NAG Library Function Document

nag_dgghrd (f08wec)

1 Purpose

nag_dgghrd (f08wec) reduces a pair of real matrices (A, B), where B is upper triangular, to the generalized upper Hessenberg form using orthogonal transformations.

2 Specification

```
#include <nag.h>
#include <nagf08.h>
```

```
void nag_dgghrd (Nag_OrderType order, Nag_ComputeQType compq,
     Nag_ComputeZType compz, Integer n, Integer ilo, Integer ihi, double a[],
     Integer pda, double b[], Integer pdb, double q[], Integer pdq,
     double z[], Integer pdz, NagError *fail)
```

3 Description

nag_dgghrd (f08wec) is the third step in the solution of the real generalized eigenvalue problem

$$Ax = \lambda Bx.$$

The (optional) first step balances the two matrices using nag_dggbal (f08whc). In the second step, matrix B is reduced to upper triangular form using the QR factorization function nag_dgeqrf (f08aec) and this orthogonal transformation Q is applied to matrix A by calling nag_dormqr (f08agc).

nag_dgghrd (f08wec) reduces a pair of real matrices (A, B), where B is upper triangular, to the generalized upper Hessenberg form using orthogonal transformations. This two-sided transformation is of the form

$$\begin{aligned} Q^{\mathrm{T}}AZ &= H \\ Q^{\mathrm{T}}BZ &= T \end{aligned}$$

where H is an upper Hessenberg matrix, T is an upper triangular matrix and Q and Z are orthogonal matrices determined as products of Givens rotations. They may either be formed explicitly, or they may be postmultiplied into input matrices Q_1 and Z_1, so that

$$\begin{aligned} Q_1AZ_1^{\mathrm{T}} &= (Q_1Q)H(Z_1Z)^{\mathrm{T}}, \\ Q_1BZ_1^{\mathrm{T}} &= (Q_1Q)T(Z_1Z)^{\mathrm{T}}. \end{aligned}$$

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Moler C B and Stewart G W (1973) An algorithm for generalized matrix eigenproblems *SIAM J. Numer. Anal.* **10** 241–256

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by

order $=$ Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: **order** $=$ Nag_RowMajor or Nag_ColMajor.

2: **compq** – Nag_ComputeQType *Input*

On entry: specifies the form of the computed orthogonal matrix Q.

compq $=$ Nag_NotQ
Do not compute Q.

compq $=$ Nag_InitQ
The orthogonal matrix Q is returned.

compq $=$ Nag_UpdateSchur
q must contain an orthogonal matrix Q_1, and the product Q_1Q is returned.

Constraint: **compq** $=$ Nag_NotQ, Nag_InitQ or Nag_UpdateSchur.

3: **compz** – Nag_ComputeZType *Input*

On entry: specifies the form of the computed orthogonal matrix Z.

compz $=$ Nag_NotZ
Do not compute Z.

compz $=$ Nag_InitZ
The orthogonal matrix Z is returned.

compz $=$ Nag_UpdateZ
z must contain an orthogonal matrix Z_1, and the product Z_1Z is returned.

Constraint: **compz** $=$ Nag_NotZ, Nag_UpdateZ or Nag_InitZ.

4: **n** – Integer *Input*

On entry: n, the order of the matrices A and B.

Constraint: **n** ≥ 0.

5: **ilo** – Integer *Input*
6: **ihi** – Integer *Input*

On entry: i_{lo} and i_{hi} as determined by a previous call to nag_dggbal (f08whc). Otherwise, they should be set to 1 and n, respectively.

Constraints:

if **n** > 0, $1 \leq$ **ilo** $\leq$ **ihi** $\leq$ **n**;
if **n** $= 0$, **ilo** $= 1$ and **ihi** $= 0$.

7: **a**[*dim*] – double *Input/Output*

Note: the dimension, *dim*, of the array **a** must be at least $\max(1, \mathbf{pda} \times \mathbf{n})$.

The (i,j)th element of the matrix A is stored in

a$[(j-1) \times \mathbf{pda} + i - 1]$ when **order** $=$ Nag_ColMajor;
a$[(i-1) \times \mathbf{pda} + j - 1]$ when **order** $=$ Nag_RowMajor.

On entry: the matrix A of the matrix pair (A,B). Usually, this is the matrix A returned by nag_dormqr (f08agc).

On exit: **a** is overwritten by the upper Hessenberg matrix H.

8: **pda** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **a**.

Constraint: **pda** $\geq \max(1, \mathbf{n})$.

9: **b**[*dim*] – double *Input/Output*

Note: the dimension, *dim*, of the array **b** must be at least $\max(1, \mathbf{pdb} \times \mathbf{n})$.

The (i, j)th element of the matrix B is stored in

b$[(j-1) \times \mathbf{pdb} + i - 1]$ when **order** = Nag_ColMajor;
b$[(i-1) \times \mathbf{pdb} + j - 1]$ when **order** = Nag_RowMajor.

On entry: the upper triangular matrix B of the matrix pair (A, B). Usually, this is the matrix B returned by the QR factorization function nag_dgeqrf (f08aec).

On exit: **b** is overwritten by the upper triangular matrix T.

10: **pdb** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **b**.

Constraint: **pdb** $\geq \max(1, \mathbf{n})$.

11: **q**[*dim*] – double *Input/Output*

Note: the dimension, *dim*, of the array **q** must be at least

$\max(1, \mathbf{pdq} \times \mathbf{n})$ when **compq** = Nag_InitQ or Nag_UpdateSchur;
1 when **compq** = Nag_NotQ.

The (i, j)th element of the matrix Q is stored in

q$[(j-1) \times \mathbf{pdq} + i - 1]$ when **order** = Nag_ColMajor;
q$[(i-1) \times \mathbf{pdq} + j - 1]$ when **order** = Nag_RowMajor.

On entry: if **compq** = Nag_UpdateSchur, **q** must contain an orthogonal matrix Q_1.

If **compq** = Nag_NotQ, **q** is not referenced.

On exit: if **compq** = Nag_InitQ, **q** contains the orthogonal matrix Q.

If **compq** = Nag_UpdateSchur, **q** is overwritten by Q_1Q.

12: **pdq** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **q**.

Constraints:

if **compq** = Nag_InitQ or Nag_UpdateSchur, **pdq** $\geq \max(1, \mathbf{n})$;
if **compq** = Nag_NotQ, **pdq** ≥ 1.

13: **z**[*dim*] – double *Input/Output*

Note: the dimension, *dim*, of the array **z** must be at least $\max(1, \mathbf{pdz} \times \mathbf{n})$ when **compz** = Nag_UpdateZ or Nag_InitZ.

The (i, j)th element of the matrix Z is stored in

z$[(j-1) \times \mathbf{pdz} + i - 1]$ when **order** = Nag_ColMajor;
z$[(i-1) \times \mathbf{pdz} + j - 1]$ when **order** = Nag_RowMajor.

On entry: if **compz** = Nag_UpdateZ, **z** must contain an orthogonal matrix Z_1.

If **compz** = Nag_NotZ, **z** is not referenced.

On exit: if **compz** = Nag_InitZ, **z** contains the orthogonal matrix Z.

If **compz** = Nag_UpdateZ, **z** is overwritten by Z_1Z.

14: **pdz** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **z**.

Constraints:

if **compz** = Nag_UpdateZ or Nag_InitZ, **pdz** $\geq \max(1, \mathbf{n})$;
if **compz** = Nag_NotZ, **pdz** ≥ 1.

15: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_ENUM_INT_2

On entry, **compq** = $\langle value \rangle$, **pdq** = $\langle value \rangle$ and **n** = $\langle value \rangle$.
Constraint: if **compq** = Nag_InitQ or Nag_UpdateSchur, **pdq** $\geq \max(1, \mathbf{n})$;
if **compq** = Nag_NotQ, **pdq** ≥ 1.

On entry, **compz** = $\langle value \rangle$, **pdz** = $\langle value \rangle$ and **n** = $\langle value \rangle$.
Constraint: if **compz** = Nag_UpdateZ or Nag_InitZ, **pdz** $\geq \max(1, \mathbf{n})$;
if **compz** = Nag_NotZ, **pdz** ≥ 1.

NE_INT

On entry, **n** = $\langle value \rangle$.
Constraint: **n** ≥ 0.

On entry, **pda** = $\langle value \rangle$.
Constraint: **pda** > 0.

On entry, **pdb** = $\langle value \rangle$.
Constraint: **pdb** > 0.

On entry, **pdq** = $\langle value \rangle$.
Constraint: **pdq** > 0.

On entry, **pdz** = $\langle value \rangle$.
Constraint: **pdz** > 0.

NE_INT_2

On entry, **pda** = $\langle value \rangle$ and **n** = $\langle value \rangle$.
Constraint: **pda** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** = $\langle value \rangle$ and **n** = $\langle value \rangle$.
Constraint: **pdb** $\geq \max(1, \mathbf{n})$.

NE_INT_3

On entry, **n** $= \langle value\rangle$, **ilo** $= \langle value\rangle$ and **ihi** $= \langle value\rangle$.
Constraint: if **n** > 0, $1 \leq$ **ilo** $\leq$ **ihi** $\leq$ **n**;
if **n** $= 0$, **ilo** $= 1$ and **ihi** $= 0$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

7 Accuracy

The reduction to the generalized Hessenberg form is implemented using orthogonal transformations which are backward stable.

8 Parallelism and Performance

nag_dgghrd (f08wec) is not threaded by NAG in any implementation.

nag_dgghrd (f08wec) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

This function is usually followed by nag_dhgeqz (f08xec) which implements the QZ algorithm for computing generalized eigenvalues of a reduced pair of matrices.

The complex analogue of this function is nag_zgghrd (f08wsc).

10 Example

See Section 10 in nag_dhgeqz (f08xec) and nag_dtgevc (f08ykc).