

# NAG Library Function Document

## nag\_dsytrd (f08fec)

### 1 Purpose

nag\_dsytrd (f08fec) reduces a real symmetric matrix to tridiagonal form.

### 2 Specification

```
#include <nag.h>
#include <nagf08.h>

void nag_dsytrd (Nag_OrderType order, Nag_UploType uplo, Integer n,
                double a[], Integer pda, double d[], double e[], double tau[],
                NagError *fail)
```

### 3 Description

nag\_dsytrd (f08fec) reduces a real symmetric matrix  $A$  to symmetric tridiagonal form  $T$  by an orthogonal similarity transformation:  $A = QTQ^T$ .

The matrix  $Q$  is not formed explicitly but is represented as a product of  $n - 1$  elementary reflectors (see the f08 Chapter Introduction for details). Functions are provided to work with  $Q$  in this representation (see Section 9).

### 4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

### 5 Arguments

- 1: **order** – Nag\_OrderType *Input*  
*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.  
*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.
- 2: **uplo** – Nag\_UploType *Input*  
*On entry:* indicates whether the upper or lower triangular part of  $A$  is stored.  
**uplo** = Nag\_Upper  
 The upper triangular part of  $A$  is stored.  
**uplo** = Nag\_Lower  
 The lower triangular part of  $A$  is stored.  
*Constraint:* **uplo** = Nag\_Upper or Nag\_Lower.
- 3: **n** – Integer *Input*  
*On entry:*  $n$ , the order of the matrix  $A$ .  
*Constraint:* **n**  $\geq$  0.

- 4: **a**[*dim*] – double *Input/Output*  
**Note:** the dimension, *dim*, of the array **a** must be at least  $\max(1, \mathbf{pda} \times \mathbf{n})$ .  
*On entry:* the *n* by *n* symmetric matrix *A*.  
If **order** = Nag\_ColMajor,  $A_{ij}$  is stored in **a**[(*j* – 1) × **pda** + *i* – 1].  
If **order** = Nag\_RowMajor,  $A_{ij}$  is stored in **a**[(*i* – 1) × **pda** + *j* – 1].  
If **uplo** = Nag\_Upper, the upper triangular part of *A* must be stored and the elements of the array below the diagonal are not referenced.  
If **uplo** = Nag\_Lower, the lower triangular part of *A* must be stored and the elements of the array above the diagonal are not referenced.  
*On exit:* **a** is overwritten by the tridiagonal matrix *T* and details of the orthogonal matrix *Q* as specified by **uplo**.
- 5: **pda** – Integer *Input*  
*On entry:* the stride separating row or column elements (depending on the value of **order**) of the matrix *A* in the array **a**.  
*Constraint:* **pda** ≥  $\max(1, \mathbf{n})$ .
- 6: **d**[*dim*] – double *Output*  
**Note:** the dimension, *dim*, of the array **d** must be at least  $\max(1, \mathbf{n})$ .  
*On exit:* the diagonal elements of the tridiagonal matrix *T*.
- 7: **e**[*dim*] – double *Output*  
**Note:** the dimension, *dim*, of the array **e** must be at least  $\max(1, \mathbf{n} - 1)$ .  
*On exit:* the off-diagonal elements of the tridiagonal matrix *T*.
- 8: **tau**[*dim*] – double *Output*  
**Note:** the dimension, *dim*, of the array **tau** must be at least  $\max(1, \mathbf{n} - 1)$ .  
*On exit:* further details of the orthogonal matrix *Q*.
- 9: **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 3.2.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument *<value>* had an illegal value.

### NE\_INT

On entry, **n** = *<value>*.

Constraint: **n** ≥ 0.

On entry, **pda** =  $\langle value \rangle$ .  
 Constraint: **pda** > 0.

### NE\_INT\_2

On entry, **pda** =  $\langle value \rangle$  and **n** =  $\langle value \rangle$ .  
 Constraint: **pda**  $\geq$  max(1, **n**).

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.  
 See Section 3.6.6 in How to Use the NAG Library and its Documentation for further information.

### NE\_NO\_LICENCE

Your licence key may have expired or may not have been installed correctly.  
 See Section 3.6.5 in How to Use the NAG Library and its Documentation for further information.

## 7 Accuracy

The computed tridiagonal matrix  $T$  is exactly similar to a nearby matrix  $(A + E)$ , where

$$\|E\|_2 \leq c(n)\epsilon\|A\|_2,$$

$c(n)$  is a modestly increasing function of  $n$ , and  $\epsilon$  is the *machine precision*.

The elements of  $T$  themselves may be sensitive to small perturbations in  $A$  or to rounding errors in the computation, but this does not affect the stability of the eigenvalues and eigenvectors.

## 8 Parallelism and Performance

nag\_dsytrd (f08fec) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag\_dsytrd (f08fec) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

The total number of floating-point operations is approximately  $\frac{4}{3}n^3$ .

To form the orthogonal matrix  $Q$  nag\_dsytrd (f08fec) may be followed by a call to nag\_dorgtr (f08ffc):

```
nag_dorgtr(order, uplo, n, &a, pda, tau, &fail)
```

To apply  $Q$  to an  $n$  by  $p$  real matrix  $C$  nag\_dsytrd (f08fec) may be followed by a call to nag\_dormtr (f08fgc). For example,

```
nag_dormtr(order, Nag_LeftSide, uplo, Nag_NoTrans, n, p, &a, pda,
tau, &c, pdc, &fail)
```

forms the matrix product  $QC$ .

The complex analogue of this function is nag\_zhetrd (f08fsc).

## 10 Example

This example reduces the matrix  $A$  to tridiagonal form, where

$$A = \begin{pmatrix} 2.07 & 3.87 & 4.20 & -1.15 \\ 3.87 & -0.21 & 1.87 & 0.63 \\ 4.20 & 1.87 & 1.15 & 2.06 \\ -1.15 & 0.63 & 2.06 & -1.81 \end{pmatrix}.$$

### 10.1 Program Text

```

/* nag_dsytrd (f08fec) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf08.h>
#include <nagf16.h>
#include <nagx04.h>

int main(void)
{
    /* Scalars */
    Integer i, j, n, pda, pdz, d_len, e_len, tau_len;
    Integer exit_status = 0;
    NagError fail;
    Nag_UploType uplo;
    Nag_OrderType order;
    /* Arrays */
    char nag_enum_arg[40];
    double *a = 0, *d = 0, *e = 0, *tau = 0, *z = 0;

#ifdef NAG_COLUMN_MAJOR
#define A(I, J) a[(J - 1) * pda + I - 1]
#define Z(I, J) z[(J - 1) * pdz + I - 1]
    order = Nag_ColMajor;
#else
#define A(I, J) a[(I - 1) * pda + J - 1]
#define Z(I, J) z[(I - 1) * pdz + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_dsytrd (f08fec) Example Program Results\n\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%*[\n] ", &n);
#else
    scanf("%" NAG_IFMT "%*[\n] ", &n);
#endif

    pda = n;
    pdz = n;
    tau_len = n - 1;
    d_len = n;

```

```

e_len = n - 1;
/* Allocate memory */
if (!(a = NAG_ALLOC(n * n, double)) ||
    !(d = NAG_ALLOC(d_len, double)) ||
    !(e = NAG_ALLOC(e_len, double)) ||
    !(tau = NAG_ALLOC(tau_len, double)) || !(z = NAG_ALLOC(n * n, double)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Read A from data file */
#ifdef _WIN32
    scanf_s(" %39s%[\n] ", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf(" %39s%[\n] ", nag_enum_arg);
#endif
/* nag_enum_name_to_value (x04nac).
 * Converts NAG enum member name to value
 */
uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);

if (uplo == Nag_Upper) {
    for (i = 1; i <= n; ++i) {
        for (j = i; j <= n; ++j)
#ifdef _WIN32
            scanf_s("%lf", &A(i, j));
#else
            scanf("%lf", &A(i, j));
#endif
    }
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
}
else {
    for (i = 1; i <= n; ++i) {
        for (j = 1; j <= i; ++j)
#ifdef _WIN32
            scanf_s("%lf", &A(i, j));
#else
            scanf("%lf", &A(i, j));
#endif
    }
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
}

/* Reduce A to tridiagonal form T = (Q^T)*A*Q */
/* nag_dsytrd (f08fec).
 * Orthogonal reduction of real symmetric matrix to
 * symmetric tridiagonal form
 */
nag_dsytrd(order, uplo, n, a, pda, d, e, tau, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_dsytrd (f08fec).\n%s\n", fail.message);
    exit_status = 1;
}

/* Copy A into Z using nag_dtr_copy (f16qec). */
nag_dtr_copy(order, uplo, Nag_NoTrans, Nag_NonUnitDiag, n, a, pda, z, pdz,
            &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from dtr_copy.\n%s\n", fail.message);
    exit_status = 1;
}

```

```

    goto END;
}

/* Form Q explicitly, storing the result in z using nag_dorgtr (f08ffc). */
nag_dorgtr(order, uplo, n, z, pdz, tau, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_dorgtr (f08ffc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Calculate all the eigenvalues and eigenvectors of matrix A */
nag_dsteqr(order, Nag_UpdateZ, n, d, e, z, pdz, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_dsteqr (f08jec).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Normalize the eigenvectors */
for (j = 1; j <= n; j++) {
    for (i = n; i >= 1; i--) {
        Z(i, j) = Z(i, j) / Z(1, j);
    }
}

/* Print eigenvalues and eigenvectors */
printf("Eigenvalues\n");
for (i = 1; i <= n; ++i)
    printf("%8.4f%s", d[i - 1], i % 8 == 0 ? "\n" : " ");
printf("\n\n");

/* nag_gen_real_mat_print (x04cac).
 * Print real general matrix (easy-to-use)
 */
fflush(stdout);
nag_gen_real_mat_print(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, n,
                       z, pdz, "Eigenvectors", 0, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_real_mat_print (x04cac).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
}
END:
NAG_FREE(a);
NAG_FREE(d);
NAG_FREE(e);
NAG_FREE(tau);
NAG_FREE(z);

return exit_status;
}

```

## 10.2 Program Data

```

nag_dsytrd (f08fec) Example Program Data
4                               :Value of N
Nag_Lower                       :Value of UPLO
2.07
3.87  -0.21
4.20  1.87  1.15
-1.15  0.63  2.06  -1.81  :End of matrix A

```

### **10.3 Program Results**

nag\_dsytrd (f08fec) Example Program Results

Eigenvalues

-5.0034 -1.9987 0.2013 8.0008

Eigenvectors

	1	2	3	4
1	1.0000	1.0000	1.0000	1.0000
2	-0.6148	-3.4333	0.4489	0.6668
3	-0.8378	1.7553	-1.3572	0.8248
4	1.0219	-1.6052	-1.8213	0.0988

---