

## NAG Library Function Document

### nag\_prob\_students\_t (g01ebc)

#### 1 Purpose

nag\_prob\_students\_t (g01ebc) returns the lower tail, upper tail or two tail probability for the Student's  $t$ -distribution with real degrees of freedom.

#### 2 Specification

```
#include <nag.h>
#include <nagg01.h>
double nag_prob_students_t (Nag_TailProbability tail, double t, double df,
                             NagError *fail)
```

#### 3 Description

The lower tail probability for the Student's  $t$ -distribution with  $\nu$  degrees of freedom,  $P(T \leq t : \nu)$  is defined by:

$$P(T \leq t : \nu) = \frac{\Gamma((\nu + 1)/2)}{\sqrt{\pi\nu}\Gamma(\nu/2)} \int_{-\infty}^t \left[1 + \frac{T^2}{\nu}\right]^{-(\nu+1)/2} dT, \quad \nu \geq 1.$$

Computationally, there are two situations:

- (i) when  $\nu < 20$ , a transformation of the beta distribution,  $P_\beta(B \leq \beta : a, b)$  is used

$$P(T \leq t : \nu) = \frac{1}{2}P_\beta\left(B \leq \frac{\nu}{\nu + t^2} : \nu/2, \frac{1}{2}\right) \quad \text{when } t < 0.0$$

or

$$P(T \leq t : \nu) = \frac{1}{2} + \frac{1}{2}P_\beta\left(B \geq \frac{\nu}{\nu + t^2} : \nu/2, \frac{1}{2}\right) \quad \text{when } t > 0.0;$$

- (ii) when  $\nu \geq 20$ , an asymptotic normalizing expansion of the Cornish–Fisher type is used to evaluate the probability, see Hill (1970).

#### 4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

Hill G W (1970) Student's  $t$ -distribution *Comm. ACM* **13**(10) 617–619

#### 5 Arguments

1: **tail** – Nag\_TailProbability *Input*

*On entry:* indicates which tail the returned probability should represent.

**tail** = Nag\_UpperTail

The upper tail probability is returned, i.e.,  $P(T \geq t : \nu)$ .

**tail** = Nag\_TwoTailSignif

The two tail (significance level) probability is returned, i.e.,  $P(T \geq |t| : \nu) + P(T \leq -|t| : \nu)$ .

**tail** = Nag\_TwoTailConfid

The two tail (confidence interval) probability is returned, i.e.,  $P(T \leq |t| : \nu) - P(T \leq -|t| : \nu)$ .

**tail** = Nag\_LowerTail

The lower tail probability is returned, i.e.,  $P(T \leq t : \nu)$ .

*Constraint:* **tail** = Nag\_UpperTail, Nag\_TwoTailSignif, Nag\_TwoTailConfid or Nag\_LowerTail.

- 2: **t** – double *Input*  
*On entry:*  $t$ , the value of the Student's  $t$  variate.
- 3: **df** – double *Input*  
*On entry:*  $\nu$ , the degrees of freedom of the Student's  $t$ -distribution.  
*Constraint:* **df**  $\geq$  1.0.
- 4: **fail** – NagError \* *Input/Output*  
 The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 3.2.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 3.6.6 in How to Use the NAG Library and its Documentation for further information.

### NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.

See Section 3.6.5 in How to Use the NAG Library and its Documentation for further information.

### NE\_REAL\_ARG\_LT

On entry, **df** =  $\langle value \rangle$ .

*Constraint:* **df**  $\geq$  1.0.

## 7 Accuracy

The computed probability should be accurate to five significant places for reasonable probabilities but there will be some loss of accuracy for very low probabilities (less than  $10^{-10}$ ), see Hastings and Peacock (1975).

## 8 Parallelism and Performance

nag\_prob\_students\_t (g01ebc) is not threaded in any implementation.

## 9 Further Comments

The probabilities could also be obtained by using the appropriate transformation to a beta distribution (see Abramowitz and Stegun (1972)) and using `nag_prob_beta_dist` (g01eec). This function allows you to set the required accuracy.

## 10 Example

This example reads values from, and degrees of freedom for Student's  $t$ -distributions along with the required tail. The probabilities are calculated and printed until the end of data is reached.

### 10.1 Program Text

```

/* nag_prob_students_t (g01ebc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{
    Integer exit_status = 0;
    double df, prob, t;
    int i;
    static Nag_TailProbability tail[4] = { Nag_LowerTail, Nag_UpperTail,
        Nag_TwoTailSignif, Nag_TwoTailConfid
    };
    static const char *tailmess[] = { "Nag_LowerTail", "Nag_UpperTail",
        "Nag_TwoTailSignif",
        "Nag_TwoTailConfid"
    };
    NagError fail;

    INIT_FAIL(fail);

    printf("nag_prob_students_t (g01ebc) Example Program Results\n\n");
    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n]");
#else
    scanf("%*[\n]");
#endif
    printf("      t      df      prob      tail\n\n");
#ifdef _WIN32
    while (scanf_s("%lf %lf %d\n", &t, &df, &i) != EOF)
#else
    while (scanf("%lf %lf %d\n", &t, &df, &i) != EOF)
#endif
    {
        /* nag_prob_students_t (g01ebc).
         * Probabilities for Student's t-distribution
         */
        prob = nag_prob_students_t(tail[i], t, df, &fail);
        if (fail.code != NE_NOERROR) {
            printf("Error from nag_prob_students_t (g01ebc).\n%s\n", fail.message);
            exit_status = 1;
            goto END;
        }
        printf(" %6.3f%8.3f%8.4f %s\n", t, df, prob, tailmess[i]);
    }
}

```

```
    }  
END:  
    return exit_status;  
}
```

## 10.2 Program Data

```
nag_prob_students_t (g01ebc) Example Program Data  
0.85  20.0  0  
0.85  20.0  2  
0.85  20.0  3  
0.85  20.0  1
```

## 10.3 Program Results

```
nag_prob_students_t (g01ebc) Example Program Results
```

t	df	prob	tail
0.850	20.000	0.7973	Nag_LowerTail
0.850	20.000	0.4054	Nag_TwoTailSignif
0.850	20.000	0.5946	Nag_TwoTailConfid
0.850	20.000	0.2027	Nag_UpperTail

---