NAG Library Function Document

nag_fft_multid_single (c06pfc)

1 Purpose

nag_fft_multid_single (c06pfc) computes the discrete Fourier transform of one variable in a multivariate sequence of complex data values.

2 Specification

```
#include <nag.h>
#include <nagc06.h>
void nag_fft_multid_single (Nag_TransformDirection direct, Integer ndim,
     Integer l, const Integer nd[], Integer n, Complex x[], NagError *fail)
```

3 Description

nag_fft_multid_single (c06pfc) computes the discrete Fourier transform of one variable (the lth say) in a multivariate sequence of complex data values $z_{j_1 j_2 \cdots j_m}$, where $j_1 = 0, 1, \ldots, n_1 - 1$, $j_2 = 0, 1, \ldots, n_2 - 1$, and so on. Thus the individual dimensions are $n_1, n_2, \ldots, n_m$, and the total number of data values is $n = n_1 \times n_2 \times \cdots \times n_m$.

The function computes n/n_l one-dimensional transforms defined by

$$\hat{z}_{j_1 \ldots k_l \ldots j_m} = \frac{1}{\sqrt{n_l}} \sum_{j_l=0}^{n_l-1} z_{j_1 \ldots j_l \ldots j_m} \times \exp\left(\pm \frac{2\pi i j_l k_l}{n_l}\right),$$

where $k_l = 0, 1, \ldots, n_l - 1$. The plus or minus sign in the argument of the exponential terms in the above definition determine the direction of the transform: a minus sign defines the **forward** direction and a plus sign defines the **backward** direction.

(Note the scale factor of $\frac{1}{\sqrt{n_l}}$ in this definition.)

A call of nag_fft_multid_single (c06pfc) with **direct** = Nag_ForwardTransform followed by a call with **direct** = Nag_BackwardTransform will restore the original data.

The data values must be supplied in a one-dimensional complex array using column-major storage ordering of multidimensional data (i.e., with the first subscript j_1 varying most rapidly).

This function uses a variant of the fast Fourier transform (FFT) algorithm (see Brigham (1974)) known as the Stockham self-sorting algorithm, which is described in Temperton (1983).

4 References

Brigham E O (1974) *The Fast Fourier Transform* Prentice–Hall

Temperton C (1983) Self-sorting mixed-radix fast Fourier transforms *J. Comput. Phys.* **52** 1–23

5 Arguments

1: **direct** – Nag_TransformDirection *Input*

On entry: if the forward transform as defined in Section 3 is to be computed, then **direct** must be set equal to Nag_ForwardTransform.

If the backward transform is to be computed then **direct** must be set equal to Nag_BackwardTransform.

Constraint: **direct** = Nag_ForwardTransform or Nag_BackwardTransform.

2: **ndim** – Integer *Input*

On entry: m, the number of dimensions (or variables) in the multivariate data.

Constraint: **ndim** ≥ 1.

3: **l** – Integer *Input*

On entry: l, the index of the variable (or dimension) on which the discrete Fourier transform is to be performed.

Constraint: $1 \leq \mathbf{l} \leq \mathbf{ndim}$.

4: **nd**[**ndim**] – const Integer *Input*

On entry: the elements of **nd** must contain the dimensions of the **ndim** variables; that is, **nd**$[i-1]$ must contain the dimension of the ith variable.

Constraint: **nd**$[i-1] \geq 1$, for $i = 1, 2, \ldots,$ **ndim**.

5: **n** – Integer *Input*

On entry: n, the total number of data values.

Constraint: **n** must equal the product of the first **ndim** elements of the array **nd**.

6: **x**[**n**] – Complex *Input/Output*

On entry: the complex data values. Data values are stored in **x** using column-major ordering for storing multidimensional arrays; that is, $z_{j_1 j_2 \cdots j_m}$ is stored in $\mathbf{x}[j_1 + n_1 j_2 + n_1 n_2 j_3 + \cdots]$.

On exit: the corresponding elements of the computed transform.

7: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in How to Use the NAG Library and its Documentation for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **l** = ⟨*value*⟩.
Constraint: **l** ≥ 1 and **l** $\leq$ **ndim**.

On entry, **ndim** = ⟨*value*⟩.
Constraint: **ndim** ≥ 1.

NE_INT_2

n must equal the product of the dimensions held in array **nd**: **n** = ⟨*value*⟩, product of **nd** elements is ⟨*value*⟩.

On entry **nd**$[I-1] = \langle value \rangle$ and $I = \langle value \rangle$.
Constraint: **nd**$[I-1] \geq 1$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in How to Use the NAG Library and its Documentation for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in How to Use the NAG Library and its Documentation for further information.

7 Accuracy

Some indication of accuracy can be obtained by performing a subsequent inverse transform and comparing the results with the original sequence (in exact arithmetic they would be identical).

8 Parallelism and Performance

nag_fft_multid_single (c06pfc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag_fft_multid_single (c06pfc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the x06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Notefor your implementation for any additional implementation-specific information.

9 Further Comments

The time taken is approximately proportional to $n \times \log n_l$, but also depends on the factorization of n_l. nag_fft_multid_single (c06pfc) is faster if the only prime factors of n_l are 2, 3 or 5; and fastest of all if n_l is a power of 2.

10 Example

This example reads in a bivariate sequence of complex data values and prints the discrete Fourier transform of the second variable. It then performs an inverse transform and prints the sequence so obtained, which may be compared with the original data values.

10.1 Program Text

```
/* nag_fft_multid_single (c06pfc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
```

```
#include <stdio.h>
#include <string.h>
#include <nag_stdlib.h>
#include <nagc06.h>
#include <nagx04.h>

int main(void)
{
  /* Scalars */
  Integer i, l, n, ndim;
  Integer exit_status = 0;
  NagError fail;
  /* Arrays */
  Complex *x = 0;
  Integer *nd = 0;
#ifdef NAG_LOAD_FP
  /* The following line is needed to force the Microsoft linker
     to load floating point support */
  float force_loading_of_ms_float_support = 0;
#endif /* NAG_LOAD_FP */

  INIT_FAIL(fail);

  printf("nag_fft_multid_single (c06pfc) Example Program Results\n");
  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif
#ifdef _WIN32
  scanf_s("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "", &ndim, &l, &n);
#else
  scanf("%" NAG_IFMT "%" NAG_IFMT "%" NAG_IFMT "", &ndim, &l, &n);
#endif
  if (n >= 1) {
    /* Allocate memory */
    if (!(x = NAG_ALLOC(n, Complex)) || !(nd = NAG_ALLOC(ndim, Integer)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }
    for (i = 0; i < ndim; ++i) {
#ifdef _WIN32
      scanf_s("%" NAG_IFMT "", &nd[i]);
#else
      scanf("%" NAG_IFMT "", &nd[i]);
#endif
    }

    /* Read in complex data and print out. */
#ifdef _WIN32
    scanf_s("%*[^\n]");
#else
    scanf("%*[^\n]");
#endif
    for (i = 0; i < n; ++i) {
#ifdef _WIN32
      scanf_s(" ( %lf, %lf ) ", &x[i].re, &x[i].im);
#else
      scanf(" ( %lf, %lf ) ", &x[i].re, &x[i].im);
#endif
    }
#ifdef _WIN32
    scanf_s("%*[^\n]");
#else
    scanf("%*[^\n]");
#endif
    printf("\n");
    fflush(stdout);
```

```
    /* nag_gen_complx_mat_print_comp (x04dbc).
     * Print complex general matrix (comprehensive)
     */
    fflush(stdout);
    nag_gen_complx_mat_print_comp(Nag_ColMajor, Nag_GeneralMatrix,
                                  Nag_NonUnitDiag, nd[0], n / nd[0], x, nd[0],
                                  Nag_BracketForm, "%6.3f", "Original data",
                                  Nag_NoLabels, 0, Nag_NoLabels, 0, 80, 0,
                                  0, &fail);
    if (fail.code != NE_NOERROR) {
      printf("Error from nag_gen_complx_mat_print_comp (x04dbc).\n%s\n",
             fail.message);
      exit_status = 1;
      goto END;
    }

    /* Compute transform */
    /* nag_fft_multid_single (c06pfc).
     * One-dimensional complex discrete Fourier transform of
     * multi-dimensional data (using complex data type)
     */
    nag_fft_multid_single(Nag_ForwardTransform, ndim, 1, nd, n, x, &fail);
    if (fail.code != NE_NOERROR) {
      printf("Error from nag_fft_multid_single (c06pfc).\n%s\n",
             fail.message);
      exit_status = 1;
      goto END;
    }
    printf("\n");
    /* nag_gen_complx_mat_print_comp (x04dbc), see above. */
    fflush(stdout);
    nag_gen_complx_mat_print_comp(Nag_ColMajor, Nag_GeneralMatrix,
                                  Nag_NonUnitDiag, nd[0], n / nd[0], x, nd[0],
                                  Nag_BracketForm, "%6.3f",
                                  "Discrete Fourier transform of variable 2",
                                  Nag_NoLabels, 0, Nag_NoLabels, 0, 80, 0,
                                  0, &fail);
    if (fail.code != NE_NOERROR) {
      printf("Error from nag_gen_complx_mat_print_comp (x04dbc).\n%s\n",
             fail.message);
      exit_status = 1;
      goto END;
    }

    /* Compute inverse transform */
    /* nag_fft_multid_single (c06pfc), see above. */
    nag_fft_multid_single(Nag_BackwardTransform, ndim, 1, nd, n, x, &fail);
    if (fail.code != NE_NOERROR) {
      printf("Error from nag_fft_multid_single (c06pfc).\n%s\n",
             fail.message);
      exit_status = 1;
      goto END;
    }
    printf("\n");
    /* nag_gen_complx_mat_print_comp (x04dbc), see above. */
    fflush(stdout);
    nag_gen_complx_mat_print_comp(Nag_ColMajor, Nag_GeneralMatrix,
                                  Nag_NonUnitDiag, nd[0], n / nd[0], x, nd[0],
                                  Nag_BracketForm, "%6.3f",
                                  "Original data as restored by inverse"
                                  " transform", Nag_NoLabels, 0,
                                  Nag_NoLabels, 0, 80, 0, 0, &fail);
    if (fail.code != NE_NOERROR) {
      printf("Error from nag_gen_complx_mat_print_comp (x04dbc).\n%s\n",
             fail.message);
      exit_status = 1;
      goto END;
    }
  }
  else
    printf("\nInvalid value of n.\n");
```

```
END:
  NAG_FREE(x);
  NAG_FREE(nd);

  return exit_status;
}
```

10.2 Program Data

```
nag_fft_multid_single (c06pfc) Example Program Data
    2    2   15
    3    5
     (1.000,0.000)      (0.994,-0.111)     (0.903,-0.430)
     (0.999,-0.040)     (0.989,-0.151)     (0.885,-0.466)
     (0.987,-0.159)     (0.963,-0.268)     (0.823,-0.568)
     (0.936,-0.352)     (0.891,-0.454)     (0.694,-0.720)
     (0.802,-0.597)     (0.731,-0.682)     (0.467,-0.884)
```

10.3 Program Results

```
nag_fft_multid_single (c06pfc) Example Program Results

 Original data
   ( 1.000, 0.000)  ( 0.999,-0.040)  ( 0.987,-0.159)  ( 0.936,-0.352)
   ( 0.994,-0.111)  ( 0.989,-0.151)  ( 0.963,-0.268)  ( 0.891,-0.454)
   ( 0.903,-0.430)  ( 0.885,-0.466)  ( 0.823,-0.568)  ( 0.694,-0.720)

   ( 0.802,-0.597)
   ( 0.731,-0.682)
   ( 0.467,-0.884)

 Discrete Fourier transform of variable 2
   ( 2.113,-0.513)  ( 0.288,-0.000)  ( 0.126, 0.130)  (-0.003, 0.190)
   ( 2.043,-0.745)  ( 0.286,-0.032)  ( 0.139, 0.115)  ( 0.018, 0.189)
   ( 1.687,-1.372)  ( 0.260,-0.125)  ( 0.170, 0.063)  ( 0.079, 0.173)

   (-0.287, 0.194)
   (-0.263, 0.225)
   (-0.176, 0.299)

 Original data as restored by inverse transform
   ( 1.000,-0.000)  ( 0.999,-0.040)  ( 0.987,-0.159)  ( 0.936,-0.352)
   ( 0.994,-0.111)  ( 0.989,-0.151)  ( 0.963,-0.268)  ( 0.891,-0.454)
   ( 0.903,-0.430)  ( 0.885,-0.466)  ( 0.823,-0.568)  ( 0.694,-0.720)

   ( 0.802,-0.597)
   ( 0.731,-0.682)
   ( 0.467,-0.884)
```