

## NAG Library Function Document

### nag\_durbin\_watson\_stat (g02fcc)

#### 1 Purpose

nag\_durbin\_watson\_stat (g02fcc) calculates the Durbin–Watson statistic, for a set of residuals, and the upper and lower bounds for its significance.

#### 2 Specification

```
#include <nag.h>
#include <nagg02.h>
void nag_durbin_watson_stat (Integer n, Integer p, const double res[],
    double *d, double *pd1, double *pdu, NagError *fail)
```

#### 3 Description

For the general linear regression model

$$y = X\beta + \epsilon,$$

where  $y$  is a vector of length  $n$  of the dependent variable,

$X$  is a  $n$  by  $p$  matrix of the independent variables,

$\beta$  is a vector of length  $p$  of unknown arguments,

and  $\epsilon$  is a vector of length  $n$  of unknown random errors.

The residuals are given by

$$r = y - \hat{y} = y - X\hat{\beta}$$

and the fitted values,  $\hat{y} = X\hat{\beta}$ , can be written as  $Hy$  for a  $n$  by  $n$  matrix  $H$ . Note that when a mean term is included in the model the sum of the residuals is zero. If the observations have been taken serially, that is  $y_1, y_2, \dots, y_n$  can be considered as a time series, the Durbin–Watson test can be used to test for serial correlation in the  $\epsilon_i$ , see Durbin and Watson (1950), Durbin and Watson (1951) and Durbin and Watson (1971).

The Durbin–Watson statistic is

$$d = \frac{\sum_{i=1}^{n-1} (r_{i+1} - r_i)^2}{\sum_{i=1}^n r_i^2}.$$

Positive serial correlation in the  $\epsilon_i$  will lead to a small value of  $d$  while for independent errors  $d$  will be close to 2. Durbin and Watson show that the exact distribution of  $d$  depends on the eigenvalues of the matrix  $HA$  where the matrix  $A$  is such that  $d$  can be written as

$$d = \frac{r^T A r}{r^T r}$$

and the eigenvalues of the matrix  $A$  are  $\lambda_j = (1 - \cos(\pi j/n))$ , for  $j = 1, 2, \dots, n - 1$ .

However bounds on the distribution can be obtained, the lower bound being

$$d_l = \frac{\sum_{i=1}^{n-p} \lambda_i u_i^2}{\sum_{i=1}^{n-p} u_i^2}$$

and the upper bound being

$$d_u = \frac{\sum_{i=1}^{n-p} \lambda_{i-1+p} u_i^2}{\sum_{i=1}^{n-p} u_i^2},$$

where the  $u_i$  are independent standard Normal variables. The lower tail probabilities associated with these bounds,  $p_l$  and  $p_u$ , are computed by `nag_prob_durbin_watson` (g01epc). The interpretation of the bounds is that, for a test of size (significance)  $\alpha$ , if  $p_l \leq \alpha$  the test is significant, if  $p_u > \alpha$  the test is not significant, while if  $p_l > \alpha$  and  $p_u \leq \alpha$  no conclusion can be reached.

The above probabilities are for the usual test of positive auto-correlation. If the alternative of negative auto-correlation is required, then a call to `nag_prob_durbin_watson` (g01epc) should be made with the argument **d** taking the value of  $4 - d$ ; see Newbold (1988).

## 4 References

Durbin J and Watson G S (1950) Testing for serial correlation in least squares regression. I *Biometrika* **37** 409–428

Durbin J and Watson G S (1951) Testing for serial correlation in least squares regression. II *Biometrika* **38** 159–178

Durbin J and Watson G S (1971) Testing for serial correlation in least squares regression. III *Biometrika* **58** 1–19

Granger C W J and Newbold P (1986) *Forecasting Economic Time Series* (2nd Edition) Academic Press

Newbold P (1988) *Statistics for Business and Economics* Prentice–Hall

## 5 Arguments

- 1: **n** – Integer *Input*  
*On entry:*  $n$ , the number of residuals.  
*Constraint:*  $n > p$ .
- 2: **p** – Integer *Input*  
*On entry:*  $p$ , the number of independent variables in the regression model, including the mean.  
*Constraint:*  $p \geq 1$ .
- 3: **res[n]** – const double *Input*  
*On entry:* the residuals,  $r_1, r_2, \dots, r_n$ .  
*Constraint:* the mean of the residuals  $\leq \sqrt{\epsilon}$ , where  $\epsilon =$  *machine precision*.
- 4: **d** – double \* *Output*  
*On exit:* the Durbin–Watson statistic,  $d$ .

- 5: **pdl** – double \* *Output*  
*On exit:* lower bound for the significance of the Durbin–Watson statistic,  $p_l$ .
- 6: **pdu** – double \* *Output*  
*On exit:* upper bound for the significance of the Durbin–Watson statistic,  $p_u$ .
- 7: **fail** – NagError \* *Input/Output*  
 The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 3.2.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INT

On entry,  $\mathbf{p} = \langle value \rangle$ .

Constraint:  $\mathbf{p} \geq 1$ .

### NE\_INT\_2

On entry,  $\mathbf{n} = \langle value \rangle$  and  $\mathbf{p} = \langle value \rangle$ .

Constraint:  $\mathbf{n} > \mathbf{p}$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 3.6.6 in How to Use the NAG Library and its Documentation for further information.

### NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.

See Section 3.6.5 in How to Use the NAG Library and its Documentation for further information.

### NE\_RESID\_IDEN

On entry, all residuals are identical.

### NE\_RESID\_MEAN

On entry, the mean of **res** is not approximately 0.0,  $\text{mean} = \langle value \rangle$ .

## 7 Accuracy

The probabilities are computed to an accuracy of at least 4 decimal places.

## 8 Parallelism and Performance

nag\_durbin\_watson\_stat (g02fcc) is not threaded in any implementation.

## 9 Further Comments

If the exact probabilities are required, then the first  $n - p$  eigenvalues of  $HA$  can be computed and `nag_prob_lin_chi_sq (g01jdc)` used to compute the required probabilities with the argument `c` set to 0.0 and the argument `d` set to the Durbin–Watson statistic  $d$ .

## 10 Example

A set of 10 residuals are read in and the Durbin–Watson statistic along with the probability bounds are computed and printed.

### 10.1 Program Text

```

/* nag_durbin_watson_stat (g02fcc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg02.h>

int main(void)
{
    /* Scalars */
    double d, pdl, pdu;
    Integer exit_status, i, p, n;
    NagError fail;

    /* Arrays */
    double *res = 0;

    INIT_FAIL(fail);

    exit_status = 0;
    printf("nag_durbin_watson_stat (g02fcc) Example Program Results\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif

#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%*[\n] ", &p);
#else
    scanf("%" NAG_IFMT "%*[\n] ", &p);
#endif
    n = 10;

    /* Allocate memory */
    if (!(res = NAG_ALLOC(n, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    for (i = 1; i <= n; ++i)
#ifdef _WIN32
        scanf_s("%lf", &res[i - 1]);
#else

```

```

        scanf("%lf", &res[i - 1]);
    #endif
    #ifdef _WIN32
        scanf_s("%*[^\\n] ");
    #else
        scanf("%*[^\\n] ");
    #endif

    /* nag_durbin_watson_stat (g02fcc).
     * Computes Durbin-Watson test statistic
     */
    nag_durbin_watson_stat(n, p, res, &d, &pdl, &pdu, &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_durbin_watson_stat (g02fcc).\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }

    printf("\n");
    printf(" Durbin-Watson statistic %10.4f\n\n", d);
    printf(" Lower and upper bound %10.4f%10.4f\n", pdl, pdu);
END:
    NAG_FREE(res);
    return exit_status;
}

```

## 10.2 Program Data

```

nag_durbin_watson_stat (g02fcc) Example Program Data
 2
3.735719 0.912755 0.683626 0.416693 1.9902
-0.444816 -1.283088 -3.666035 -0.426357 -1.918697

```

## 10.3 Program Results

```

nag_durbin_watson_stat (g02fcc) Example Program Results

Durbin-Watson statistic      0.9238

Lower and upper bound      0.0610      0.0060

```

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