NAG Library Function Document

nag_2_sample_t_test (g07cac)

1 Purpose

nag_2_sample_t_test (g07cac) computes a t-test statistic to test for a difference in means between two Normal populations, together with a confidence interval for the difference between the means.

2 Specification

```
#include <nag.h>
#include <nagg07.h>
```

```
void nag_2_sample_t_test (Nag_TailProbability tail, Nag_PopVar equal,
     Integer nx, Integer ny, double xmean, double ymean, double xstd,
     double ystd, double clevel, double *t, double *df, double *prob,
     double *dl, double *du, NagError *fail)
```

3 Description

Consider two independent samples, denoted by X and Y, of size n_x and n_y drawn from two Normal populations with means μ_x and μ_y, and variances σ_x^2 and σ_y^2 respectively. Denote the sample means by $\bar{x}$ and $\bar{y}$ and the sample variances by s_x^2 and s_y^2 respectively.

nag_2_sample_t_test (g07cac) calculates a test statistic and its significance level to test the null hypothesis $H_0 : \mu_x = \mu_y$, together with upper and lower confidence limits for $\mu_x - \mu_y$. The test used depends on whether or not the two population variances are assumed to be equal.

1. It is assumed that the two variances are equal, that is $\sigma_x^2 = \sigma_y^2$.

   The test used is the two sample t-test. The test statistic t is defined by;

   $$t_{\text{obs}} = \frac{\bar{x} - \bar{y}}{s\sqrt{(1/n_x) + \left(1/n_y\right)}}$$

   where $s^2 = \dfrac{(n_x - 1)s_x^2 + \left(n_y - 1\right)s_y^2}{n_x + n_y - 2}$ is the pooled variance of the two samples.

   Under the null hypothesis H_0 this test statistic has a t-distribution with $\left(n_x + n_y - 2\right)$ degrees of freedom.

   The test of H_0 is carried out against one of three possible alternatives:

   (i) $H_1 : \mu_x \neq \mu_y$; the significance level, $p = P(t \geq |t_{\text{obs}}|)$, i.e., a two tailed probability.

   (ii) $H_1 : \mu_x > \mu_y$; the significance level, $p = P(t \geq t_{\text{obs}})$, i.e., an upper tail probability.

   (iii) $H_1 : \mu_x < \mu_y$; the significance level, $p = P(t \leq t_{\text{obs}})$, i.e., a lower tail probability.

   Upper and lower $100(1 - \alpha)\%$ confidence limits for $\mu_x - \mu_y$ are calculated as:

   $$(\bar{x} - \bar{y}) \pm t_{1-\alpha/2}s\sqrt{(1/n_x) + \left(1/n_y\right)},$$

   where $t_{1-\alpha/2}$ is the $100(1 - \alpha/2)$ percentage point of the t-distribution with $\left(n_x + n_y - 2\right)$ degrees of freedom.

2. It is not assumed that the two variances are equal.

   If the population variances are not equal the usual two sample t-statistic no longer has a t-distribution and an approximate test is used.

This problem is often referred to as the Behrens–Fisher problem, see Kendall and Stuart (1979). The test used here is based on Satterthwaites procedure. To test the null hypothesis the test statistic t' is used where

$$t'_{\text{obs}} = \frac{\bar{x} - \bar{y}}{\text{se}(\bar{x} - \bar{y})}$$

where $\text{se}(\bar{x} - \bar{y})(\bar{x} - \bar{y}) = \sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}$.

A t-distribution with f degrees of freedom is used to approximate the distribution of t' where

$$f = \frac{\text{se}\,(\bar{x} - \bar{y})^4}{\frac{s_x^2/n_x^2}{(n_x - 1)} + \frac{s_y^2/n_y^2}{(n_y - 1)}}.$$

The test of H_0 is carried out against one of the three alternative hypotheses described above, replacing t by t' and t_{obs} by t'_{obs}.

Upper and lower $100(1 - \alpha)\%$ confidence limits for $\mu_x - \mu_y$ are calculated as:

$$(\bar{x} - \bar{y}) \pm t_{1-\alpha/2}\,\text{se}(x - \bar{y}).$$

where $t_{1-\alpha/2}$ is the $100(1 - \alpha/2)$ percentage point of the t-distribution with f degrees of freedom.

4 References

Johnson M G and Kotz A (1969) *The Encyclopedia of Statistics* **2** Griffin

Kendall M G and Stuart A (1979) *The Advanced Theory of Statistics (3 Volumes)* (4th Edition) Griffin

Snedecor G W and Cochran W G (1967) *Statistical Methods* Iowa State University Press

5 Arguments

1: **tail** – Nag_TailProbability *Input*

On entry: indicates which tail probability is to be calculated, and thus which alternative hypothesis is to be used.

tail = Nag_TwoTail
The two tail probability, i.e., $H_1 : \mu_x \neq \mu_y$.

tail = Nag_UpperTail
The upper tail probability, i.e., $H_1 : \mu_x > \mu_y$.

tail = Nag_LowerTail
The lower tail probability, i.e., $H_1 : \mu_x < \mu_y$.

Constraint: **tail** = Nag_UpperTail, Nag_LowerTail or Nag_TwoTail.

2: **equal** – Nag_PopVar *Input*

On entry: indicates whether the population variances are assumed to be equal or not.

equal = Nag_PopVarEqual
The population variances are assumed to be equal, that is $\sigma_x^2 = \sigma_y^2$.

equal = Nag_PopVarNotEqual
The population variances are not assumed to be equal.

Constraint: **equal** = Nag_PopVarEqual or Nag_PopVarNotEqual.

3: **nx** – Integer *Input*

On entry: the size of the X sample, n_x.

Constraint: $\mathbf{nx} \geq 2$.

4: **ny** – Integer *Input*

On entry: the size of the Y sample, n_y.

Constraint: $\mathbf{ny} \geq 2$.

5: **xmean** – double *Input*

On entry: the mean of the X sample, $\bar{x}$.

6: **ymean** – double *Input*

On entry: the mean of the Y sample, $\bar{y}$.

7: **xstd** – double *Input*

On entry: the standard deviation of the X sample, s_x.

Constraint: $\mathbf{xstd} > 0.0$.

8: **ystd** – double *Input*

On entry: the standard deviation of the Y sample, s_y.

Constraint: $\mathbf{ystd} > 0.0$.

9: **clevel** – double *Input*

On entry: the confidence level, $1 - \alpha$, for the specified tail. For example $\mathbf{clevel} = 0.95$ will give a 95% confidence interval.

Constraint: $0.0 < \mathbf{clevel} < 1.0$.

10: **t** – double * *Output*

On exit: contains the test statistic, t_{obs} or t'_{obs}.

11: **df** – double * *Output*

On exit: contains the degrees of freedom for the test statistic.

12: **prob** – double * *Output*

On exit: contains the significance level, that is the tail probability, p, as defined by **tail**.

13: **dl** – double * *Output*

On exit: contains the lower confidence limit for $\mu_x - \mu_y$.

14: **du** – double * *Output*

On exit: contains the upper confidence limit for $\mu_x - \mu_y$.

15: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_BAD_PARAM

On entry, argument **equal** had an illegal value.

On entry, argument **tail** had an illegal value.

NE_INT_ARG_LT

On entry, **nx** = ⟨*value*⟩.
Constraint: **nx** ≥ 2.

On entry, **ny** = ⟨*value*⟩.
Constraint: **ny** ≥ 2.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL_ARG_GE

On entry, **clevel** must not be greater than or equal to 1.0: **clevel** = ⟨*value*⟩.

NE_REAL_ARG_LE

On entry, **clevel** must not be less than or equal to 0.0: **clevel** = ⟨*value*⟩.

On entry, **xstd** must not be less than or equal to 0.0: **xstd** = ⟨*value*⟩.

On entry, **ystd** must not be less than or equal to 0.0: **ystd** = ⟨*value*⟩.

7 Accuracy

The computed probability and the confidence limits should be accurate to approximately five significant figures.

8 Parallelism and Performance

Not applicable.

9 Further Comments

The sample means and standard deviations can be computed using nag_summary_stats_onevar (g01atc).

10 Example

The following example program reads the two sample sizes and the sample means and standard deviations for two independent samples. The data is taken from page 116 of Snedecor and Cochran (1967) from a test to compare two methods of estimating the concentration of a chemical in a vat. A test of the equality of the means is carried out first assuming that the two population variances are equal and then making no assumption about the equality of the population variances.

10.1 Program Text

```
/* nag_2_sample_t_test (g07cac) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 4, 1996.
 *
 * Mark 6 revised, 2000.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg07.h>

int main(void)
{
  /* Local variables */
  double   prob, xstd, ystd;
  double   t;
  double   xmean, ymean, df, dl, du;
  double   clevel;

  Integer  exit_status = 0, nx, ny;
  NagError fail;

  INIT_FAIL(fail);

  printf("nag_2_sample_t_test (g07cac) Example Program Results\n");
  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif
#ifdef _WIN32
  scanf_s("%"NAG_IFMT" %"NAG_IFMT"", &nx, &ny);
#else
  scanf("%"NAG_IFMT" %"NAG_IFMT"", &nx, &ny);
#endif
#ifdef _WIN32
  scanf_s("%lf %lf %lf %lf", &xmean, &ymean, &xstd, &ystd);
#else
  scanf("%lf %lf %lf %lf", &xmean, &ymean, &xstd, &ystd);
#endif
#ifdef _WIN32
  scanf_s("%lf", &clevel);
#else
  scanf("%lf", &clevel);
#endif

  /* nag_2_sample_t_test (g07cac).
   * Computes t-test statistic for a difference in means
   * between two Normal populations, confidence interval
   */
  nag_2_sample_t_test(Nag_TwoTail, Nag_PopVarEqual, nx, ny, xmean, ymean, xstd,
                      ystd, clevel, &t, &df, &prob, &dl, &du, &fail);

  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_2_sample_t_test (g07cac).\n%s\n",
              fail.message);
      exit_status = 1;
      goto END;
    }
  printf("\nAssuming population variances are equal.\n\n");
  printf("t test statistic = %10.4f\n", t);
  printf("Degrees of freedom = %8.1f\n", df);
  printf("Significance level = %8.4f\n", prob);
  printf(
          "Lower confidence limit for difference in means = %10.4f\n", dl);
  printf(
          "Upper confidence limit for difference in means = %10.4f\n\n", du);

  /* nag_2_sample_t_test (g07cac), see above. */
  nag_2_sample_t_test(Nag_TwoTail, Nag_PopVarNotEqual, nx, ny, xmean, ymean,
                      xstd, ystd, clevel, &t, &df, &prob, &dl, &du,
                      &fail);
  if (fail.code != NE_NOERROR)
```

```
    {
      printf("Error from nag_2_sample_t_test (g07cac).\n%s\n",
              fail.message);
      exit_status = 1;
      goto END;
    }

  printf("\nNo assumptions about population variances.\n\n");
  printf("t test statistic = %10.4f\n", t);
  printf("Degrees of freedom = %8.4f\n", df);
  printf("Significance level = %8.4f\n", prob);
  printf("Lower confidence limit for difference in means = %10.4f\n",
          dl);
  printf("Upper confidence limit for difference in means = %10.4f\n",
          du);
 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_2_sample_t_test (g07cac) Example Program Data
4  8
25.0 21.0
0.8185  4.2083
0.95
```

10.3 Program Results

```
nag_2_sample_t_test (g07cac) Example Program Results

Assuming population variances are equal.

t test statistic =     1.8403
Degrees of freedom =     10.0
Significance level =   0.0955
Lower confidence limit for difference in means =    -0.8429
Upper confidence limit for difference in means =     8.8429


No assumptions about population variances.

t test statistic =     2.5922
Degrees of freedom =   7.9925
Significance level =   0.0320
Lower confidence limit for difference in means =     0.4410
Upper confidence limit for difference in means =     7.5590
```