NAG Library Function Document

nag_deviates_students_t (g01fbc)

1 Purpose

nag_deviates_students_t (g01fbc) returns the deviate associated with the given tail probability of Student's t-distribution with real degrees of freedom.

2 Specification

```
#include <nag.h>
#include <nagg01.h>
double nag_deviates_students_t (Nag_TailProbability tail, double p,
       double df, NagError *fail)
```

3 Description

The deviate, t_p associated with the lower tail probability, p, of the Student's t-distribution with ν degrees of freedom is defined as the solution to

$$P\big(T < t_p : \nu\big) = p = \frac{\Gamma((\nu+1)/2)}{\sqrt{\nu\pi}\Gamma(\nu/2)} \int_{-\infty}^{t_p} \left(1 + \frac{T^2}{\nu}\right)^{-(\nu+1)/2} dT, \quad \nu \geq 1; \; -\infty < t_p < \infty.$$

For $\nu = 1$ or 2 the integral equation is easily solved for t_p.

For other values of $\nu < 3$ a transformation to the beta distribution is used and the result obtained from nag_deviates_beta (g01fec).

For $\nu \geq 3$ an inverse asymptotic expansion of Cornish–Fisher type is used. The algorithm is described by Hill (1970).

4 References

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

Hill G W (1970) Student's t-distribution *Comm. ACM* **13(10)** 617–619

5 Arguments

1: **tail** – Nag_TailProbability *Input*

On entry: indicates which tail the supplied probability represents.

tail = Nag_UpperTail
The upper tail probability, i.e., $P\big(T \geq t_p : \nu\big)$.

tail = Nag_LowerTail
The lower tail probability, i.e., $P\big(T \leq t_p : \nu\big)$.

tail = Nag_TwoTailSignif
The two tail (significance level) probability, i.e., $P\big(T \geq \big|t_p\big| : \nu\big) + P\big(T \leq -\big|t_p\big| : \nu\big)$.

tail = Nag_TwoTailConfid
The two tail (confidence interval) probability, i.e., $P\big(T \leq \big|t_p\big| : \nu\big) - P\big(T \leq -\big|t_p\big| : \nu\big)$.

Constraint: **tail** = Nag_UpperTail, Nag_LowerTail, Nag_TwoTailSignif or Nag_TwoTailConfid.

2: **p** – double *Input*

On entry: p, the probability from the required Student's t-distribution as defined by **tail**.

Constraint: $0.0 < \mathbf{p} < 1.0$.

3: **df** – double *Input*

On entry: ν, the degrees of freedom of the Student's t-distribution.

Constraint: $\mathbf{df} \geq 1.0$.

4: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

On any of the error conditions listed below except **fail**.**code** = NE_SOL_NOT_CONV nag_deviates_students_t (g01fbc) returns 0.0.

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in How to Use the NAG Library and its Documentation for further information.

NE_REAL_ARG_GE

On entry, $\mathbf{p} = \langle value \rangle$.
Constraint: $\mathbf{p} < 1.0$.

NE_REAL_ARG_LE

On entry, $\mathbf{p} = \langle value \rangle$.
Constraint: $\mathbf{p} > 0.0$.

NE_REAL_ARG_LT

On entry, $\mathbf{df} = \langle value \rangle$.
Constraint: $\mathbf{df} \geq 1.0$.

NE_SOL_NOT_CONV

The solution has failed to converge. However, the result should be a reasonable approximation.

7 Accuracy

The results should be accurate to five significant digits, for most argument values. The error behaviour for various argument values is discussed in Hill (1970).

8 Parallelism and Performance

nag_deviates_students_t (g01fbc) is not threaded in any implementation.

9 Further Comments

The value t_p may be calculated by using the transformation described in Section 3 and using nag_deviates_beta (g01fec). This function allows you to set the required accuracy.

10 Example

This example reads the probability, the tail that probability represents and the degrees of freedom for a number of Student's t-distributions and computes the corresponding deviates.

10.1 Program Text

```
/* nag_deviates_students_t (g01fbc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 *
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{
  Integer exit_status = 0;
  double df, p, t;
  int i;
  static Nag_TailProbability tail[] = { Nag_LowerTail, Nag_UpperTail,
    Nag_TwoTailSignif, Nag_TwoTailConfid
  };
  static const char *tailmess[] = { "Nag_LowerTail", "Nag_UpperTail",
    "Nag_TwoTailSignif",
    "Nag_TwoTailConfid"
  };
  NagError fail;

  INIT_FAIL(fail);

  printf("nag_deviates_students_t (g01fbc) Example Program Results\n\n");
  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif
  printf("    p      df          tail                  t\n\n");
#ifdef _WIN32
  while (scanf_s("%lf %lf %d", &p, &df, &i) != EOF)
#else
  while (scanf("%lf %lf %d", &p, &df, &i) != EOF)
#endif
  {
```

```
    /* nag_deviates_students_t (g01fbc).
     * Deviates for Student's t-distribution
     */
    t = nag_deviates_students_t(tail[i], p, df, &fail);
    if (fail.code != NE_NOERROR) {
      printf("Error from nag_deviates_students_t (g01fbc).\n%s\n",
             fail.message);
      exit_status = 1;
      goto END;
    }
    printf("%8.3f%8.3f   %-19s  %8.3f\n", p, df, tailmess[i], t);
  }

END:
  return exit_status;
}
```

10.2 Program Data

```
nag_deviates_students_t (g01fbc) Example Program Data
0.0100  20.0  2
0.01     7.5  0
0.99    45.0  3
```

10.3 Program Results

```
nag_deviates_students_t (g01fbc) Example Program Results

    p      df          tail                  t

   0.010  20.000   Nag_TwoTailSignif       2.845
   0.010   7.500   Nag_LowerTail          -2.943
   0.990  45.000   Nag_TwoTailConfid       2.690
```