NAG Library Function Document

nag_zggglm (f08zpc)

1 Purpose

nag_zggglm (f08zpc) solves a complex general Gauss–Markov linear (least squares) model problem.

2 Specification

```
#include <nag.h>
#include <nagf08.h>
```

```
void nag_zggglm (Nag_OrderType order, Integer m, Integer n, Integer p,
     Complex a[], Integer pda, Complex b[], Integer pdb, Complex d[],
     Complex x[], Complex y[], NagError *fail)
```

3 Description

nag_zggglm (f08zpc) solves the complex general Gauss–Markov linear model (GLM) problem

$$\underset{x}{\text{minimize}}\|y\|_2 \quad \text{subject to} \quad d = Ax + By$$

where A is an m by n matrix, B is an m by p matrix and d is an m element vector. It is assumed that $n \leq m \leq n + p$, $\text{rank}(A) = n$ and $\text{rank}(E) = m$, where $E = \begin{pmatrix} A & B \end{pmatrix}$. Under these assumptions, the problem has a unique solution x and a minimal 2-norm solution y, which is obtained using a generalized QR factorization of the matrices A and B.

In particular, if the matrix B is square and nonsingular, then the GLM problem is equivalent to the weighted linear least squares problem

$$\underset{x}{\text{minimize}}\left\|B^{-1}(d - Ax)\right\|_2.$$

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia

Anderson E, Bai Z and Dongarra J (1992) Generalized *QR* factorization and its applications *Linear Algebra Appl. (Volume 162–164)* 243–271

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **m** – Integer *Input*

On entry: m, the number of rows of the matrices A and B.

Constraint: **m** ≥ 0.

3: **n** – Integer *Input*

On entry: n, the number of columns of the matrix A.

Constraint: $0 \leq \mathbf{n} \leq \mathbf{m}$.

4: **p** – Integer *Input*

On entry: p, the number of columns of the matrix B.

Constraint: $\mathbf{p} \geq \mathbf{m} - \mathbf{n}$.

5: **a**[*dim*] – Complex *Input/Output*

Note: the dimension, *dim*, of the array **a** must be at least

$\max(1, \mathbf{pda} \times \mathbf{n})$ when **order** = Nag_ColMajor;
$\max(1, \mathbf{m} \times \mathbf{pda})$ when **order** = Nag_RowMajor.

The (i,j)th element of the matrix A is stored in

$\mathbf{a}[(j-1) \times \mathbf{pda} + i - 1]$ when **order** = Nag_ColMajor;
$\mathbf{a}[(i-1) \times \mathbf{pda} + j - 1]$ when **order** = Nag_RowMajor.

On entry: the m by n matrix A.

On exit: **a** is overwritten.

6: **pda** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **a**.

Constraints:

if **order** = Nag_ColMajor, $\mathbf{pda} \geq \max(1, \mathbf{m})$;
if **order** = Nag_RowMajor, $\mathbf{pda} \geq \max(1, \mathbf{n})$.

7: **b**[*dim*] – Complex *Input/Output*

Note: the dimension, *dim*, of the array **b** must be at least

$\max(1, \mathbf{pdb} \times \mathbf{p})$ when **order** = Nag_ColMajor;
$\max(1, \mathbf{m} \times \mathbf{pdb})$ when **order** = Nag_RowMajor.

The (i,j)th element of the matrix B is stored in

$\mathbf{b}[(j-1) \times \mathbf{pdb} + i - 1]$ when **order** = Nag_ColMajor;
$\mathbf{b}[(i-1) \times \mathbf{pdb} + j - 1]$ when **order** = Nag_RowMajor.

On entry: the m by p matrix B.

On exit: **b** is overwritten.

8: **pdb** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **b**.

Constraints:

if **order** = Nag_ColMajor, $\mathbf{pdb} \geq \max(1, \mathbf{m})$;
if **order** = Nag_RowMajor, $\mathbf{pdb} \geq \max(1, \mathbf{p})$.

9: **d**[**m**] – Complex *Input/Output*

On entry: the left-hand side vector d of the GLM equation.

On exit: **d** is overwritten.

10: **x**[**n**] – Complex *Output*

On exit: the solution vector x of the GLM problem.

11: **y**[**p**] – Complex *Output*

On exit: the solution vector y of the GLM problem.

12: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_INT

On entry, $\mathbf{m} = \langle value \rangle$.
Constraint: $\mathbf{m} \geq 0$.

On entry, $\mathbf{pda} = \langle value \rangle$.
Constraint: $\mathbf{pda} > 0$.

On entry, $\mathbf{pdb} = \langle value \rangle$.
Constraint: $\mathbf{pdb} > 0$.

NE_INT_2

On entry, $\mathbf{m} = \langle value \rangle$ and $\mathbf{n} = \langle value \rangle$.
Constraint: $0 \leq \mathbf{n} \leq \mathbf{m}$.

On entry, $\mathbf{pda} = \langle value \rangle$ and $\mathbf{m} = \langle value \rangle$.
Constraint: $\mathbf{pda} \geq \max(1, \mathbf{m})$.

On entry, $\mathbf{pda} = \langle value \rangle$ and $\mathbf{n} = \langle value \rangle$.
Constraint: $\mathbf{pda} \geq \max(1, \mathbf{n})$.

On entry, $\mathbf{pdb} = \langle value \rangle$ and $\mathbf{m} = \langle value \rangle$.
Constraint: $\mathbf{pdb} \geq \max(1, \mathbf{m})$.

On entry, $\mathbf{pdb} = \langle value \rangle$ and $\mathbf{p} = \langle value \rangle$.
Constraint: $\mathbf{pdb} \geq \max(1, \mathbf{p})$.

NE_INT_3

On entry, $\mathbf{p} = \langle value \rangle$, $\mathbf{m} = \langle value \rangle$ and $\mathbf{n} = \langle value \rangle$.
Constraint: $\mathbf{p} \geq \mathbf{m} - \mathbf{n}$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

NE_SINGULAR

The bottom $(N - M)$ by $(N - M)$ part of the upper trapezoidal factor T associated with B in the generalized QR factorization of the pair (A, B) is singular, so that $\text{rank}\begin{pmatrix} A & B \end{pmatrix} < n$; the least squares solutions could not be computed.

The $(N - P)$ by $(N - P)$ part of the upper trapezoidal factor T associated with A in the generalized RQ factorization of the pair (B, A) is singular, so that $\text{rank}\begin{pmatrix} B & A \end{pmatrix} < n$; the least squares solutions could not be computed.

7 Accuracy

For an error analysis, see Anderson *et al.* (1992). See also Section 4.6 of Anderson *et al.* (1999).

8 Parallelism and Performance

nag_zggglm (f08zpc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag_zggglm (f08zpc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

When $p = m \geq n$, the total number of real floating-point operations is approximately $\frac{8}{3}(2m^3 - n^3) + 16nm^2$; when $p = m = n$, the total number of real floating-point operations is approximately $\frac{56}{3}m^3$.

10 Example

This example solves the weighted least squares problem

$$\underset{x}{\text{minimize}} \left\| B^{-1}(d - Ax) \right\|_2,$$

where

$$B = \begin{pmatrix} 0.5 - 1.0i & & & \\ & 1.0 - 2.0i & & \\ & & 2.0 - 3.0i & \\ & & & 5.0 - 4.0i \end{pmatrix},$$

$$d = \begin{pmatrix} 6.00 - 0.40i \\ -5.27 + 0.90i \\ 2.72 - 2.13i \\ -1.30 - 2.80i \end{pmatrix}$$

and

$$A = \begin{pmatrix} 0.96 - 0.81i & -0.03 + 0.96i & -0.91 + 2.06i \\ -0.98 + 1.98i & -1.20 + 0.19i & -0.66 + 0.42i \\ 0.62 - 0.46i & 1.01 + 0.02i & 0.63 - 0.17i \\ 1.08 - 0.28i & 0.20 - 0.12i & -0.07 + 1.23i \end{pmatrix}.$$

10.1 Program Text

```
/* nag_zggglm (f08zpc) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 9, 2009.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf08.h>
#include <nagf16.h>

int main(void)
{
  /* Scalars */
  double        rnorm;
  Integer       i, j, m, n, p, pda, pdb;
  Integer       exit_status = 0;
  NagError      fail;
  Nag_OrderType order;
  /* Arrays */
  Complex       *a = 0, *b = 0, *d = 0, *x = 0, *y = 0;

#ifdef NAG_COLUMN_MAJOR
#define A(I, J) a[(J-1)*pda + I - 1]
#define B(I, J) b[(J-1)*pdb + I - 1]
  order = Nag_ColMajor;
#else
#define A(I, J) a[(I-1)*pda + J - 1]
#define B(I, J) b[(I-1)*pdb + J - 1]
  order = Nag_RowMajor;
#endif

  INIT_FAIL(fail);

  printf("nag_zggglm (f08zpc) Example Program Results\n\n");

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
#endif
#ifdef _WIN32
  scanf_s("%"NAG_IFMT"%"NAG_IFMT"%"NAG_IFMT"%*[^\n] ", &n, &m, &p);
#else
  scanf("%"NAG_IFMT"%"NAG_IFMT"%"NAG_IFMT"%*[^\n] ", &n, &m, &p);
#endif

#ifdef NAG_COLUMN_MAJOR
  pda = n;
  pdb = n;
#else
  pda = m;
  pdb = p;
#endif

  /* Allocate memory */
  if (!(a = NAG_ALLOC(n*m, Complex)) ||
      !(b = NAG_ALLOC(n*p, Complex)) ||
```

```
      !(d = NAG_ALLOC(n, Complex)) ||
      !(x = NAG_ALLOC(m, Complex)) ||
      !(y = NAG_ALLOC(p, Complex)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Read A, B and D from data file */
  for (i = 1; i <= n; ++i)
    {
      for (j = 1; j <= m; ++j)
#ifdef _WIN32
        scanf_s(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
#else
        scanf(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
#endif
    }
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
#endif

  for (i = 1; i <= n; ++i)
    {
      for (j = 1; j <= p; ++j)
#ifdef _WIN32
        scanf_s(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
#else
        scanf(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
#endif
    }
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
#endif

  for (i = 1; i <= n; ++i)
#ifdef _WIN32
    scanf_s(" ( %lf , %lf )", &d[i - 1].re, &d[i - 1].im);
#else
    scanf(" ( %lf , %lf )", &d[i - 1].re, &d[i - 1].im);
#endif
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
#endif

  /* Solve the weighted least-squares problem      */
  /* minimize ||inv(B)*(d - A*x)|| (in the 2-norm) */
  nag_zggglm(order, n, m, p, a, pda, b, pdb, d, x, y, &fail);

  if (fail.code == NE_NOERROR)
    {
      /* Print least-squares solution */
      printf("Weighted least-squares solution\n");
      for (i = 1; i <= m; ++i)
        printf("(%9.4f, %9.4f)%s", x[i - 1].re, x[i - 1].im,
                i%3 == 0 || i == m?"\n":" ");

      /* Print residual vector y = inv(B)*(d - A*x) */
      printf("\nResidual vector\n");
      for (i = 1; i <= p; ++i)
        printf("(%11.2e, %11.2e)%s", y[i - 1].re, y[i - 1].im,
                i%3 == 0 || i == p?"\n":" ");

      /* Compute and print the square root of the residual sum of */
```

```
      /* squares                                              */
      nag_zge_norm(Nag_ColMajor, Nag_FrobeniusNorm, 1, p, y, 1, &rnorm, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_zge_norm (f16uac).\n%s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }

      printf("\nSquare root of the residual sum of squares\n");
      printf("%11.2e\n", rnorm);
    }
  else
    {
      printf("Error from nag_zggglm (f08zpc).\n%s\n", fail.message);
      exit_status = 1;
    }

 END:
  NAG_FREE(a);
  NAG_FREE(b);
  NAG_FREE(d);
  NAG_FREE(x);
  NAG_FREE(y);

  return exit_status;
}
```

10.2 Program Data

```
nag_zggglm (f08zpc) Example Program Data

  4             3             4                          :Values of M, N and P

( 0.96,-0.81) (-0.03, 0.96) (-0.91, 2.06)
(-0.98, 1.98) (-1.20, 0.19) (-0.66, 0.42)
( 0.62,-0.46) ( 1.01, 0.02) ( 0.63,-0.17)
( 1.08,-0.28) ( 0.20,-0.12) (-0.07, 1.23)                :End of matrix A

( 0.50,-1.00) ( 0.00, 0.00) ( 0.00, 0.00) ( 0.00, 0.00)
( 0.00, 0.00) ( 1.00,-2.00) ( 0.00, 0.00) ( 0.00, 0.00)
( 0.00, 0.00) ( 0.00, 0.00) ( 2.00,-3.00) ( 0.00, 0.00)
( 0.00, 0.00) ( 0.00, 0.00) ( 0.00, 0.00) ( 5.00,-4.00) :End of matrix B

( 6.00,-0.40)
(-5.27, 0.90)
( 2.72,-2.13)
(-1.30,-2.80)                                            :End of vector d
```

10.3 Program Results

```
nag_zggglm (f08zpc) Example Program Results

Weighted least-squares solution
(  -0.9846,    1.9950) (   3.9929,   -4.9748) (  -3.0026,    0.9994)

Residual vector
(   1.26e-04,   -4.66e-04) (   1.11e-03,   -8.61e-04) (   3.84e-03,   -1.82e-03)
(   2.03e-03,    3.02e-03)

Square root of the residual sum of squares
   5.79e-03
```