

# NAG Library Function Document

## nag\_zhetri (f07mwc)

### 1 Purpose

nag\_zhetri (f07mwc) computes the inverse of a complex Hermitian indefinite matrix  $A$ , where  $A$  has been factorized by nag\_zhetrf (f07mrc).

### 2 Specification

```
#include <nag.h>
#include <nagf07.h>
```

```
void nag_zhetri (Nag_OrderType order, Nag_UploType uplo, Integer n,
                Complex a[], Integer pda, const Integer ipiv[], NagError *fail)
```

### 3 Description

nag\_zhetri (f07mwc) is used to compute the inverse of a complex Hermitian indefinite matrix  $A$ , the function must be preceded by a call to nag\_zhetrf (f07mrc), which computes the Bunch–Kaufman factorization of  $A$ .

If **uplo** = Nag\_Upper,  $A = PUDU^H P^T$  and  $A^{-1}$  is computed by solving  $U^H P^T X P U = D^{-1}$  for  $X$ .

If **uplo** = Nag\_Lower,  $A = PLDL^H P^T$  and  $A^{-1}$  is computed by solving  $L^H P^T X P L = D^{-1}$  for  $X$ .

### 4 References

Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer. Anal.* **12** 1–19

### 5 Arguments

1: **order** – Nag\_OrderType *Input*

*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 2.3.1.3 in How to Use the NAG Library and its Documentation for a more detailed explanation of the use of this argument.

*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.

2: **uplo** – Nag\_UploType *Input*

*On entry:* specifies how  $A$  has been factorized.

**uplo** = Nag\_Upper  
 $A = PUDU^H P^T$ , where  $U$  is upper triangular.

**uplo** = Nag\_Lower  
 $A = PLDL^H P^T$ , where  $L$  is lower triangular.

*Constraint:* **uplo** = Nag\_Upper or Nag\_Lower.

3: **n** – Integer *Input*

*On entry:*  $n$ , the order of the matrix  $A$ .

*Constraint:*  $n \geq 0$ .

- 4: **a**[*dim*] – Complex *Input/Output*  
**Note:** the dimension, *dim*, of the array **a** must be at least  $\max(1, \mathbf{pda} \times \mathbf{n})$ .  
*On entry:* details of the factorization of *A*, as returned by nag\_zhetrf (f07mrc).  
*On exit:* the factorization is overwritten by the *n* by *n* Hermitian matrix  $A^{-1}$ .  
 If **uplo** = Nag\_Upper, the upper triangle of  $A^{-1}$  is stored in the upper triangular part of the array.  
 If **uplo** = Nag\_Lower, the lower triangle of  $A^{-1}$  is stored in the lower triangular part of the array.
- 5: **pda** – Integer *Input*  
*On entry:* the stride separating row or column elements (depending on the value of **order**) of the matrix in the array **a**.  
**Constraint:**  $\mathbf{pda} \geq \max(1, \mathbf{n})$ .
- 6: **ipiv**[*dim*] – const Integer *Input*  
**Note:** the dimension, *dim*, of the array **ipiv** must be at least  $\max(1, \mathbf{n})$ .  
*On entry:* details of the interchanges and the block structure of *D*, as returned by nag\_zhetrf (f07mrc).
- 7: **fail** – NagError \* *Input/Output*  
 The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 3.2.1.2 in How to Use the NAG Library and its Documentation for further information.

### NE\_BAD\_PARAM

On entry, argument  $\langle \text{value} \rangle$  had an illegal value.

### NE\_INT

On entry,  $\mathbf{n} = \langle \text{value} \rangle$ .

Constraint:  $\mathbf{n} \geq 0$ .

On entry,  $\mathbf{pda} = \langle \text{value} \rangle$ .

Constraint:  $\mathbf{pda} > 0$ .

### NE\_INT\_2

On entry,  $\mathbf{pda} = \langle \text{value} \rangle$  and  $\mathbf{n} = \langle \text{value} \rangle$ .

Constraint:  $\mathbf{pda} \geq \max(1, \mathbf{n})$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 3.6.6 in How to Use the NAG Library and its Documentation for further information.

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.  
See Section 3.6.5 in How to Use the NAG Library and its Documentation for further information.

**NE\_SINGULAR**

Element  $\langle value \rangle$  of the diagonal is exactly zero.  $D$  is singular and the inverse of  $A$  cannot be computed.

**7 Accuracy**

The computed inverse  $X$  satisfies a bound of the form

if **uplo** = Nag\_Upper,  $|DU^H P^T X P U - I| \leq c(n)\epsilon(|D||U^H|P^T|X|P|U| + |D||D^{-1}|)$ ;

if **uplo** = Nag\_Lower,  $|DL^H P^T X P L - I| \leq c(n)\epsilon(|D||L^H|P^T|X|P|L| + |D||D^{-1}|)$ ,

$c(n)$  is a modest linear function of  $n$ , and  $\epsilon$  is the *machine precision*.

**8 Parallelism and Performance**

nag\_zhetri (f07mwc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

**9 Further Comments**

The total number of real floating-point operations is approximately  $\frac{8}{3}n^3$ .

The real analogue of this function is nag\_dsytri (f07mjc).

**10 Example**

This example computes the inverse of the matrix  $A$ , where

$$A = \begin{pmatrix} -1.36 + 0.00i & 1.58 + 0.90i & 2.21 - 0.21i & 3.91 + 1.50i \\ 1.58 - 0.90i & -8.87 + 0.00i & -1.84 - 0.03i & -1.78 + 1.18i \\ 2.21 + 0.21i & -1.84 + 0.03i & -4.63 + 0.00i & 0.11 + 0.11i \\ 3.91 - 1.50i & -1.78 - 1.18i & 0.11 - 0.11i & -1.84 + 0.00i \end{pmatrix}.$$

Here  $A$  is Hermitian indefinite and must first be factorized by nag\_zhetrf (f07mrc).

**10.1 Program Text**

```
/* nag_zhetri (f07mwc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>
```

```

int main(void)
{
    /* Scalars */
    Integer i, j, n, pda;
    Integer exit_status = 0;
    NagError fail;
    Nag_UploType uplo;
    Nag_MatrixType matrix;
    Nag_OrderType order;
    /* Arrays */
    Integer *ipiv = 0;
    char nag_enum_arg[40];
    Complex *a = 0;
#ifdef NAG_LOAD_FP
    /* The following line is needed to force the Microsoft linker
       to load floating point support */
    float force_loading_of_ms_float_support = 0;
#endif /* NAG_LOAD_FP */

#ifdef NAG_COLUMN_MAJOR
#define A(I, J) a[(J-1)*pda + I - 1]
    order = Nag_ColMajor;
#else
#define A(I, J) a[(I-1)*pda + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_zhetri (f07mwc) Example Program Results\n\n");

    /* Skip heading in data file */
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
#ifdef _WIN32
    scanf_s("%" NAG_IFMT "%*[\n] ", &n);
#else
    scanf("%" NAG_IFMT "%*[\n] ", &n);
#endif
#ifdef NAG_COLUMN_MAJOR
    pda = n;
#else
    pda = n;
#endif

    /* Allocate memory */
    if (!(ipiv = NAG_ALLOC(n, Integer)) || !(a = NAG_ALLOC(n * n, Complex)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    /* Read A from data file */
#ifdef _WIN32
    scanf_s(" %39s%*[\n] ", nag_enum_arg, (unsigned)_countof(nag_enum_arg));
#else
    scanf(" %39s%*[\n] ", nag_enum_arg);
#endif
    /* nag_enum_name_to_value (x04nac).
       * Converts NAG enum member name to value
       */
    uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);

    if (uplo == Nag_Upper) {
        matrix = Nag_UpperMatrix;
        for (i = 1; i <= n; ++i) {
            for (j = i; j <= n; ++j)

```

```

#ifdef _WIN32
    scanf_s(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
#else
    scanf(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
#endif
}
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
}
else {
    matrix = Nag_LowerMatrix;
    for (i = 1; i <= n; ++i) {
        for (j = 1; j <= i; ++j)
#ifdef _WIN32
            scanf_s(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
#else
            scanf(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
#endif
    }
#ifdef _WIN32
    scanf_s("%*[\n] ");
#else
    scanf("%*[\n] ");
#endif
}

/* Factorize A */
/* nag_zhetrf (f07mrc).
 * Bunch-Kaufman factorization of complex Hermitian
 * indefinite matrix
 */
nag_zhetrf(order, uplo, n, a, pda, ipiv, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_zhetrf (f07mrc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Compute inverse of A */
/* nag_zhetri (f07mwc).
 * Inverse of complex Hermitian indefinite matrix, matrix
 * already factorized by nag_zhetrf (f07mrc)
 */
nag_zhetri(order, uplo, n, a, pda, ipiv, &fail);
if (fail.code != NE_NOERROR) {
    printf("Error from nag_zhetri (f07mwc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Print inverse */
/* nag_gen_complx_mat_print_comp (x04dbc).
 * Print complex general matrix (comprehensive)
 */
fflush(stdout);
nag_gen_complx_mat_print_comp(order, matrix, Nag_NonUnitDiag, n, n, a, pda,
                             Nag_BracketForm, "%7.4f", "Inverse",
                             Nag_IntegerLabels, 0, Nag_IntegerLabels, 0,
                             80, 0, 0, &fail);

if (fail.code != NE_NOERROR) {
    printf("Error from nag_gen_complx_mat_print_comp (x04dbc).\n%s\n",
          fail.message);
    exit_status = 1;
    goto END;
}
}
END:
NAG_FREE(ipiv);
NAG_FREE(a);
return exit_status;
}

```

## 10.2 Program Data

```
nag_zhetri (f07mwc) Example Program Data
4                                     :Value of n
Nag_Lower                           :Value of uplo
(-1.36, 0.00)
( 1.58,-0.90) (-8.87, 0.00)
( 2.21, 0.21) (-1.84, 0.03) (-4.63, 0.00)
( 3.91,-1.50) (-1.78,-1.18) ( 0.11,-0.11) (-1.84, 0.00) :End of matrix A
```

## 10.3 Program Results

```
nag_zhetri (f07mwc) Example Program Results

Inverse
          1          2          3          4
1 ( 0.0826, 0.0000)
2 (-0.0335, 0.0440) (-0.1408, 0.0000)
3 ( 0.0603,-0.0105) ( 0.0422,-0.0222) (-0.2007,-0.0000)
4 ( 0.2391,-0.0926) ( 0.0304, 0.0203) ( 0.0982,-0.0635) ( 0.0073, 0.0000)
```

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