

# NAG Library Function Document

## nag\_zgehrd (f08nsc)

### 1 Purpose

nag\_zgehrd (f08nsc) reduces a complex general matrix to Hessenberg form.

### 2 Specification

```
#include <nag.h>
#include <nagf08.h>

void nag_zgehrd (Nag_OrderType order, Integer n, Integer ilo, Integer ihi,
                 Complex a[], Integer pda, Complex tau[], NagError *fail)
```

### 3 Description

nag\_zgehrd (f08nsc) reduces a complex general matrix  $A$  to upper Hessenberg form  $H$  by a unitary similarity transformation:  $A = QHQ^H$ .  $H$  has real subdiagonal elements.

The matrix  $Q$  is not formed explicitly, but is represented as a product of elementary reflectors (see the f08 Chapter Introduction for details). Functions are provided to work with  $Q$  in this representation (see Section 9).

The function can take advantage of a previous call to nag\_zgebal (f08nvc), which may produce a matrix with the structure:

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ & A_{22} & A_{23} \\ & & A_{33} \end{pmatrix}$$

where  $A_{11}$  and  $A_{33}$  are upper triangular. If so, only the central diagonal block  $A_{22}$ , in rows and columns  $i_{lo}$  to  $i_{hi}$ , needs to be reduced to Hessenberg form (the blocks  $A_{12}$  and  $A_{23}$  will also be affected by the reduction). Therefore the values of  $i_{lo}$  and  $i_{hi}$  determined by nag\_zgebal (f08nvc) can be supplied to the function directly. If nag\_zgebal (f08nvc) has not previously been called however, then  $i_{lo}$  must be set to 1 and  $i_{hi}$  to  $n$ .

### 4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

### 5 Arguments

1: **order** – Nag\_OrderType *Input*

*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.

2: **n** – Integer *Input*

*On entry:*  $n$ , the order of the matrix  $A$ .

*Constraint:*  $n \geq 0$ .

- 3: **ilo** – Integer *Input*  
 4: **ihi** – Integer *Input*

*On entry:* if  $A$  has been output by nag\_zgebal (f08nvc), then **ilo** and **ihi** **must** contain the values returned by that function. Otherwise, **ilo** must be set to 1 and **ihi** to **n**.

*Constraints:*

if  $n > 0$ ,  $1 \leq \mathbf{ilo} \leq \mathbf{ihi} \leq n$ ;  
 if  $n = 0$ ,  $\mathbf{ilo} = 1$  and  $\mathbf{ihi} = 0$ .

- 5: **a**[*dim*] – Complex *Input/Output*

**Note:** the dimension, *dim*, of the array **a** must be at least  $\max(1, \mathbf{pda} \times n)$ .

The (*i*, *j*)th element of the matrix  $A$  is stored in

$\mathbf{a}[(j-1) \times \mathbf{pda} + i - 1]$  when **order** = Nag\_ColMajor;  
 $\mathbf{a}[(i-1) \times \mathbf{pda} + j - 1]$  when **order** = Nag\_RowMajor.

*On entry:* the  $n$  by  $n$  general matrix  $A$ .

*On exit:* **a** is overwritten by the upper Hessenberg matrix  $H$  and details of the unitary matrix  $Q$ . The subdiagonal elements of  $H$  are real.

- 6: **pda** – Integer *Input*

*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **a**.

*Constraint:*  $\mathbf{pda} \geq \max(1, n)$ .

- 7: **tau**[*dim*] – Complex *Output*

**Note:** the dimension, *dim*, of the array **tau** must be at least  $\max(1, n - 1)$ .

*On exit:* further details of the unitary matrix  $Q$ .

- 8: **fail** – NagError \* *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 3.2.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle \text{value} \rangle$  had an illegal value.

### NE\_INT

On entry,  $n = \langle \text{value} \rangle$ .

Constraint:  $n \geq 0$ .

On entry,  $\mathbf{pda} = \langle \text{value} \rangle$ .

Constraint:  $\mathbf{pda} > 0$ .

**NE\_INT\_2**

On entry, **pda** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .  
 Constraint: **pda**  $\geq$   $\max(1, \mathbf{n})$ .

**NE\_INT\_3**

On entry, **n** =  $\langle value \rangle$ , **ilo** =  $\langle value \rangle$  and **ihi** =  $\langle value \rangle$ .  
 Constraint: if **n** > 0,  $1 \leq \mathbf{ilo} \leq \mathbf{ihi} \leq \mathbf{n}$ ;  
 if **n** = 0, **ilo** = 1 and **ihi** = 0.

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.  
 See Section 3.6.6 in How to Use the NAG Library and its Documentation for further information.

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.  
 See Section 3.6.5 in How to Use the NAG Library and its Documentation for further information.

**7 Accuracy**

The computed Hessenberg matrix  $H$  is exactly similar to a nearby matrix  $(A + E)$ , where

$$\|E\|_2 \leq c(n)\epsilon\|A\|_2,$$

$c(n)$  is a modestly increasing function of  $n$ , and  $\epsilon$  is the *machine precision*.

The elements of  $H$  themselves may be sensitive to small perturbations in  $A$  or to rounding errors in the computation, but this does not affect the stability of the eigenvalues, eigenvectors or Schur factorization.

**8 Parallelism and Performance**

nag\_zgehrd (f08nsc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

**9 Further Comments**

The total number of real floating-point operations is approximately  $\frac{8}{3}q^2(2q + 3n)$ , where  $q = i_{hi} - i_{lo}$ ; if  $i_{lo} = 1$  and  $i_{hi} = n$ , the number is approximately  $\frac{40}{3}n^3$ .

To form the unitary matrix  $Q$  nag\_zgehrd (f08nsc) may be followed by a call to nag\_zunghr (f08ntc):

```
nag_zunghr(order, n, ilo, ihi, &a, pda, tau, &fail)
```

To apply  $Q$  to an  $m$  by  $n$  complex matrix  $C$  nag\_zgehrd (f08nsc) may be followed by a call to nag\_zunmhr (f08nuc). For example,

```
nag_zunmhr(order, Nag_LeftSide, Nag_NoTrans, m, n, ilo, ihi, &a, pda,
tau, &c, pdc, &fail)
```

forms the matrix product  $QC$ .

The real analogue of this function is nag\_dgehrd (f08nec).

## 10 Example

This example computes the upper Hessenberg form of the matrix  $A$ , where

$$A = \begin{pmatrix} -3.97 - 5.04i & -4.11 + 3.70i & -0.34 + 1.01i & 1.29 - 0.86i \\ 0.34 - 1.50i & 1.52 - 0.43i & 1.88 - 5.38i & 3.36 + 0.65i \\ 3.31 - 3.85i & 2.50 + 3.45i & 0.88 - 1.08i & 0.64 - 1.48i \\ -1.10 + 0.82i & 1.81 - 1.59i & 3.25 + 1.33i & 1.57 - 3.44i \end{pmatrix}.$$

### 10.1 Program Text

```

/* nag_zgehrd (f08nsc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf08.h>
#include <nagx04.h>

int main(void)
{
    /* Scalars */
    Integer i, j, n, pda, tau_len;
    Integer exit_status = 0;
    NagError fail;
    Nag_OrderType order;
    /* Arrays */
    Complex *a = 0, *tau = 0;

#ifdef NAG_COLUMN_MAJOR
#define A(I, J) a[(J - 1) * pda + I - 1]
    order = Nag_ColMajor;
#else
#define A(I, J) a[(I - 1) * pda + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_zgehrd (f08nsc) Example Program Results\n\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%*[\n] ", &n);
#else
    scanf("%" NAG_IFMT "%*[\n] ", &n);
#endif
#ifdef NAG_COLUMN_MAJOR
    pda = n;
#else
    pda = n;
#endif
    tau_len = n - 1;

    /* Allocate memory */
    if (!(a = NAG_ALLOC(n * n, Complex)) ||
        !(tau = NAG_ALLOC(tau_len, Complex)))

```

```

{
  printf("Allocation failure\n");
  exit_status = -1;
  goto END;
}

/* Read A from data file */
for (i = 1; i <= n; ++i) {
  for (j = 1; j <= n; ++j)
#ifdef _WIN32
    scanf_s(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
#else
    scanf(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
#endif
  }
#ifdef _WIN32
  scanf_s("%*[\n] ");
#else
  scanf("%*[\n] ");
#endif

/* Reduce A to upper Hessenberg form */
/* nag_zghehd (f08nsc).
 * Unitary reduction of complex general matrix to upper
 * Hessenberg form
 */
nag_zghehd(order, n, 1, n, a, pda, tau, &fail);
if (fail.code != NE_NOERROR) {
  printf("Error from nag_zghehd (f08nsc).\n%s\n", fail.message);
  exit_status = 1;
  goto END;
}

/* Set the elements below the first subdiagonal to zero */
for (i = 1; i <= n - 2; ++i) {
  for (j = i + 2; j <= n; ++j)
    A(j, i).re = 0.0, A(j, i).im = 0.0;
}

/* Print upper Hessenberg form */
/* nag_gen_complx_mat_print_comp (x04dbc).
 * Print complex general matrix (comprehensive)
 */
fflush(stdout);
nag_gen_complx_mat_print_comp(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n,
                              n, a, pda, Nag_BracketForm, "%7.4f",
                              "Upper Hessenberg form", Nag_IntegerLabels, 0,
                              Nag_IntegerLabels, 0, 80, 0, 0, &fail);
if (fail.code != NE_NOERROR) {
  printf("Error from nag_gen_complx_mat_print_comp (x04dbc).\n%s\n",
        fail.message);
  exit_status = 1;
  goto END;
}
END:
  NAG_FREE(a);
  NAG_FREE(tau);
  return exit_status;
}

```

## 10.2 Program Data

```

nag_zghehd (f08nsc) Example Program Data
4                                     :Value of N
(-3.97,-5.04) (-4.11, 3.70) (-0.34, 1.01) ( 1.29,-0.86)
( 0.34,-1.50) ( 1.52,-0.43) ( 1.88,-5.38) ( 3.36, 0.65)
( 3.31,-3.85) ( 2.50, 3.45) ( 0.88,-1.08) ( 0.64,-1.48)
(-1.10, 0.82) ( 1.81,-1.59) ( 3.25, 1.33) ( 1.57,-3.44) :End of matrix A

```

### 10.3 Program Results

nag\_zgehrd (f08nsc) Example Program Results

Upper Hessenberg form

	1	2	3	4
1	(-3.9700, -5.0400)	(-1.1318, -2.5693)	(-4.6027, -0.1426)	(-1.4249, 1.7330)
2	(-5.4797, 0.0000)	( 1.8585, -1.5502)	( 4.4145, -0.7638)	(-0.4805, -1.1976)
3	( 0.0000, 0.0000)	( 6.2673, 0.0000)	(-0.4504, -0.0290)	(-1.3467, 1.6579)
4	( 0.0000, 0.0000)	( 0.0000, 0.0000)	(-3.5000, 0.0000)	( 2.5619, -3.3708)

---