

## NAG Library Function Document

### nag\_zero\_cont\_func\_brent\_binsrch (c05auc)

#### 1 Purpose

nag\_zero\_cont\_func\_brent\_binsrch (c05auc) locates a simple zero of a continuous function from a given starting value. It uses a binary search to locate an interval containing a zero of the function, then Brent's method, which is a combination of nonlinear interpolation, linear extrapolation and bisection, to locate the zero precisely.

#### 2 Specification

```
#include <nag.h>
#include <nagc05.h>

void nag_zero_cont_func_brent_binsrch (double *x, double h, double eps,
    double eta,
    double (*f)(double x, Nag_Comm *comm),
    double *a, double *b, Nag_Comm *comm, NagError *fail)
```

#### 3 Description

nag\_zero\_cont\_func\_brent\_binsrch (c05auc) attempts to locate an interval  $[a, b]$  containing a simple zero of the function  $f(x)$  by a binary search starting from the initial point  $x = \mathbf{x}$  and using repeated calls to nag\_interval\_zero\_cont\_func (c05auc). If this search succeeds, then the zero is determined to a user-specified accuracy by a call to nag\_zero\_cont\_func\_brent (c05auc). The specifications of functions nag\_interval\_zero\_cont\_func (c05auc) and nag\_zero\_cont\_func\_brent (c05auc) should be consulted for details of the methods used.

The approximation  $x$  to the zero  $\alpha$  is determined so that at least one of the following criteria is satisfied:

- (i)  $|x - \alpha| \leq \mathbf{eps}$ ,
- (ii)  $|f(x)| \leq \mathbf{eta}$ .

#### 4 References

Brent R P (1973) *Algorithms for Minimization Without Derivatives* Prentice–Hall

#### 5 Arguments

- 1: **x** – double \* *Input/Output*  
*On entry:* an initial approximation to the zero.  
*On exit:* if **fail.code** = NE\_NOERROR or NW\_TOO\_MUCH\_ACC\_REQUESTED, **x** is the final approximation to the zero.  
 If **fail.code** = NE\_PROBABLE\_POLE, **x** is likely to be a pole of  $f(x)$ .  
 Otherwise, **x** contains no useful information.
- 2: **h** – double *Input*  
*On entry:* a step length for use in the binary search for an interval containing the zero. The maximum interval searched is  $[\mathbf{x} - 256.0 \times \mathbf{h}, \mathbf{x} + 256.0 \times \mathbf{h}]$ .  
*Constraint:* **h** must be sufficiently large that  $\mathbf{x} + \mathbf{h} \neq \mathbf{x}$  on the computer.

- 3: **eps** – double *Input*  
*On entry:* the termination tolerance on  $x$  (see Section 3).  
*Constraint:* **eps** > 0.0.
- 4: **eta** – double *Input*  
*On entry:* a value such that if  $|f(x)| \leq \mathbf{eta}$ ,  $x$  is accepted as the zero. **eta** may be specified as 0.0 (see Section 7).
- 5: **f** – function, supplied by the user *External Function*  
**f** must evaluate the function  $f$  whose zero is to be determined.

The specification of **f** is:

```
double f (double x, Nag_Comm *comm)
```

1: **x** – double *Input*

*On entry:* the point at which the function must be evaluated.

2: **comm** – Nag\_Comm \*

Pointer to structure of type Nag\_Comm; the following members are relevant to **f**.

**user** – double \*

**iuser** – Integer \*

**p** – Pointer

The type Pointer will be void \*. Before calling nag\_zero\_cont\_func\_brent\_binsrch (c05auc) you may allocate memory and initialize these pointers with various quantities for use by **f** when called from nag\_zero\_cont\_func\_brent\_binsrch (c05auc) (see Section 2.3.1.1 in How to Use the NAG Library and its Documentation).

- 6: **a** – double \* *Output*
- 7: **b** – double \* *Output*
- On exit:* the lower and upper bounds respectively of the interval resulting from the binary search. If the zero is determined exactly such that  $f(x) = 0.0$  or is determined so that  $|f(x)| \leq \mathbf{eta}$  at any stage in the calculation, then on exit **a** = **b** =  $x$ .
- 8: **comm** – Nag\_Comm \*  
The NAG communication argument (see Section 2.3.1.1 in How to Use the NAG Library and its Documentation).
- 9: **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 2.7 in How to Use the NAG Library and its Documentation).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

See Section 3.2.1.2 in How to Use the NAG Library and its Documentation for further information.

**NE\_BAD\_PARAM**

On entry, argument  $\langle value \rangle$  had an illegal value.

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.  
See Section 3.6.6 in How to Use the NAG Library and its Documentation for further information.

**NE\_NO\_LICENCE**

Your licence key may have expired or may not have been installed correctly.  
See Section 3.6.5 in How to Use the NAG Library and its Documentation for further information.

**NE\_PROBABLE\_POLE**

Solution may be a pole rather than a zero.

**NE\_REAL**

On entry,  $\mathbf{eps} = \langle value \rangle$ .  
Constraint:  $\mathbf{eps} > 0.0$ .

**NE\_REAL\_2**

On entry,  $\mathbf{x} = \langle value \rangle$  and  $\mathbf{h} = \langle value \rangle$ .  
Constraint:  $\mathbf{x} + \mathbf{h} \neq \mathbf{x}$  (to machine accuracy).

**NE\_ZERO\_NOT\_FOUND**

An interval containing the zero could not be found. Increasing  $\mathbf{h}$  and calling `nag_zero_cont_func_brent_binsrch (c05auc)` again will increase the range searched for the zero. Decreasing  $\mathbf{h}$  and calling `nag_zero_cont_func_brent_binsrch (c05auc)` again will refine the mesh used in the search for the zero.

**NW\_TOO\_MUCH\_ACC\_REQUESTED**

The tolerance  $\mathbf{eps}$  has been set too small for the problem being solved. However, the value  $\mathbf{x}$  returned is a good approximation to the zero.  $\mathbf{eps} = \langle value \rangle$ .

**7 Accuracy**

The levels of accuracy depend on the values of  $\mathbf{eps}$  and  $\mathbf{eta}$ . If full machine accuracy is required, they may be set very small, resulting in an exit with `fail.code = NW_TOO_MUCH_ACC_REQUESTED`, although this may involve many more iterations than a lesser accuracy. You are recommended to set  $\mathbf{eta} = 0.0$  and to use  $\mathbf{eps}$  to control the accuracy, unless you have considerable knowledge of the size of  $f(x)$  for values of  $x$  near the zero.

**8 Parallelism and Performance**

`nag_zero_cont_func_brent_binsrch (c05auc)` is not threaded in any implementation.

**9 Further Comments**

The time taken by `nag_zero_cont_func_brent_binsrch (c05auc)` depends primarily on the time spent evaluating  $\mathbf{f}$  (see Section 5). The accuracy of the initial approximation  $\mathbf{x}$  and the value of  $\mathbf{h}$  will have a somewhat unpredictable effect on the timing.

If it is important to determine an interval of relative length less than  $2 \times \mathbf{eps}$  containing the zero, or if  $\mathbf{f}$  is expensive to evaluate and the number of calls to  $\mathbf{f}$  is to be restricted, then use of

`nag_interval_zero_cont_func` (c05avc) followed by `nag_zero_cont_func_brent_rcomm` (c05azc) is recommended. Use of this combination is also recommended when the structure of the problem to be solved does not permit a simple `f` to be written: the reverse communication facilities of these functions are more flexible than the direct communication of `f` required by `nag_zero_cont_func_brent_binsrch` (c05auc).

If the iteration terminates with successful exit and  $\mathbf{a} = \mathbf{b} = \mathbf{x}$  there is no guarantee that the value returned in `x` corresponds to a simple zero and you should check whether it does.

One way to check this is to compute the derivative of  $f$  at the point `x`, preferably analytically, or, if this is not possible, numerically, perhaps by using a central difference estimate. If  $f'(x) = 0.0$ , then `x` must correspond to a multiple zero of  $f$  rather than a simple zero.

## 10 Example

This example calculates an approximation to the zero of  $x - e^{-x}$  using a tolerance of `eps = 1.0e-5` starting from `x = 1.0` and using an initial search step `h = 0.1`.

### 10.1 Program Text

```

/* nag_zero_cont_func_brent_binsrch (c05auc) Example Program.
 *
 * NAGPRODCODE Version.
 *
 * Copyright 2016 Numerical Algorithms Group.
 *
 * Mark 26, 2016.
 */

#include <nag.h>
#include <nagx04.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <math.h>
#include <nagc05.h>

#ifdef __cplusplus
extern "C"
{
#endif
    static double NAG_CALL f(double x, Nag_Comm *comm);
#ifdef __cplusplus
}
#endif

int main(void)
{
    /* Scalars */
    Integer exit_status = 0;
    double a, b, eps, eta, h, x;
    NagError fail;
    Nag_Comm comm;
    /* Arrays */
    static double ruser[1] = { -1.0 };

    INIT_FAIL(fail);

    printf("nag_zero_cont_func_brent_binsrch (c05auc) Example Program Results\n");

    x = 1.0;
    h = 0.1;
    eps = 1e-05;
    eta = 0.0;

    /* For communication with user-supplied functions: */
    comm.user = ruser;

    /* nag_zero_cont_func_brent_binsrch (c05auc).

```

```

    * Locates a simple zero of a continuous function of one variable,
    * binary search for an interval containing a zero.
    */
nag_zero_cont_func_brent_binsrch(&x, h, eps, eta, f, &a, &b, &comm, &fail);
if (fail.code == NE_NOERROR) {
    printf("Root is %13.5f\n", x);
    printf("Interval searched is [%8.5f,%8.5f]\n", a, b);
}
else {
    printf("%s\n", fail.message);
    if (fail.code == NE_PROBABLE_POLE ||
        fail.code == NW_TOO_MUCH_ACC_REQUESTED)
        printf("Final value = %13.5f\n", x);
    exit_status = 1;
    goto END;
}

END:
    return exit_status;
}

static double NAG_CALL f(double x, Nag_Comm *comm)
{
    if (comm->user[0] == -1.0) {
        printf("(User-supplied callback f, first invocation.)\n");
        comm->user[0] = 0.0;
    }
    return x - exp(-x);
}

```

## 10.2 Program Data

None.

## 10.3 Program Results

```

nag_zero_cont_func_brent_binsrch (c05auc) Example Program Results
(User-supplied callback f, first invocation.)
Root is          0.56714
Interval searched is [ 0.50000, 0.90000]

```

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