NAG Library Function Document

nag_tsa_transf_filter (g13bbc)

1 Purpose

nag_tsa_transf_filter (g13bbc) filters a time series by a transfer function model.

2 Specification

```
#include <nag.h>
#include <nagg13.h>
void nag_tsa_transf_filter (const double y[], Integer ny,
     Nag_TransfOrder *transfv, Nag_ArimaOrder *arimas, const double par[],
     Integer npar, double cy, double b[], Integer nb, NagError *fail)
```

3 Description

From a given series $y_1, y_2, \ldots, y_n$ a new series $b_1, b_2, \ldots, b_n$ is calculated using a supplied (filtering) transfer function model according to the equation

$$b_t = \delta_1 b_{t-1} + \delta_2 b_{t-2} + \cdots + \delta_p b_{t-p} + \omega_0 y_{t-b} - \omega_1 y_{t-b-1} - \cdots - \omega_q y_{t-b-q}. \tag{1}$$

As in the use of nag_tsa_arma_filter (g13bac), large transient errors may arise in the early values of b_t due to ignorance of y_t for $t < 0$, and two possibilities are allowed.

(i) The equation (1) is applied from $t = 1 + b + q, \ldots, n$ so all terms in y_t on the right-hand side of (1) are known, the unknown set of values b_t for $t = b + q, \ldots, b + q + 1 - p$ being taken as zero.

(ii) The unknown values of y_t for $t \leq 0$ are estimated by backforecasting exactly as for nag_tsa_arma_filter (g13bac).

4 References

Box G E P and Jenkins G M (1976) *Time Series Analysis: Forecasting and Control* (Revised Edition) Holden–Day

5 Arguments

1: **y**[**ny**] – const double *Input*

On entry: the Q'_y backforecasts starting with backforecast at time $1 - Q'_y$ to backforecast at time 0 followed by the time series starting at time 1, where $Q'_y =$ **arimas.q** + **arimas.bigq** × **arimas.s**. If there are no backforecasts either because the ARIMA model for the time series is not known or because it is known but has no moving average terms, then the time series starts at the beginning of **y**.

2: **ny** – Integer *Input*

On entry: the total number of backforecasts and time series data points in array **y**.

Constraint: **ny** $\geq \max\left(1 + Q'_y, \textbf{npar}\right)$.

3: **transfv** – Nag_TransfOrder * *Input*

On entry: the orders of the transfer function model where the triplet (**transfv.nag_b**, **transfv.nag_q**, **transfv.nag_p**) corresponds to the triplet (b, q, p) as described in Section 2.3.1 in the g13 Chapter Introduction.

Constraints:

transfv.**nag_b** ≥ 0;
transfv.**nag_q** ≥ 0;
transfv.**nag_p** ≥ 0.

4: **arimas** – Nag_ArimaOrder * *Input*

On entry: if available, the orders for the filtering ARIMA model for the time series as a pointer to structure of type Nag_ArimaOrder with the following members:

p – Integer
d – Integer *Input*
q – Integer *Input*
bigp – Integer *Input*
bigd – Integer *Input*
bigq – Integer *Input*
s – Integer *Input*

On entry: these seven members of **arimas** must specify the orders vector (p, d, q, P, D, Q, s), respectively, of the ARIMA model for the output noise component.

p, q, P and Q refer, respectively, to the number of autoregressive (ϕ), moving average (θ), seasonal autoregressive (Φ) and seasonal moving average (Θ) parameters.

d, D and s refer, respectively, to the order of non-seasonal differencing, the order of seasonal differencing and the seasonal period.

If no ARIMA model for the series is to be supplied **arimas** should be set to a **NULL** pointer.

Constraints:

arimas.**p** ≥ 0;
arimas.**d** ≥ 0;
arimas.**q** ≥ 0;
arimas.**bigp** ≥ 0;
arimas.**bigd** ≥ 0;
arimas.**bigq** ≥ 0;
arimas.**s** ≥ 0;
arimas.**s** $\neq 1$;
if **arimas**.**s** $= 0$, **arimas**.**bigp** + **arimas**.**bigd** + **arimas**.**bigq** $= 0$;
if **arimas**.**s** $\neq 0$, **arimas**.**bigp** + **arimas**.**bigd** + **arimas**.**bigq** $\neq 0$.

5: **par**[**npar**] – const double *Input*

On entry: the parameters of the filtering transfer function model followed by the parameters of the ARIMA model for the time series. In the transfer function model the parameters are in the standard order of MA-like followed by AR-like operator parameters. In the ARIMA model the parameters are in the standard order of non-seasonal AR and MA followed by seasonal AR and MA.

6: **npar** – Integer *Input*

On entry: the total number of parameters held in array **par**.

Constraints:

if **arimas** is not **NULL**, **npar** = **transfv**.**nag_q** + **transfv**.**nag_p** + 1;
if **arimas** is **NULL**, **npar** = **transfv**.**nag_q** + **transfv**.**nag_p** + 1 + **arimas**.**p** + **arimas**.**q** + **arimas**.**bigp** + **arimas**.**bigq**.

7: **cy** – double *Input*

On entry: if the ARIMA model is known (i.e., **arimas** is **NULL**), **cy** must specify the constant term of the ARIMA model for the time series. If this model is not known (i.e., **arimas** is not **NULL**) then **cy** is not used.

8: **b**[**nb**] – double *Output*

On exit: the filtered output series. If the ARIMA model for the time series was known, and hence Q'_y backforecasts were supplied in **y**, then **b** contains Q'_y 'filtered' backforecasts followed by the filtered series. Otherwise, the filtered series begins at the start of **b** just as the original series began at the start of **y**. In either case, if the value of the series at time t is held in **y**$[t-1]$, then the filtered value at time t is held in **b**$[t-1]$.

9: **nb** – Integer *Input*

On entry: the dimension of the array **b**.

In addition to holding the returned filtered series, **b** is also used as an intermediate work array if the ARIMA model for the time series is known.

Constraints:

if **arimas** is not **NULL**, **nb** $\geq$ **ny**;
if **arimas** is **NULL**, **nb** $\geq$ **ny** $+ \max($**transfv**.**nag_b** $+$ **transfv**.**nag_q**, **transfv**.**nag_p**$)$.

10: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_ARRAY_SIZE

The array **b** is too small. Minimum required size: ⟨*value*⟩.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_CONSTRAINT

On entry, **arimas** $=$ ⟨*value*⟩.
Constraint: **arimas**.**bigd** ≥ 0.

On entry, **arimas** $=$ ⟨*value*⟩.
Constraint: **arimas**.**bigp** ≥ 0.

On entry, **arimas** $=$ ⟨*value*⟩.
Constraint: **arimas**.**bigq** ≥ 0.

On entry, **arimas** $=$ ⟨*value*⟩.
Constraint: **arimas**.**d** ≥ 0.

On entry, **arimas** $=$ ⟨*value*⟩.
Constraint: **arimas**.**p** ≥ 0.

On entry, **arimas** = ⟨*value*⟩.
Constraint: **arimas.q** ≥ 0.

On entry, **arimas** = ⟨*value*⟩.
Constraint: **arimas.s** $\neq 1$.

On entry, **arimas** = ⟨*value*⟩.
Constraint: **arimas.s** ≥ 0.

On entry, **arimas** = ⟨*value*⟩.
Constraint: if **arimas.s** $= 0$, **arimas.bigp** + **arimas.bigd** + **arimas.bigq** $= 0$.

On entry, **arimas** = ⟨*value*⟩.
Constraint: if **arimas.s** $\neq 0$, **arimas.bigp** + **arimas.bigd** + **arimas.bigq** $\neq 0$.

On entry, **transfv** = ⟨*value*⟩.
Constraint: **transfv.nag_b** ≥ 0.

On entry, **transfv** = ⟨*value*⟩.
Constraint: **transfv.nag_p** ≥ 0.

On entry, **transfv** = ⟨*value*⟩.
Constraint: **transfv.nag_q** ≥ 0.

NE_INT

On entry, **npar** is inconsistent with **transfv** and **arimas**: **npar** = ⟨*value*⟩.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in How to Use the NAG Library and its Documentation for further information.

NE_MODEL_PARAMS

A supplied model has invalid parameters.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in How to Use the NAG Library and its Documentation for further information.

NE_SINGULAR

The matrix used to solve for starting values for MA is singular.

NE_TIME_SERIES

The supplied time series is too short.

7 Accuracy

Accuracy and stability are high except when the AR-like parameters are close to the invertibility boundary. All calculations are performed in ***basic precision*** except for one inner product type calculation which on machines of low precision is performed in ***additional precision***.

8 Parallelism and Performance

nag_tsa_transf_filter (g13bbc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag_tsa_transf_filter (g13bbc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Notefor your implementation for any additional implementation-specific information.

9 Further Comments

If an ARIMA model is supplied, local workspace arrays of fixed lengths are allocated internally by nag_tsa_transf_filter (g13bbc). The total size of these arrays amounts to K Integer elements and $K \times (K+2)$ double elements, where $K = \textbf{transfv.nag\_p} + \textbf{arimas.p} + \textbf{arimas.d} + (\textbf{arimas.bigp} + \textbf{arimas.bigd}) \times \textbf{arimas.s}$.

The time taken by nag_tsa_transf_filter (g13bbc) is roughly proportional to the product of the length of the series and number of parameters in the filtering model with appreciable increase if an ARIMA model is supplied for the time series.

10 Example

This example reads a time series of length 296. It reads one univariate ARIMA $(1,1,0,0,1,1,12)$ model for the series and the $(0,13,12)$ filtering transfer function model. 12 initial backforecasts are required and these are calculated by a call to nag_tsa_multi_inp_model_forecast (g13bjc). The backforecasts are inserted at the start of the series and nag_tsa_transf_filter (g13bbc) is called to perform the filtering.

10.1 Program Text

```
/* nag_tsa_transf_filter (g13bbc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg13.h>

int main(void)
{
  /* Scalars */
  double a1, a2, cx, cy;
  Integer i, ii, ij, iqxd, j, k, n, nb, ni, npar, nparx, nx;
  Integer nser, npara, tdxxy, tdmrx, ldparx, tdparx;
  Integer exit_status = 0, idd = 0, ny = 0;

  /* Arrays */
  double *b = 0, *fsd = 0, *fva = 0, *par = 0, *parx = 0;
  double *x = 0, *y = 0, *rms = 0, *parxx = 0;
  Integer mr[10], mrx[7], *mrxx = 0;

  Nag_TransfOrder transfj, transfv;
  Nag_ArimaOrder arimaj, arimas;
  Nag_G13_Opt options;
  NagError fail;

  INIT_FAIL(fail);

  exit_status = 0;
```

```
  /* Initialize the options structure used by nag_tsa_multi_inp_model_forecast
   * (g13bjc) */
  /* nag_tsa_options_init (g13bxc).
   * Initialization function for option setting
   */
  nag_tsa_options_init(&options);

  printf("nag_tsa_transf_filter (g13bbc) Example Program Results\n");

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
#endif

#ifdef _WIN32
  scanf_s("%" NAG_IFMT "%*[^\n] ", &nx);
#else
  scanf("%" NAG_IFMT "%*[^\n] ", &nx);
#endif

  printf("\n");
  if (nx > 0) {
    /* Allocate array x */
    if (!(x = NAG_ALLOC(nx + 2, double)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

    for (i = 1; i <= nx; ++i)
#ifdef _WIN32
      scanf_s("%lf", &x[i - 1]);
#else
      scanf("%lf", &x[i - 1]);
#endif
#ifdef _WIN32
    scanf_s("%*[^\n] ");
#else
    scanf("%*[^\n] ");
#endif

    /* Read univariate ARIMA for series */
    for (i = 1; i <= 7; ++i)
#ifdef _WIN32
      scanf_s("%" NAG_IFMT "", &mrx[i - 1]);
#else
      scanf("%" NAG_IFMT "", &mrx[i - 1]);
#endif
#ifdef _WIN32
    scanf_s("%*[^\n] ");
#else
    scanf("%*[^\n] ");
#endif

#ifdef _WIN32
    scanf_s("%lf%*[^\n] ", &cx);
#else
    scanf("%lf%*[^\n] ", &cx);
#endif

    nparx = mrx[0] + mrx[2] + mrx[3] + mrx[5];

    arimaj.p = mrx[0];
    arimaj.d = mrx[1];
    arimaj.q = mrx[2];
    arimaj.bigp = mrx[3];
    arimaj.bigd = mrx[4];
    arimaj.bigq = mrx[5];
```

```
    arimaj.s = mrx[6];

    nser = 1;

    if (nparx > 0) {
      /* Allocate array parx */
      if (!(parx = NAG_ALLOC(nparx + nser, double)))
      {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
      }
      for (i = 1; i <= nparx; ++i)
#ifdef _WIN32
        scanf_s("%lf", &parx[i - 1]);
#else
        scanf("%lf", &parx[i - 1]);
#endif
#ifdef _WIN32
      scanf_s("%*[^\n] ");
#else
      scanf("%*[^\n] ");
#endif

      /* Read model by which to filter series */
      for (i = 1; i <= 3; ++i)
#ifdef _WIN32
        scanf_s("%" NAG_IFMT "", &mr[i - 1]);
#else
        scanf("%" NAG_IFMT "", &mr[i - 1]);
#endif
#ifdef _WIN32
      scanf_s("%*[^\n] ");
#else
      scanf("%*[^\n] ");
#endif

      transfv.nag_b = mr[0];
      transfv.nag_q = mr[1];
      transfv.nag_p = mr[2];

      npar = mr[1] + mr[2] + 1;
      if (npar > 0) {
        /* Allocate array par */
        if (!(par = NAG_ALLOC(npar + nparx, double)))
        {
          printf("Allocation failure\n");
          exit_status = -1;
          goto END;
        }
        for (i = 1; i <= npar; ++i)
#ifdef _WIN32
          scanf_s("%lf", &par[i - 1]);
#else
          scanf("%lf", &par[i - 1]);
#endif
#ifdef _WIN32
        scanf_s("%*[^\n] ");
#else
        scanf("%*[^\n] ");
#endif

        /* Initially backforecast QY values */
        /* (1) Reverse series in situ */
        n = nx / 2;
        ni = nx;
        for (i = 1; i <= n; ++i) {
          a1 = x[i - 1];
          a2 = x[ni - 1];
          x[i - 1] = a2;
          x[ni - 1] = a1;
```

```
  --ni;
}
idd = mrx[1] + mrx[4];
/* (2) Possible sign reversal for ARIMA constant */
if (idd % 2 != 0)
  cx = -cx;

/* (3) Calculate number of backforecasts required */
iqxd = mrx[2] + mrx[5] * mrx[6];
if (iqxd != 0) {
  if (!(fsd = NAG_ALLOC(iqxd, double)) ||
      !(fva = NAG_ALLOC(iqxd, double)))
  {
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
  }
  npara = nparx + nser;
  parx[npara - 1] = cx;
  tdxxy = nser;
  tdmrx = nser - 1;
  ldparx = nser - 1;
  tdparx = nser - 1;
  if (!(rms = NAG_ALLOC(nser, double)) ||
      !(parxx = NAG_ALLOC(nser, double)) ||
      !(mrxx = NAG_ALLOC(7 * nser, Integer)))
  {
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
  }

  /* nag_tsa_transf_orders (g13byc).
   * Allocates memory to transfer function model orders
   */
  nag_tsa_transf_orders(nser, &transfj, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_tsa_transf_orders (g13byc)"
           ".\n%s\n", fail.message);
    exit_status = 1;
    goto END;
  }

  rms[0] = 0;
  transfj.nag_b = 0;
  transfj.nag_q = 0;
  transfj.nag_p = 0;
  transfj.nag_r = 1;
  for (i = 1; i <= 7; ++i)
    mrxx[i - 1] = 0;
  parxx[0] = 0;

  /* Tell nag_tsa_multi_inp_model_forecast (g13bjc) not to
   * print parameters on entry */
  options.list = Nag_FALSE;

  /* nag_tsa_multi_inp_model_forecast (g13bjc).
   * Forecasting function
   */
  nag_tsa_multi_inp_model_forecast(&arimaj, nser, &transfj,
                                   parx, npara, nx, iqxd, x,
                                   tdxxy, rms, mrxx, tdmrx,
                                   parxx, ldparx, tdparx,
                                   fva, fsd, &options, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_tsa_multi_inp_model_forecast "
           "(g13bjc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
  }
}
```

```
  /* Calculate series length */
  ny = nx + iqxd;

  /* Allocate array y */
  if (!(y = NAG_ALLOC(ny, double)))
  {
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
  }

  /* Move backforecasts to start of y array */
  j = iqxd;
  for (i = 1; i <= iqxd; ++i) {
    y[i - 1] = fva[j - 1];
    --j;
  }

  /* Move series into y */
  j = iqxd + 1;
  k = nx;
  for (i = 1; i <= nx; ++i) {
    if (j > 215)
      goto END;
    y[j - 1] = x[k - 1];
    ++j;
    --k;
  }
}

/* Move ARIMA for series into mr */
for (i = 1; i <= 7; ++i)
  mr[i + 2] = mrx[i - 1];

arimas.p = mr[3];
arimas.d = mr[4];
arimas.q = mr[5];
arimas.bigp = mr[6];
arimas.bigd = mr[7];
arimas.bigq = mr[8];
arimas.s = mr[9];

/* Move parameters of ARIMA for y into par */
for (i = 1; i <= nparx; ++i)
  par[npar + i - 1] = parx[i - 1];
npar += nparx;

/* Move constant and reset sign reversal */
cy = cx;
if (idd % 2 != 0)
  cy = -cy;

/* Set parameters for call to filter routine
 * nag_tsa_transf_filter (g13bbc) */
nb = ny + MAX(mr[0] + mr[1], mr[2]);

/* Allocate array b */
if (!(b = NAG_ALLOC(nb, double)))
{
  printf("Allocation failure\n");
  exit_status = -1;
  goto END;
}

/* Filter series by call to nag_tsa_transf_filter (g13bbc) */
/* nag_tsa_transf_filter (g13bbc).
 * Multivariate time series, filtering by a transfer
 * function model
 */
nag_tsa_transf_filter(y, ny, &transfv, &arimas, par, npar, cy, b, nb,
```

```
                          &fail);
      if (fail.code != NE_NOERROR) {
        printf("Error from nag_tsa_transf_filter (g13bbc).\n%s\n",
               fail.message);
        exit_status = 1;
        goto END;
      }

      printf("                  Original        Filtered\n");
      printf(" Backforecasts    y-series         series\n");
      if (iqxd != 0) {
        ij = -iqxd;
        for (i = 1; i <= iqxd; ++i) {
          printf("%8" NAG_IFMT "%17.1f%16.1f\n", ij, y[i - 1], b[i - 1]);
          ++ij;
        }

        printf("\n");
        printf("        Filtered       Filtered"
               "       Filtered       Filtered\n");
        printf("          series         series"
               "         series         series\n");
        for (i = iqxd + 1; i <= ny; i += 4) {
          for (ii = i; ii <= MIN(ny, i + 3); ++ii) {
            printf("%5" NAG_IFMT "", ii - iqxd);
            printf("%10.1f", b[ii - 1]);
          }
          printf("\n");
        }
      }
    }
  }

END:

  /* Free the options structure used by nag_tsa_multi_inp_model_forecast
   * (g13bjc) */
  /* nag_tsa_free (g13xzc).
   * Freeing function for use with g13 option setting
   */
  nag_tsa_free(&options);

  NAG_FREE(b);
  NAG_FREE(fsd);
  NAG_FREE(fva);
  NAG_FREE(par);
  NAG_FREE(parx);
  NAG_FREE(x);
  NAG_FREE(y);
  NAG_FREE(rms);
  NAG_FREE(parxx);
  NAG_FREE(mrxx);

  return exit_status;
}
```

10.2 Program Data

```
nag_tsa_transf_filter (g13bbc) Example Program Data
   158
5312. 5402. 4960. 4717. 4383. 3828. 3665. 3718.
3744. 3994. 4150. 4064. 4324. 4256. 3986. 3670.
3292. 2952. 2765. 2813. 2850. 3085. 3256. 3213.
3514. 3386. 3205. 3124. 2804. 2536. 2445. 2649.
2761. 3183. 3456. 3529. 4067. 4079. 4082. 4029.
3887. 3684. 3707. 3923. 4068. 4557. 4975. 5197.
6054. 6471. 6277. 5529. 5059. 4539. 4236. 4305.
4299. 4478. 4561. 4470. 4712. 4512. 4129. 3942.
3572. 3149. 3026. 3141. 3145. 3322. 3384. 3373.
3630. 3555. 3413. 3127. 2966. 2685. 2642. 2789.
```

```
2867. 3032. 3125. 3176. 3359. 3265. 3053. 2915.
2690. 2518. 2523. 2737. 3074. 3671. 4355. 4648.
5232. 5349. 5228. 5172. 4932. 4637. 4642. 4930.
5033. 5223. 5482. 5560. 5960. 5929. 5697. 5583.
5316. 5039. 4972. 5169. 5138. 5316. 5409. 5375.
5803. 5736. 5643. 5416. 5059. 4810. 4937. 5166.
5187. 5348. 5483. 5626. 6077. 6033. 5996. 5860.
5499. 5210. 5421. 5609. 5586. 3663. 5829. 6005.
6693. 6792. 6966. 7227. 7089. 6823. 7286. 7621.
7758. 8000. 8393. 8592. 9186. 9175.
     1     1     0     0     1     1    12
  0.000
  0.620  0.820
     0    13    12
 1.0131  0.0806 -0.0150 -0.0150 -0.0150 -0.0150
-0.0150 -0.0150 -0.0150 -0.0150 -0.0150 -0.0150
 0.9981 -0.0956  0.0000  0.0000  0.0000  0.0000
 0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
 0.0000  0.8200
```

10.3 Program Results

```
nag_tsa_transf_filter (g13bbc) Example Program Results

                 Original        Filtered
 Backforecasts   y-series         series
     -12          5159.0          4549.2
     -11          5165.9          4550.9
     -10          4947.5          4552.8
      -9          4729.8          4554.9
      -8          4424.5          4557.4
      -7          4072.5          4560.7
      -6          3995.5          4565.0
      -5          4142.7          4571.1
      -4          4219.7          4580.0
      -3          4452.1          4593.5
      -2          4758.0          4614.3
      -1          4834.6          4647.1

        Filtered        Filtered        Filtered        Filtered
         series          series          series          series
    1    4699.2     2    4782.2     3    4552.8     4    4550.4
    5    4525.7     6    4324.8     7    4256.9     8    4169.7
    9    4127.9    10    4154.6    11    4011.3    12    3878.7
   13    3705.1    14    3619.1    15    3603.1    16    3496.1
   17    3422.6    18    3463.5    19    3349.8    20    3262.1
   21    3225.9    22    3218.1    23    3103.6    24    3023.5
   25    2905.9    26    2758.5    27    2828.2    28    2958.4
   29    2926.2    30    3019.8    31    3010.7    32    3082.8
   33    3111.7    34    3286.3    35    3279.3    36    3324.4
   37    3461.7    38    3468.3    39    3709.0    40    3839.6
   41    4004.4    42    4146.3    43    4265.3    44    4344.6
   45    4419.8    46    4647.2    47    4802.6    48    4999.5
   49    5446.0    50    5861.0    51    5855.9    52    5310.7
   53    5202.5    54    5046.6    55    4857.1    56    4812.3
   57    4740.7    58    4631.1    59    4447.5    60    4317.7
   61    4079.8    62    3833.7    63    3667.7    64    3774.8
   65    3709.9    66    3648.5    67    3645.3    68    3619.8
   69    3549.4    70    3439.2    71    3250.3    72    3209.2
   73    3005.2    74    2912.4    75    2994.1    76    2947.9
   77    3103.7    78    3168.1    79    3226.0    80    3224.1
   81    3233.0    82    3119.2    83    2992.5    84    3014.8
   85    2763.7    86    2671.3    87    2664.9    88    2778.2
   89    2823.8    90    2989.0    91    3072.2    92    3132.1
   93    3394.6    94    3717.4    95    4180.5    96    4405.9
   97    4605.2    98    4733.0    99    4830.9   100    5030.8
  101    5079.0   102    5125.0   103    5236.7   104    5392.7
  105    5396.7   106    5300.7   107    5312.1   108    5336.6
  109    5347.9   110    5331.2   111    5322.0   112    5444.8
  113    5468.7   114    5532.9   115    5555.9   116    5603.4
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 117    5483.2  118    5406.8  119    5250.5  120    5171.9
 121    5217.4  122    5162.3  123    5296.1  124    5268.2
 125    5204.9  126    5290.7  127    5500.0  128    5552.3
 129    5503.3  130    5419.2  131    5335.6  132    5447.6
 133    5495.1  134    5475.1  135    5643.8  136    5713.1
 137    5655.1  138    5691.9  139    5958.4  140    5959.0
 141    5884.8  142    3714.7  143    5877.8  144    5814.1
 145    6095.6  146    6210.7  147    6560.5  148    7013.9
 149    7174.8  150    7230.8  151    7726.7  152    7880.0
 153    7997.4  154    8428.5  155    8264.1  156    8443.1
 157    8615.4  158    8644.6
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