

## NAG Library Function Document

### nag\_deviates\_chi\_sq (g01fcc)

#### 1 Purpose

nag\_deviates\_chi\_sq (g01fcc) returns the deviate associated with the given lower tail probability of the  $\chi^2$ -distribution with real degrees of freedom.

#### 2 Specification

```
#include <nag.h>
#include <nagg01.h>
double nag_deviates_chi_sq (double p, double df, NagError *fail)
```

#### 3 Description

The deviate,  $x_p$ , associated with the lower tail probability  $p$  of the  $\chi^2$ -distribution with  $\nu$  degrees of freedom is defined as the solution to

$$P(X \leq x_p : \nu) = p = \frac{1}{2^{\nu/2} \Gamma(\nu/2)} \int_0^{x_p} e^{-X/2} X^{\nu/2-1} dX, \quad 0 \leq x_p < \infty; \nu > 0.$$

The required  $x_p$  is found by using the relationship between a  $\chi^2$ -distribution and a gamma distribution, i.e., a  $\chi^2$ -distribution with  $\nu$  degrees of freedom is equal to a gamma distribution with scale parameter 2 and shape parameter  $\nu/2$ .

For very large values of  $\nu$ , greater than  $10^5$ , Wilson and Hilferty's normal approximation to the  $\chi^2$  is used; see Kendall and Stuart (1969).

#### 4 References

Best D J and Roberts D E (1975) Algorithm AS 91. The percentage points of the  $\chi^2$  distribution *Appl. Statist.* **24** 385–388

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

Kendall M G and Stuart A (1969) *The Advanced Theory of Statistics (Volume 1)* (3rd Edition) Griffin

#### 5 Arguments

- 1: **p** – double *Input*  
*On entry:*  $p$ , the lower tail probability from the required  $\chi^2$ -distribution.  
*Constraint:*  $0.0 \leq \mathbf{p} < 1.0$ .
- 2: **df** – double *Input*  
*On entry:*  $\nu$ , the degrees of freedom of the  $\chi^2$ -distribution.  
*Constraint:* **df** > 0.0.
- 3: **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

On any of the error conditions listed below except **fail.code** = NE\_ALG\_NOT\_CONV nag\_deviates\_chi\_sq (g01fcc) returns 0.0.

### NE\_ALG\_NOT\_CONV

The algorithm has failed to converge in  $\langle value \rangle$  iterations. The result should be a reasonable approximation.

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 3.2.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_GAM\_NOT\_CONV

The series used to calculate the gamma function has failed to converge. This is an unlikely error exit.

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 3.6.6 in How to Use the NAG Library and its Documentation for further information.

### NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.

See Section 3.6.5 in How to Use the NAG Library and its Documentation for further information.

### NE\_PROBAB\_CLOSE\_TO\_TAIL

The probability is too close to 0.0 or 1.0.

### NE\_REAL\_ARG\_GE

On entry,  $p = \langle value \rangle$ .

Constraint:  $p < 1.0$ .

### NE\_REAL\_ARG\_LE

On entry,  $df = \langle value \rangle$ .

Constraint:  $df > 0.0$ .

### NE\_REAL\_ARG\_LT

On entry,  $p = \langle value \rangle$ .

Constraint:  $p \geq 0.0$ .

## 7 Accuracy

The results should be accurate to five significant digits for most argument values. Some accuracy is lost for  $p$  close to 0.0.

## 8 Parallelism and Performance

nag\_deviates\_chi\_sq (g01fcc) is not threaded in any implementation.

## 9 Further Comments

For higher accuracy the relationship described in Section 3 may be used and a direct call to `nag_deviates_gamma_dist` (g01fcc) made.

## 10 Example

This example reads lower tail probabilities for several  $\chi^2$ -distributions, and calculates and prints the corresponding deviates until the end of data is reached.

### 10.1 Program Text

```

/* nag_deviates_chi_sq (g01fcc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{
    Integer exit_status = 0;
    double df, p, x;
    NagError fail;

    INIT_FAIL(fail);

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
    printf("nag_deviates_chi_sq (g01fcc) Example Program Results\n");
    printf("      p          df          x\n\n");
#ifdef _WIN32
    while (scanf_s("%lf %lf", &p, &df) != EOF)
#else
    while (scanf("%lf %lf", &p, &df) != EOF)
#endif
    {
        /* nag_deviates_chi_sq (g01fcc).
         * Deviates for the chi^2 distribution
         */
        x = nag_deviates_chi_sq(p, df, &fail);
        if (fail.code != NE_NOERROR) {
            printf("Error from nag_deviates_chi_sq (g01fcc).\n%s\n", fail.message);
            exit_status = 1;
            goto END;
        }
        printf("%8.3f%8.3f%8.3f\n", p, df, x);
    }

END:
    return exit_status;
}

```

## 10.2 Program Data

```
nag_deviates_chi_sq (g01fcc) Example Program Data
0.0100 20.0
0.4279 7.50
0.8694 45.0
```

## 10.3 Program Results

```
nag_deviates_chi_sq (g01fcc) Example Program Results
      p      df      x
0.010  20.000   8.260
0.428   7.500   6.200
0.869  45.000  55.759
```

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