NAG Library Function Document

nag_ode_bvp_ps_lin_grid_vals (d02uwc)

1   Purpose

nag_ode_bvp_ps_lin_grid_vals (d02uwc) interpolates from a set of function values on a supplied grid onto a set of values for a uniform grid on the same range. The interpolation is performed using barycentric Lagrange interpolation. nag_ode_bvp_ps_lin_grid_vals (d02uwc) is primarily a utility function to map a set of function values specified on a Chebyshev Gauss–Lobatto grid onto a uniform grid.

2   Specification

```
#include <nag.h>
#include <nagd02.h>
void nag_ode_bvp_ps_lin_grid_vals (Integer n, Integer nip, const double x[],
      const double f[], double xip[], double fip[], NagError *fail)
```

3   Description

nag_ode_bvp_ps_lin_grid_vals (d02uwc) interpolates from a set of $n+1$ function values, $f(x_i)$, on a supplied grid, x_i, for $i = 0, 1, \ldots, n$, onto a set of m values, $\hat{f}(\hat{x}_j)$, on a uniform grid, $\hat{x}_j$, for $j = 1, 2, \ldots, m$. The image $\hat{x}$ has the same range as x, so that $\hat{x}_j = x_{\min} + ((j-1)/(m-1)) \times (x_{\max} - x_{\min})$, for $j = 1, 2, \ldots, m$. The interpolation is performed using barycentric Lagrange interpolation as described in Berrut and Trefethen (2004).

nag_ode_bvp_ps_lin_grid_vals (d02uwc) is primarily a utility function to map a set of function values specified on a Chebyshev Gauss–Lobatto grid computed by nag_ode_bvp_ps_lin_cgl_grid (d02ucc) onto an evenly-spaced grid with the same range as the original grid.

4   References

Berrut J P and Trefethen L N (2004) Barycentric lagrange interpolation *SIAM Rev.* **46(3)** 501–517

5   Arguments

1:    **n** – Integer   *Input*

   On entry: n, where the number of grid points for the input data is $n+1$.

   Constraint: **n** > 0 and **n** is even.

2:    **nip** – Integer   *Input*

   On entry: the number, m, of grid points in the uniform mesh $\hat{x}$ onto which function values are interpolated. If **nip** $= 1$ then on successful exit from nag_ode_bvp_ps_lin_grid_vals (d02uwc), **fip**[0] will contain the value $f(x_n)$.

   Constraint: **nip** > 0.

3:    **x**[**n** + **1**] – const double   *Input*

   On entry: the grid points, x_i, for $i = 0, 1, \ldots, n$, at which the function is specified.

   Usually this should be the array of Chebyshev Gauss–Lobatto points returned in nag_ode_bvp_ps_lin_cgl_grid (d02ucc).

4:   **f**[**n** + **1**] – const double   *Input*

On entry: the function values, $f(x_i)$, for $i = 0, 1, \ldots, n$.

5:   **xip**[**nip**] – double   *Output*

On exit: the evenly-spaced grid points, $\hat{x}_j$, for $j = 1, 2, \ldots, m$.

6:   **fip**[**nip**] – double   *Output*

On exit: the set of interpolated values $\hat{f}(\hat{x}_j)$, for $j = 1, 2, \ldots, m$. Here $\hat{f}(\hat{x}_j) \approx f(x = \hat{x}_j)$.

7:   **fail** – NagError *   *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6   Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: **n** > 0.

On entry, **n** = ⟨*value*⟩.
Constraint: **n** is even.

On entry, **nip** = ⟨*value*⟩.
Constraint: **nip** > 0.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in How to Use the NAG Library and its Documentation for further information.

7   Accuracy

nag_ode_bvp_ps_lin_grid_vals (d02uwc) is intended, primarily, for use with Chebyshev Gauss–Lobatto input grids. For such input grids and for well-behaved functions (no discontinuities, peaks or cusps), the accuracy should be a small multiple of ***machine precision***.

8   Parallelism and Performance

nag_ode_bvp_ps_lin_grid_vals (d02uwc) is not threaded in any implementation.

9   Further Comments

None.

10   Example

This example interpolates the function $x + \cos(5x)$, as specified on a 65-point Gauss–Lobatto grid on $[-1, 1]$, onto a coarse uniform grid.

10.1 Program Text

```
/* nag_ode_bvp_ps_lin_grid_vals (d02uwc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */
#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagd02.h>
#include <nagx02.h>

#ifdef __cplusplus
extern "C"
{
#endif
  static double NAG_CALL exact(double x);
#ifdef __cplusplus
}
#endif

int main(void)
{
  /*  Scalars */
  Integer exit_status = 0;
  Integer i, n, nip;
  double a = -1.0, b = 1.0;
  double uerr = 0.0;
  double teneps = 10.0 * nag_machine_precision;
  /*  Arrays */
  double *f = 0, *fip = 0, *x = 0, *xip = 0;
  /* NAG types */
  Nag_Boolean reqerr = Nag_FALSE;
  NagError fail;

  INIT_FAIL(fail);

  printf("nag_ode_bvp_ps_lin_grid_vals (d02uwc) Example Program Results\n\n");

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
#endif

#ifdef _WIN32
  scanf_s("%" NAG_IFMT "%*[^\n] ", &n);
#else
  scanf("%" NAG_IFMT "%*[^\n] ", &n);
#endif
#ifdef _WIN32
  scanf_s("%" NAG_IFMT "%*[^\n] ", &nip);
#else
  scanf("%" NAG_IFMT "%*[^\n] ", &nip);
```

```
#endif
  if (!(f = NAG_ALLOC((n + 1), double)) ||
      !(fip = NAG_ALLOC((nip), double)) ||
      !(xip = NAG_ALLOC((nip), double)) || !(x = NAG_ALLOC((n + 1), double))
        )
  {
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
  }

  /* Set up solution grid:
   * nag_ode_bvp_ps_lin_cgl_grid (d02ucc).
   * Generate Chebyshev Gauss-Lobatto grid.
   */
  nag_ode_bvp_ps_lin_cgl_grid(n, a, b, x, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_ode_bvp_ps_lin_cgl_grid (d02ucc).\n%s\n",
           fail.message);
    exit_status = 1;
    goto END;
  }

  /* Set up problem right hand sides for grid. */
  for (i = 0; i < n + 1; i++)
    f[i] = exact(x[i]);

  /* Map to an equally spaced grid:
   * nag_ode_bvp_ps_lin_grid_vals (d02uwc).
   * Interpolate a function from Chebyshev grid to uniform grid
   * using barycentric Lagrange interpolation.
   */
  nag_ode_bvp_ps_lin_grid_vals(n, nip, x, f, xip, fip, &fail);
  if (fail.code != NE_NOERROR) {
    printf("Error from nag_ode_bvp_ps_lin_grid_vals (d02uwc).\n%s\n",
           fail.message);
    exit_status = 1;
    goto END;
  }

  /* Print solution. */
  printf("Numerical solution f\n\n");
  printf("        x           f\n");
  for (i = 0; i < nip; i++)
    printf("%10.4f %10.4f\n", xip[i], fip[i]);

  if (reqerr) {
    for (i = 0; i < nip; i++)
      uerr = MAX(uerr, fabs(fip[i] - exact(xip[i])));
    printf("f is within a multiple %" NAG_IFMT " of machine precision.\n",
           10 * ((Integer) (uerr / teneps) + 1));
  }
END:
  NAG_FREE(f);
  NAG_FREE(fip);
  NAG_FREE(x);
  NAG_FREE(xip);
  return exit_status;
}

static double NAG_CALL exact(double x)
{
  return x + cos(5.0 * x);
}
```

10.2 Program Data

```
nag_ode_bvp_ps_lin_grid_vals (d02uwc) Example Program Data
  64              : n
  17              : nip
```

10.3 Program Results

```
nag_ode_bvp_ps_lin_grid_vals (d02uwc) Example Program Results
```

Numerical solution f

```
      x            f
  -1.0000     -0.7163
  -0.8750     -1.2060
  -0.7500     -1.5706
  -0.6250     -1.6249
  -0.5000     -1.3011
  -0.3750     -0.6745
  -0.2500      0.0653
  -0.1250      0.6860
   0.0000      1.0000
   0.1250      0.9360
   0.2500      0.5653
   0.3750      0.0755
   0.5000     -0.3011
   0.6250     -0.3749
   0.7500     -0.0706
   0.8750      0.5440
   1.0000      1.2837
```