f** is expensive to evaluate and the number of calls to **f** is to be restricted, then use of nag_interval_zero_cont_func (c05avc) followed by nag_zero_cont_func_brent_rcomm (c05azc) is recommended. Use of this combination is also recommended when the structure of the problem to be solved does not permit a simple **f** to be written: the reverse communication facilities of these functions are more flexible than the direct communication of **f** required by nag_zero_cont_func_brent_binsrch (c05auc).

If the iteration terminates with successful exit and $\mathbf{a} = \mathbf{b} = \mathbf{x}$ there is no guarantee that the value returned in **x** corresponds to a simple zero and you should check whether it does.

One way to check this is to compute the derivative of f at the point **x**, preferably analytically, or, if this is not possible, numerically, perhaps by using a central difference estimate. If $f'(\mathbf{x}) = 0.0$, then **x** must correspond to a multiple zero of f rather than a simple zero.

10 Example

This example calculates an approximation to the zero of $x - e^{-x}$ using a tolerance of $\mathbf{eps} = 1.0\text{e}{-5}$ starting from $\mathbf{x} = 1.0$ and using an initial search step $\mathbf{h} = 0.1$.

10.1 Program Text

```
/* nag_zero_cont_func_brent_binsrch (c05auc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <nagx04.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <math.h>
#include <nagc05.h>

#ifdef __cplusplus
extern "C"
{
#endif
  static double NAG_CALL f(double x, Nag_Comm *comm);
#ifdef __cplusplus
}
#endif

int main(void)
{
  /* Scalars */
  Integer exit_status = 0;
  double a, b, eps, eta, h, x;
  NagError fail;
  Nag_Comm comm;
  /* Arrays */
  static double ruser[1] = { -1.0 };

  INIT_FAIL(fail);

  printf("nag_zero_cont_func_brent_binsrch (c05auc) Example Program Results\n");

  x = 1.0;
  h = 0.1;
  eps = 1e-05;
  eta = 0.0;

  /* For communication with user-supplied functions: */
  comm.user = ruser;

  /* nag_zero_cont_func_brent_binsrch (c05auc).
```

```
   * Locates a simple zero of a continuous function of one variable,
   * binary search for an interval containing a zero.
   */
  nag_zero_cont_func_brent_binsrch(&x, h, eps, eta, f, &a, &b, &comm, &fail);
  if (fail.code == NE_NOERROR) {
    printf("Root is %13.5f\n", x);
    printf("Interval searched is [%8.5f,%8.5f]\n", a, b);
  }
  else {
    printf("%s\n", fail.message);
    if (fail.code == NE_PROBABLE_POLE ||
        fail.code == NW_TOO_MUCH_ACC_REQUESTED)
      printf("Final value = %13.5f\n", x);
    exit_status = 1;
    goto END;
  }

END:
  return exit_status;
}

static double NAG_CALL f(double x, Nag_Comm *comm)
{
  if (comm->user[0] == -1.0) {
    printf("(User-supplied callback f, first invocation.)\n");
    comm->user[0] = 0.0;
  }
  return x - exp(-x);
}
```

10.2 Program Data

None.

10.3 Program Results

```
nag_zero_cont_func_brent_binsrch (c05auc) Example Program Results
(User-supplied callback f, first invocation.)
Root is       0.56714
Interval searched is [ 0.50000, 0.90000]
```