

NAG Library Function Document

nag_tsa_transf_filter (g13bbc)

1 Purpose

nag_tsa_transf_filter (g13bbc) filters a time series by a transfer function model.

2 Specification

```
#include <nag.h>
#include <nagg13.h>

void nag_tsa_transf_filter (const double y[], Integer ny,
                           Nag_TransfOrder *transfv, Nag_ArimaOrder *arimas, const double par[],
                           Integer npar, double cy, double b[], Integer nb, NagError *fail)
```

3 Description

From a given series y_1, y_2, \dots, y_n a new series b_1, b_2, \dots, b_n is calculated using a supplied (filtering) transfer function model according to the equation

$$b_t = \delta_1 b_{t-1} + \delta_2 b_{t-2} + \dots + \delta_p b_{t-p} + \omega_0 y_{t-b} - \omega_1 y_{t-b-1} - \dots - \omega_q y_{t-b-q}. \quad (1)$$

As in the use of nag_tsa_arma_filter (g13bac), large transient errors may arise in the early values of b_t due to ignorance of y_t for $t < 0$, and two possibilities are allowed.

- (i) The equation (1) is applied from $t = 1 + b + q, \dots, n$ so all terms in y_t on the right-hand side of (1) are known, the unknown set of values b_t for $t = b + q, \dots, b + q + 1 - p$ being taken as zero.
- (ii) The unknown values of y_t for $t \leq 0$ are estimated by backforecasting exactly as for nag_tsa_arma_filter (g13bac).

4 References

Box G E P and Jenkins G M (1976) *Time Series Analysis: Forecasting and Control* (Revised Edition) Holden-Day

5 Arguments

1: **y[ny]** – const double *Input*

On entry: the Q'_y backforecasts starting with backforecast at time $1 - Q'_y$ to backforecast at time 0 followed by the time series starting at time 1, where $Q'_y = \text{arimas.q} + \text{arimas.bigq} \times \text{arimas.s}$. If there are no backforecasts either because the ARIMA model for the time series is not known or because it is known but has no moving average terms, then the time series starts at the beginning of **y**.

2: **ny** – Integer *Input*

On entry: the total number of backforecasts and time series data points in array **y**.

Constraint: $\text{ny} \geq \max(1 + Q'_y, \text{npar})$.

3: **transfv** – Nag_TransfOrder * *Input*

On entry: the orders of the transfer function model where the triplet (**transfv.nag_b**, **transfv.nag_q**, **transfv.nag_p**) corresponds to the triplet (b, q, p) as described in Section 2.3.1 in the g13 Chapter Introduction.

Constraints:

transfv.nag_b ≥ 0 ;
transfv.nag_q ≥ 0 ;
transfv.nag_p ≥ 0 .

4: **arimas** – Nag_ArimaOrder * *Input*

On entry: if available, the orders for the filtering ARIMA model for the time series as a pointer to structure of type Nag_ArimaOrder with the following members:

p – Integer	<i>Input</i>
d – Integer	<i>Input</i>
q – Integer	<i>Input</i>
bigp – Integer	<i>Input</i>
bigd – Integer	<i>Input</i>
bigq – Integer	<i>Input</i>
s – Integer	<i>Input</i>

On entry: these seven members of **arimas** must specify the orders vector (p, d, q, P, D, Q, s), respectively, of the ARIMA model for the output noise component.

p, q, P and Q refer, respectively, to the number of autoregressive (ϕ), moving average (θ), seasonal autoregressive (Φ) and seasonal moving average (Θ) parameters.

d, D and s refer, respectively, to the order of non-seasonal differencing, the order of seasonal differencing and the seasonal period.

If no ARIMA model for the series is to be supplied **arimas** should be set to a **NULL** pointer.

Constraints:

arimas.p ≥ 0 ;
arimas.d ≥ 0 ;
arimas.q ≥ 0 ;
arimas.bigp ≥ 0 ;
arimas.bigd ≥ 0 ;
arimas.bigq ≥ 0 ;
arimas.s ≥ 0 ;
arimas.s $\neq 1$;
if **arimas.s** = 0, **arimas.bigp + arimas.bigd + arimas.bigq** = 0;
if **arimas.s** $\neq 0$, **arimas.bigp + arimas.bigd + arimas.bigq** $\neq 0$.

5: **par[npar]** – const double *Input*

On entry: the parameters of the filtering transfer function model followed by the parameters of the ARIMA model for the time series. In the transfer function model the parameters are in the standard order of MA-like followed by AR-like operator parameters. In the ARIMA model the parameters are in the standard order of non-seasonal AR and MA followed by seasonal AR and MA.

6: **npar** – Integer *Input*

On entry: the total number of parameters held in array **par**.

Constraints:

if **arimas** is not **NULL**, **npar = transfv.nag_q + transfv.nag_p + 1**;
if **arimas** is **NULL**, **npar = transfv.nag_q + transfv.nag_p + 1 + arimas.p + arimas.q + arimas.bigp + arimas.bigq**.

7: **cy** – double *Input*

On entry: if the ARIMA model is known (i.e., **arimas** is NULL), **cy** must specify the constant term of the ARIMA model for the time series. If this model is not known (i.e., **arimas** is not NULL) then **cy** is not used.

8: **b[nb]** – double *Output*

On exit: the filtered output series. If the ARIMA model for the time series was known, and hence Q_y' backforecasts were supplied in **y**, then **b** contains Q_y' ‘filtered’ backforecasts followed by the filtered series. Otherwise, the filtered series begins at the start of **b** just as the original series began at the start of **y**. In either case, if the value of the series at time t is held in **y**[$t - 1$], then the filtered value at time t is held in **b**[$t - 1$].

9: **nb** – Integer *Input*

On entry: the dimension of the array **b**.

In addition to holding the returned filtered series, **b** is also used as an intermediate work array if the ARIMA model for the time series is known.

Constraints:

if **arimas** is not NULL, **nb** \geq **ny**;
 if **arimas** is NULL, **nb** \geq **ny** + max(**transfv.nag_b** + **transfv.nag_q**, **transfv.nag_p**).

10: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

See Section 3.2.1.2 in the Essential Introduction for further information.

NE_ARRAY_SIZE

The array **b** is too small. Minimum required size: $\langle value \rangle$.

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_CONSTRAINT

On entry, **arimas** = $\langle value \rangle$.
 Constraint: **arimas.bigd** \geq 0.

On entry, **arimas** = $\langle value \rangle$.
 Constraint: **arimas.biggp** \geq 0.

On entry, **arimas** = $\langle value \rangle$.
 Constraint: **arimas.biggq** \geq 0.

On entry, **arimas** = $\langle value \rangle$.
 Constraint: **arimas.d** \geq 0.

On entry, **arimas** = $\langle value \rangle$.
 Constraint: **arimas.p** \geq 0.

On entry, **arimas** = $\langle value \rangle$.
 Constraint: **arimas.q** \geq 0.

On entry, **arimas** = $\langle value \rangle$.

Constraint: **arimas.s** $\neq 1$.

On entry, **arimas** = $\langle value \rangle$.

Constraint: **arimas.s** ≥ 0 .

On entry, **arimas** = $\langle value \rangle$.

Constraint: if **arimas.s** = 0, **arimas.bigp** + **arimas.bigd** + **arimas.bigq** = 0.

On entry, **arimas** = $\langle value \rangle$.

Constraint: if **arimas.s** $\neq 0$, **arimas.bigp** + **arimas.bigd** + **arimas.bigq** $\neq 0$.

On entry, **transfv** = $\langle value \rangle$.

Constraint: **transfv.nag.b** ≥ 0 .

On entry, **transfv** = $\langle value \rangle$.

Constraint: **transfv.nag.p** ≥ 0 .

On entry, **transfv** = $\langle value \rangle$.

Constraint: **transfv.nag.q** ≥ 0 .

NE_INT

On entry, **npar** is inconsistent with **transfv** and **arimas**: **npar** = $\langle value \rangle$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.

See Section 3.6.6 in the Essential Introduction for further information.

NE_MODEL_PARAMS

A supplied model has invalid parameters.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.

See Section 3.6.5 in the Essential Introduction for further information.

NE_SINGULAR

The matrix used to solve for starting values for MA is singular.

NE_TIME_SERIES

The supplied time series is too short.

7 Accuracy

Accuracy and stability are high except when the AR-like parameters are close to the invertibility boundary. All calculations are performed in **basic precision** except for one inner product type calculation which on machines of low precision is performed in **additional precision**.

8 Parallelism and Performance

`nag_tsa_transf_filter` (g13bbc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

`nag_tsa_transf_filter` (g13bbc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

If an ARIMA model is supplied, local workspace arrays of fixed lengths are allocated internally by nag_tsa_transf_filter (g13bbc). The total size of these arrays amounts to K Integer elements and $K \times (K + 2)$ double elements, where $K = \text{transfv.nag.p} + \text{arimas.p} + \text{arimas.d} + (\text{arimas.bigp} + \text{arimas.bigd}) \times \text{arimas.s}$.

The time taken by nag_tsa_transf_filter (g13bbc) is roughly proportional to the product of the length of the series and number of parameters in the filtering model with appreciable increase if an ARIMA model is supplied for the time series.

10 Example

This example reads a time series of length 296. It reads one univariate ARIMA (1, 1, 0, 0, 1, 1, 12) model for the series and the (0, 13, 12) filtering transfer function model. 12 initial backforecasts are required and these are calculated by a call to nag_tsa_multi_inp_model_forecast (g13bjc). The backforecasts are inserted at the start of the series and nag_tsa_transf_filter (g13bbc) is called to perform the filtering.

10.1 Program Text

```
/* nag_tsa_transf_filter (g13bbc) Example Program.
*
* Copyright 2014 Numerical Algorithms Group.
*
* Mark 7, 2002.
* Mark 7b revised, 2004.
*/
#include <stdio.h>
#include <nag.h>
#include <nag_stdl�.h>
#include <nagg13.h>

int main(void)
{
    /* Scalars */
    double          a1, a2, cx, cy;
    Integer         i, ii, ij, iqxd, j, k, n, nb, ni, npar, nparsx, nx;
    Integer         nser, npara, tdxxy, tdmrx, ldparsx, tdparsx;
    Integer         exit_status = 0, idd = 0, ny = 0;

    /* Arrays */
    double          *b = 0, *fsd = 0, *fva = 0, *par = 0, *parx = 0;
    double          *x = 0, *y = 0, *rms = 0, *parxx = 0;
    Integer         mr[10], mrx[7], *mrxx = 0;

    Nag_TrafoOrder transfj, transfv;
    Nag_ArimaOrder  arimaj, arimas;
    Nag_G13_Opt     options;
    NagError        fail;

    INIT_FAIL(fail);

    exit_status = 0;

    /* Initialise the options structure used by nag_tsa_multi_inp_model_forecast
     * (g13bjc) */
    /* nag_tsa_options_init (g13bxc).
     * Initialization function for option setting
     */
    nag_tsa_options_init(&options);
```

```

printf("nag_tsa_transf_filter (g13bbc) Example Program Results\n");

/* Skip heading in data file */
#ifndef _WIN32
    scanf_s("%*[^\n] ");
#else
    scanf("%*[^\n] ");
#endif

#ifndef _WIN32
    scanf_s("%"NAG_IFMT"%*[^\n] ", &nx);
#else
    scanf("%"NAG_IFMT"%*[^\n] ", &nx);
#endif

printf("\n");
if (nx > 0)
{
    /* Allocate array x */
    if (!(x = NAG_ALLOC(nx+2, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    for (i = 1; i <= nx; ++i)
#ifndef _WIN32
        scanf_s("%lf", &x[i-1]);
#else
        scanf("%lf", &x[i-1]);
#endif
#ifndef _WIN32
    scanf_s("%*[^\n] ");
#else
    scanf("%*[^\n] ");
#endif
}

/* Read univariate ARIMA for series */
for (i = 1; i <= 7; ++i)
#ifndef _WIN32
    scanf_s("%"NAG_IFMT"", &mrx[i-1]);
#else
    scanf("%"NAG_IFMT"", &mrx[i-1]);
#endif
#ifndef _WIN32
    scanf_s("%*[^\n] ");
#else
    scanf("%*[^\n] ");
#endif
}

#ifndef _WIN32
    scanf_s("%lf%*[^\n] ", &cx);
#else
    scanf("%lf%*[^\n] ", &cx);
#endif

nparx = mrx[0] + mrx[2] + mrx[3] + mrx[5];

arimaj.p = mrx[0];
arimaj.d = mrx[1];
arimaj.q = mrx[2];
arimaj.bigr = mrx[3];
arimaj.bigrd = mrx[4];
arimaj.bigrq = mrx[5];
arimaj.s = mrx[6];

nser = 1;

if (nparx > 0)
{

```

```

/* Allocate array parx */
if (!(parx = NAG_ALLOC(nparx+nser, double)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}
for (i = 1; i <= nparx; ++i)
#endif _WIN32
    scanf_s("%lf", &parx[i-1]);
#else
    scanf("%lf", &parx[i-1]);
#endif
#endif _WIN32
    scanf_s("%*[^\n] ");
#else
    scanf("%*[^\n] ");
#endif

/* Read model by which to filter series */
for (i = 1; i <= 3; ++i)
#endif _WIN32
    scanf_s("%"NAG_IFMT"", &mr[i-1]);
#else
    scanf("%"NAG_IFMT"", &mr[i-1]);
#endif
#endif _WIN32
    scanf_s("%*[^\n] ");
#else
    scanf("%*[^\n] ");
#endif

transfv.nag_b = mr[0];
transfv.nag_q = mr[1];
transfv.nag_p = mr[2];

npar = mr[1] + mr[2] + 1;
if (npar > 0)
{
    /* Allocate array par */
    if (!(par = NAG_ALLOC(npar + nparx, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
    for (i = 1; i <= npar; ++i)
#endif _WIN32
        scanf_s("%lf", &par[i-1]);
#else
        scanf("%lf", &par[i-1]);
#endif
#endif _WIN32
        scanf_s("%*[^\n] ");
#else
        scanf("%*[^\n] ");
#endif

/* Initially backforecast QY values */
/* (1) Reverse series in situ */
n = nx / 2;
ni = nx;
for (i = 1; i <= n; ++i)
{
    a1 = x[i-1];
    a2 = x[ni-1];
    x[i-1] = a2;
    x[ni-1] = a1;
    --ni;
}
idd = mrx[1] + mrx[4];

```

```

/* (2) Possible sign reversal for ARIMA constant */
if (idd % 2 != 0)
    cx = -cx;

/* (3) Calculate number of backforecasts required */
iqxd = mrx[2] + mrx[5] * mrx[6];
if (iqxd != 0)
{
    if (!(fsd = NAG_ALLOC(iqxd, double)) ||
        !(fva = NAG_ALLOC(iqxd, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
    npara = nparx+nser;
    parx[npara-1] = cx;
    tdxxy = nser;
    tdmrx = nser-1;
    ldparx = nser-1;
    tdpars = nser-1;
    if (!(rms = NAG_ALLOC(nser, double)) ||
        !(parxx = NAG_ALLOC(nser, double)) ||
        !(mrxx = NAG_ALLOC(7*nser, Integer)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

/* nag_tsa_transf_orders (g13byc).
 * Allocates memory to transfer function model orders
 */
nag_tsa_transf_orders(nser, &transfj, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_tsa_transf_orders (g13byc)"
           ".\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

rms[0] = 0;
transfj.nag_b = 0;
transfj.nag_q = 0;
transfj.nag_p = 0;
transfj.nag_r = 1;
for (i = 1; i <= 7; ++i)
    mrxx[i-1] = 0;
parxx[0] = 0;

/* Tell nag_tsa_multi_inp_model_forecast (g13bjc) not to
 * print parameters on entry */
options.list = Nag_FALSE;

/* nag_tsa_multi_inp_model_forecast (g13bjc).
 * Forecasting function
 */
nag_tsa_multi_inp_model_forecast(&arimaj, nser, &transfj,
                                  parx, npara, nx, iqxd, x,
                                  tdxxy, rms, mrxx, tdmrx,
                                  parxx, ldparx, tdpars,
                                  fva, fsd, &options, &fail);

if (fail.code != NE_NOERROR)
{
    printf(
        "Error from nag_tsa_multi_inp_model_forecast "
        "(g13bjc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

```

```

    }

/* Calculate series length */
ny = nx + iqxd;

/* Allocate array y */
if (!(y = NAG_ALLOC(ny, double)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Move backforecasts to start of y array */
j = iqxd;
for (i = 1; i <= iqxd; ++i)
{
    y[i-1] = fva[j-1];
    --j;
}

/* Move series into y */
j = iqxd + 1;
k = nx;
for (i = 1; i <= nx; ++i)
{
    if (j > 215)
        goto END;
    y[j-1] = x[k-1];
    ++j;
    --k;
}
}

/* Move ARIMA for series into mr */
for (i = 1; i <= 7; ++i)
    mr[i+2] = mrx[i-1];

arimas.p = mr[3];
arimas.d = mr[4];
arimas.q = mr[5];
arimas.bigp = mr[6];
arimas.bigd = mr[7];
arimas.bigg = mr[8];
arimas.s = mr[9];

/* Move parameters of ARIMA for y into par */
for (i = 1; i <= nparx; ++i)
    par[npar+i-1] = parx[i-1];
npar += nparx;

/* Move constant and reset sign reversal */
cy = cx;
if (idd % 2 != 0)
    cy = -cy;

/* Set parameters for call to filter routine
 * nag_tsa_transf_filter (g13bbc) */
nb = ny + MAX(mr[0] + mr[1], mr[2]);

/* Allocate array b */
if (!(b = NAG_ALLOC(nb, double)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Filter series by call to nag_tsa_transf_filter (g13bbc) */
/* nag_tsa_transf_filter (g13bbc).
 * Multivariate time series, filtering by a transfer

```

```

    * function model
    */
nag_tsa_transf_filter(y, ny, &transfv, &arimas, par, npar, cy, b, nb,
                      &fail);
if (fail.code != NE_NOERROR)
{
    printf(
        "Error from nag_tsa_transf_filter (g13bbc).\n%s\n",
        fail.message);
    exit_status = 1;
    goto END;
}

printf("          Original          Filtered\n");
printf(" Backforecasts      y-series      series\n");
if (iqxd != 0)
{
    ij = -iqxd;
    for (i = 1; i <= iqxd; ++i)
    {
        printf("%8"NAG_IFMT"%17.1f%16.1f\n", ij, y[i-1],
               b[i-1]);
        ++ij;
    }

    printf("\n");
    printf("          Filtered          Filtered\n");
    printf("          series          series\n");
    printf("          series          series\n");
    for (i = iqxd + 1; i <= ny; i += 4)
    {
        for (ii = i; ii <= MIN(ny, i+3); ++ii)
        {
            printf("%5"NAG_IFMT"", ii-iqxd);
            printf("%10.1f", b[ii-1]);
        }
        printf("\n");
    }
}
}

END:

/* Free the options structure used by nag_tsa_multi_inp_model_forecast
 * (g13bjc) */
/* nag_tsa_free (g13xzc).
 * Freeing function for use with g13 option setting
 */
nag_tsa_free(&options);

NAG_FREE(b);
NAG_FREE(fsd);
NAG_FREE(fva);
NAG_FREE(par);
NAG_FREE(parx);
NAG_FREE(x);
NAG_FREE(y);
NAG_FREE(rms);
NAG_FREE(parxx);
NAG_FREE(mrxx);

return exit_status;
}

```

10.2 Program Data

```
nag_tsa_transf_filter (g13bbc) Example Program Data
 158
5312. 5402. 4960. 4717. 4383. 3828. 3665. 3718.
3744. 3994. 4150. 4064. 4324. 4256. 3986. 3670.
3292. 2952. 2765. 2813. 2850. 3085. 3256. 3213.
3514. 3386. 3205. 3124. 2804. 2536. 2445. 2649.
2761. 3183. 3456. 3529. 4067. 4079. 4082. 4029.
3887. 3684. 3707. 3923. 4068. 4557. 4975. 5197.
6054. 6471. 6277. 5529. 5059. 4539. 4236. 4305.
4299. 4478. 4561. 4470. 4712. 4512. 4129. 3942.
3572. 3149. 3026. 3141. 3145. 3322. 3384. 3373.
3630. 3555. 3413. 3127. 2966. 2685. 2642. 2789.
2867. 3032. 3125. 3176. 3359. 3265. 3053. 2915.
2690. 2518. 2523. 2737. 3074. 3671. 4355. 4648.
5232. 5349. 5228. 5172. 4932. 4637. 4642. 4930.
5033. 5223. 5482. 5560. 5960. 5929. 5697. 5583.
5316. 5039. 4972. 5169. 5138. 5316. 5409. 5375.
5803. 5736. 5643. 5416. 5059. 4810. 4937. 5166.
5187. 5348. 5483. 5626. 6077. 6033. 5996. 5860.
5499. 5210. 5421. 5609. 5586. 3663. 5829. 6005.
6693. 6792. 6966. 7227. 7089. 6823. 7286. 7621.
7758. 8000. 8393. 8592. 9186. 9175.

      1      1      0      0      1      1     12
 0.000
 0.620  0.820
      0     13     12
 1.0131  0.0806 -0.0150 -0.0150 -0.0150 -0.0150
-0.0150 -0.0150 -0.0150 -0.0150 -0.0150 -0.0150
 0.9981 -0.0956  0.0000  0.0000  0.0000  0.0000
 0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
 0.0000  0.8200
```

10.3 Program Results

```
nag_tsa_transf_filter (g13bbc) Example Program Results
```

Backforecasts	Original		Filtered	
	y-series	series	series	series
-12	5159.0		4549.2	
-11	5165.9		4550.9	
-10	4947.5		4552.8	
-9	4729.8		4554.9	
-8	4424.5		4557.4	
-7	4072.5		4560.7	
-6	3995.5		4565.0	
-5	4142.7		4571.1	
-4	4219.7		4580.0	
-3	4452.1		4593.5	
-2	4758.0		4614.3	
-1	4834.6		4647.1	
 Filtered				
	series	series	series	series
1	4699.2	2	4782.2	3
5	4525.7	6	4324.8	7
9	4127.9	10	4154.6	11
13	3705.1	14	3619.1	15
17	3422.6	18	3463.5	19
21	3225.9	22	3218.1	23
25	2905.9	26	2758.5	27
29	2926.2	30	3019.8	31
33	3111.7	34	3286.3	35
37	3461.7	38	3468.3	39
41	4004.4	42	4146.3	43
45	4419.8	46	4647.2	47
49	5446.0	50	5861.0	51
53	5202.5	54	5046.6	55
57	4740.7	58	4631.1	59
			4447.5	60
				4317.7

61	4079.8	62	3833.7	63	3667.7	64	3774.8
65	3709.9	66	3648.5	67	3645.3	68	3619.8
69	3549.4	70	3439.2	71	3250.3	72	3209.2
73	3005.2	74	2912.4	75	2994.1	76	2947.9
77	3103.7	78	3168.1	79	3226.0	80	3224.1
81	3233.0	82	3119.2	83	2992.5	84	3014.8
85	2763.7	86	2671.3	87	2664.9	88	2778.2
89	2823.8	90	2989.0	91	3072.2	92	3132.1
93	3394.6	94	3717.4	95	4180.5	96	4405.9
97	4605.2	98	4733.0	99	4830.9	100	5030.8
101	5079.0	102	5125.0	103	5236.7	104	5392.7
105	5396.7	106	5300.7	107	5312.1	108	5336.6
109	5347.9	110	5331.2	111	5322.0	112	5444.8
113	5468.7	114	5532.9	115	5555.9	116	5603.4
117	5483.2	118	5406.8	119	5250.5	120	5171.9
121	5217.4	122	5162.3	123	5296.1	124	5268.2
125	5204.9	126	5290.7	127	5500.0	128	5552.3
129	5503.3	130	5419.2	131	5335.6	132	5447.6
133	5495.1	134	5475.1	135	5643.8	136	5713.1
137	5655.1	138	5691.9	139	5958.4	140	5959.0
141	5884.8	142	3714.7	143	5877.8	144	5814.1
145	6095.6	146	6210.7	147	6560.5	148	7013.9
149	7174.8	150	7230.8	151	7726.7	152	7880.0
153	7997.4	154	8428.5	155	8264.1	156	8443.1
157	8615.4	158	8644.6				
