NAG Library Function Document

nag_general_elliptic_integral_f (s21dac)

1 Purpose

nag_general_elliptic_integral_f (s21dac) returns the value of the general elliptic integral of the second kind $F(z, k', a, b)$ for a complex argument z.

2 Specification

```
#include <nag.h>
#include <nags.h>
```

```
Complex nag_general_elliptic_integral_f (Complex z, double akp, double a,
        double b, NagError *fail)
```

3 Description

nag_general_elliptic_integral_f (s21dac) evaluates an approximation to the general elliptic integral of the second kind $F(z, k', a, b)$ given by

$$F(z, k', a, b) = \int_0^z \frac{a + b\zeta^2}{(1 + \zeta^2)\sqrt{(1 + \zeta^2)\left(1 + k'^2\zeta^2\right)}} d\zeta,$$

where a and b are real arguments, z is a complex argument whose real part is non-negative and k' is a real argument (the *complementary modulus*). The evaluation of F is based on the Gauss transformation. Further details, in particular for the conformal mapping provided by F, can be found in Bulirsch (1960).

Special values include

$$F(z, k', 1, 1) = \int_0^z \frac{d\zeta}{\sqrt{(1 + \zeta^2)(1 + k'2\zeta^2)}},$$

or $F_1(z, k')$ (the *elliptic integral of the first kind*) and

$$F\left(z, k', 1, k'^2\right) = \int_0^z \frac{\sqrt{1 + k'^2\zeta^2}}{(1 + \zeta^2)\sqrt{1 + \zeta^2}} d\zeta,$$

or $F_2(z, k')$ (the *elliptic integral of the second kind*). Note that the values of $F_1(z, k')$ and $F_2(z, k')$ are equal to $\tan^{-1}(z)$ in the trivial case $k' = 1$.

nag_general_elliptic_integral_f (s21dac) is derived from an Algol 60 procedure given by Bulirsch (1960). Constraints are placed on the values of z and k' in order to avoid the possibility of machine overflow.

4 References

Bulirsch R (1960) Numerical calculation of elliptic integrals and elliptic functions *Numer. Math.* **7** 76–90

5 Arguments

1: **z** – Complex *Input*

On entry: the argument z of the function.

Constraints:

$0.0 \leq \mathbf{z}.re \leq \lambda$;
$\mathrm{abs}(\mathbf{z}.im) \leq \lambda$, where $\lambda^6 = 1/\text{nag\_real\_safe\_small\_number}$.

2: **akp** – double *Input*

On entry: the argument k' of the function.

Constraint: $\mathrm{abs}(\mathbf{akp}) \leq \lambda$.

3: **a** – double *Input*

On entry: the argument a of the function.

4: **b** – double *Input*

On entry: the argument b of the function.

5: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_COMPLEX

On entry, $|\mathbf{z}.im|$ is too large: $|\mathbf{z}.im| = \langle value \rangle$. It must not exceed $\langle value \rangle$.

On entry, $\mathbf{z}.re < 0.0$: $\mathbf{z}.re = \langle value \rangle$.

On entry, $\mathbf{z}.re$ is too large: $\mathbf{z}.re = \langle value \rangle$. It must not exceed $\langle value \rangle$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

NE_REAL

On entry, $|\mathbf{akp}|$ is too large: $|\mathbf{akp}| = \langle value \rangle$. It must not exceed $\langle value \rangle$.

NE_S21_CONV

The iterative procedure used to evaluate the integral has failed to converge.

7 Accuracy

In principle the function is capable of achieving full relative precision in the computed values. However, the accuracy obtainable in practice depends on the accuracy of the standard elementary functions such as atan2 and log.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example evaluates the elliptic integral of the first kind $F_1(z,k')$ given by

$$F_1(z,k') = \int_0^z \frac{d\zeta}{\sqrt{(1+\zeta^2)(1+k'^2\zeta^2)}},$$

where $z = 1.2 + 3.7i$ and $k' = 0.5$, and prints the results.

10.1 Program Text

```
/* nag_general_elliptic_integral_f (s21dac) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * NAG C Library
 *
 * Mark 6, 2000.
 */

#include <stdio.h>

#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  Complex  y, z;
  Integer  exit_status = 0;
  NagError fail;
  double   a, akp, b;

  INIT_FAIL(fail);

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
#endif
  printf("nag_general_elliptic_integral_f (s21dac) Example Program "
         "Results\n");
  printf("      z          akp      a       b                 y\n");
#ifdef _WIN32
  while (scanf_s(" (%lf,%lf) %lf %lf %lf%*[^\n] ",
                &z.re, &z.im, &akp, &a, &b) != EOF)
    {
#else
  while (scanf(" (%lf,%lf) %lf %lf %lf%*[^\n] ",
                &z.re, &z.im, &akp, &a, &b) != EOF)
    {
#endif
      /* nag_general_elliptic_integral_f (s21dac).
       * Elliptic integrals of the second kind with complex
       * arguments
       */
      y = nag_general_elliptic_integral_f(z, akp, a, b, &fail);
```

```
      if (fail.code == NE_NOERROR)
        printf("(%4.1f, %4.1f) %7.1f %7.1f %7.1f  (%13.4e, %13.4e)\n",
               z.re, z.im, akp, a, b, y.re, y.im);
      else
        {
          printf("Error from nag_general_elliptic_integral_f (s21dac)."
                 "\n%s\n", fail.message);
          exit_status = 1;
          goto END;
        }
    }
 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_general_elliptic_integral_f (s21dac) Example Program Data
 (1.2, 3.7)  0.5  1.0  1.0
 (9.2,-3.4)  0.8  0.4  2.7  : Values of z, akp, a and b
```

10.3 Program Results

```
nag_general_elliptic_integral_f (s21dac) Example Program Results
      z          akp      a       b                y
( 1.2,  3.7)     0.5     1.0     1.0  (  1.9713e+00,   5.0538e-01)
( 9.2, -3.4)     0.8     0.4     2.7  (  2.5042e+00,  -1.1709e-01)
```