NAG Library Function Document

nag_shifted_log (s01bac)

1 Purpose

nag_shifted_log (s01bac) returns a value of the shifted logarithmic function, $\ln(1+x)$.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_shifted_log (double x, NagError *fail)
```

3 Description

nag_shifted_log (s01bac) computes values of $\ln(1+x)$, retaining full relative precision even when $|x|$ is small. The function is based on the Chebyshev expansion

$$\ln\frac{1+p^2+2p\bar{x}}{1+p^2-2p\bar{x}} = 4\sum_{k=0}^{\infty}\frac{p^{2k+1}}{2k+1}T_{2k+1}(\bar{x}).$$

Setting $\bar{x} = \dfrac{x(1+p^2)}{2p(x+2)}$, and choosing $p = \dfrac{q-1}{q+1}$, $q = \sqrt[4]{2}$ the expansion is valid in the domain $x \in \left[\dfrac{1}{\sqrt{2}} - 1, \sqrt{2} - 1\right]$.

Outside this domain, $\ln(1+x)$ is computed by the standard logarithmic function.

4 References

Lyusternik L A, Chervonenkis O A and Yanpolskii A R (1965) *Handbook for Computing Elementary Functions* p. 57 Pergamon Press

5 Arguments

1: **x** – double *Input*

On entry: the argument x of the function.

Constraint: $\mathbf{x} > -1.0$.

2: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_REAL_ARG_LE

On entry, $\mathbf{x} = \langle value \rangle$.
Constraint: $\mathbf{x} > -1.0$.

7 Accuracy

The returned result should be accurate almost to ***machine precision***, with a limit of about 20 significant figures due to the precision of internal constants. Note however that if x lies very close to -1.0 and is not exact (for example if x is the result of some previous computation and has been rounded), then precision will be lost in the computation of $1+x$, and hence $\ln(1+x)$, in nag_shifted_log (s01bac).

8 Parallelism and Performance

Not applicable.

9 Further Comments

Empirical tests show that the time taken for a call of nag_shifted_log (s01bac) usually lies between about 1.25 and 2.5 times the time for a call to the standard logarithm function.

10 Example

The example program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
/* nag_shifted_log (s01bac) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 7, 2002.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  Integer  exit_status = 0;
  double   x, y;
  NagError fail;

  INIT_FAIL(fail);

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif
  printf("nag_shifted_log (s01bac) Example Program Results\n");
  printf("       x            y\n");
#ifdef _WIN32
  while (scanf_s("%lf", &x) != EOF)
#else
  while (scanf("%lf", &x) != EOF)
#endif
    {
      /* nag_shifted_log (s01bac).
       * ln(1+x)
       */
      y = nag_shifted_log(x, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_shifted_log (s01bac).\n%s\n",
                  fail.message);
```

```
          exit_status = 1;
          goto END;
        }
      printf("%13.4e %13.4e\n", x, y);
    }

 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_shifted_log (s01bac) Example Program Data
  2.50e+0
  1.25e-1
 -9.06e-1
  1.29e-3
 -7.83e-6
  1.00e-9
```

10.3 Program Results

```
nag_shifted_log (s01bac) Example Program Results
       x            y
  2.5000e+00   1.2528e+00
  1.2500e-01   1.1778e-01
 -9.0600e-01  -2.3645e+00
  1.2900e-03   1.2892e-03
 -7.8300e-06  -7.8300e-06
  1.0000e-09   1.0000e-09
```