NAG Library Function Document

nag_dsptrd (f08gec)

1 Purpose

nag_dsptrd (f08gec) reduces a real symmetric matrix to tridiagonal form, using packed storage.

2 Specification

```
#include <nag.h>
#include <nagf08.h>
```

```
void nag_dsptrd (Nag_OrderType order, Nag_UploType uplo, Integer n,
     double ap[], double d[], double e[], double tau[], NagError *fail)
```

3 Description

nag_dsptrd (f08gec) reduces a real symmetric matrix A, held in packed storage, to symmetric tridiagonal form T by an orthogonal similarity transformation: $A = QTQ^{\mathrm{T}}$.

The matrix Q is not formed explicitly but is represented as a product of $n-1$ elementary reflectors (see the f08 Chapter Introduction for details). Functions are provided to work with Q in this representation (see Section 9).

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **uplo** – Nag_UploType *Input*

On entry: indicates whether the upper or lower triangular part of A is stored.

uplo = Nag_Upper
The upper triangular part of A is stored.

uplo = Nag_Lower
The lower triangular part of A is stored.

Constraint: **uplo** = Nag_Upper or Nag_Lower.

3: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: **n** ≥ 0.

4: **ap**$[dim]$ – double *Input/Output*

Note: the dimension, *dim*, of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n} + 1)/2)$.

On entry: the upper or lower triangle of the n by n symmetric matrix A, packed by rows or columns.

The storage of elements A_{ij} depends on the **order** and **uplo** arguments as follows:

if **order** = Nag_ColMajor and **uplo** = Nag_Upper,
A_{ij} is stored in **ap**$[(j-1) \times j/2 + i - 1]$, for $i \leq j$;
if **order** = Nag_ColMajor and **uplo** = Nag_Lower,
A_{ij} is stored in **ap**$[(2n-j) \times (j-1)/2 + i - 1]$, for $i \geq j$;
if **order** = Nag_RowMajor and **uplo** = Nag_Upper,
A_{ij} is stored in **ap**$[(2n-i) \times (i-1)/2 + j - 1]$, for $i \leq j$;
if **order** = Nag_RowMajor and **uplo** = Nag_Lower,
A_{ij} is stored in **ap**$[(i-1) \times i/2 + j - 1]$, for $i \geq j$.

On exit: **ap** is overwritten by the tridiagonal matrix T and details of the orthogonal matrix Q.

5: **d**[**n**] – double *Output*

On exit: the diagonal elements of the tridiagonal matrix T.

6: **e**[**n** − **1**] – double *Output*

On exit: the off-diagonal elements of the tridiagonal matrix T.

7: **tau**[**n** − **1**] – double *Output*

On exit: further details of the orthogonal matrix Q.

8: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 0.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

7 Accuracy

The computed tridiagonal matrix T is exactly similar to a nearby matrix $(A+E)$, where

$$\|E\|_2 \leq c(n)\epsilon\|A\|_2,$$

$c(n)$ is a modestly increasing function of n, and ϵ is the ***machine precision***.

The elements of T themselves may be sensitive to small perturbations in A or to rounding errors in the computation, but this does not affect the stability of the eigenvalues and eigenvectors.

8 Parallelism and Performance

nag_dsptrd (f08gec) is not threaded by NAG in any implementation.

nag_dsptrd (f08gec) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The total number of floating-point operations is approximately $\frac{4}{3}n^3$.

To form the orthogonal matrix Q nag_dsptrd (f08gec) may be followed by a call to nag_dopgtr (f08gfc):

```
nag_dopgtr(order,uplo,n,ap,tau,&q,pdq,&fail)
```

To apply Q to an n by p real matrix C nag_dsptrd (f08gec) may be followed by a call to nag_dopmtr (f08ggc). For example,

```
nag_dopmtr(order,Nag_LeftSide,uplo,Nag_NoTrans,n,p,ap,tau,&c,
  pdc,&fail)
```

forms the matrix product QC.

The complex analogue of this function is nag_zhptrd (f08gsc).

10 Example

This example reduces the matrix A to tridiagonal form, where

$$A = \begin{pmatrix} 2.07 & 3.87 & 4.20 & -1.15 \\ 3.87 & -0.21 & 1.87 & 0.63 \\ 4.20 & 1.87 & 1.15 & 2.06 \\ -1.15 & 0.63 & 2.06 & -1.81 \end{pmatrix},$$

using packed storage.

10.1 Program Text

```
/* nag_dsptrd (f08gec) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf08.h>
#include <nagx04.h>
```

```
int main(void)
{
  /* Scalars */
  Integer       ap_len, i, j, n, pdz, d_len, e_len, tau_len;
  Integer       exit_status = 0;
  NagError      fail;
  Nag_UploType  uplo;
  Nag_OrderType order;
  /* Arrays */
  char          nag_enum_arg[40];
  double        *ap = 0, *d = 0, *e = 0, *tau = 0, *z = 0;

#ifdef NAG_COLUMN_MAJOR
#define A_UPPER(I, J) ap[J * (J - 1) / 2 + I - 1]
#define A_LOWER(I, J) ap[(2 * n - J) * (J - 1) / 2 + I - 1]
#define Z(I, J) z[(J - 1) * pdz + I - 1]
  order = Nag_ColMajor;
#else
#define A_LOWER(I, J) ap[I * (I - 1) / 2 + J - 1]
#define A_UPPER(I, J) ap[(2 * n - I) * (I - 1) / 2 + J - 1]
#define Z(I, J) z[(I - 1) * pdz + J - 1]
  order = Nag_RowMajor;
#endif

  INIT_FAIL(fail);

  printf("nag_dsptrd (f08gec) Example Program Results\n\n");

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n] ");
#else
  scanf("%*[^\n] ");
#endif
#ifdef _WIN32
  scanf_s("%"NAG_IFMT"%*[^\n] ", &n);
#else
  scanf("%"NAG_IFMT"%*[^\n] ", &n);
#endif
#ifdef NAG_COLUMN_MAJOR
  pdz = n;
#else
  pdz = n;
#endif
  ap_len = n*(n+1)/2;
  tau_len = n-1;
  d_len = n;
  e_len = n-1;
  /* Allocate memory */
  if (!(ap = NAG_ALLOC(ap_len, double)) ||
      !(d = NAG_ALLOC(d_len, double)) ||
      !(e = NAG_ALLOC(e_len, double)) ||
      !(tau = NAG_ALLOC(tau_len, double)) ||
      !(z = NAG_ALLOC(n * n, double)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Read A from data file */
#ifdef _WIN32
  scanf_s("%39s%*[^\n] ", nag_enum_arg, _countof(nag_enum_arg));
#else
  scanf("%39s%*[^\n] ", nag_enum_arg);
#endif
  /* nag_enum_name_to_value (x04nac).
   * Converts NAG enum member name to value
   */
  uplo = (Nag_UploType) nag_enum_name_to_value(nag_enum_arg);
  if (uplo == Nag_Upper)
```

```
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = i; j <= n; ++j)
#ifdef _WIN32
            scanf_s("%lf", &A_UPPER(i, j));
#else
            scanf("%lf", &A_UPPER(i, j));
#endif
        }
#ifdef _WIN32
      scanf_s("%*[^\n] ");
#else
      scanf("%*[^\n] ");
#endif
    }
  else
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = 1; j <= i; ++j)
#ifdef _WIN32
            scanf_s("%lf", &A_LOWER(i, j));
#else
            scanf("%lf", &A_LOWER(i, j));
#endif
        }
#ifdef _WIN32
      scanf_s("%*[^\n] ");
#else
      scanf("%*[^\n] ");
#endif
    }

  /* Reduce A to tridiagonal form T = (Q**T)*A*Q */
  /* nag_dsptrd (f08gec).
   * Orthogonal reduction of real symmetric matrix to
   * symmetric tridiagonal form, packed storage
   */
  nag_dsptrd(order, uplo, n, ap, d, e, tau, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_dsptrd (f08gec).\n%s\n", fail.message);
      exit_status = 1;
    }

  /* Form Q explicitly, storing the result in Z */
  /* nag_dopgtr (f08gfc).
   * Generate orthogonal transformation matrix from reduction
   * to tridiagonal form determined by nag_dsptrd (f08gec)
   */
  nag_dopgtr(order, uplo, n, ap, tau, z, pdz, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_dopgtr (f08gfc).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

  /* Calculate all the eigenvalues and eigenvectors of A */
  /* nag_dsteqr (f08jec).
   * All eigenvalues and eigenvectors of real symmetric
   * tridiagonal matrix, reduced from real symmetric matrix
   * using implicit QL or QR
   */
  nag_dsteqr(order, Nag_UpdateZ, n, d, e, z, pdz, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_dsteqr (f08jec).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
```

```
    }
  /* Normalize the eigenvectors */
  for(j=1; j<=n; j++)
    {
      for(i=n; i>=1; i--)
        {
          Z(i, j) = Z(i, j) / Z(1,j);
        }
    }
  /* Print eigenvalues and eigenvectors */
  printf("Eigenvalues\n");
  for (i = 1; i <= n; ++i)
    printf("%8.4f%s", d[i-1], i%8 == 0?"\n":" ");
  printf("\n\n");
  /* nag_gen_real_mat_print (x04cac).
   * Print real general matrix (easy-to-use)
   */
  fflush(stdout);
  nag_gen_real_mat_print(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, n, z,
                         pdz, "Eigenvectors", 0, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_gen_real_mat_print (x04cac).\n%s\n",
              fail.message);
      exit_status = 1;
      goto END;
    }
 END:
  NAG_FREE(ap);
  NAG_FREE(d);
  NAG_FREE(e);
  NAG_FREE(tau);
  NAG_FREE(z);

  return exit_status;
}
```

10.2 Program Data

```
nag_dsptrd (f08gec) Example Program Data
  4                           :Value of n
  Nag_Lower                   :Value of uplo
  2.07
  3.87  -0.21
  4.20   1.87   1.15
 -1.15   0.63   2.06  -1.81   :End of matrix A
```

10.3 Program Results

```
nag_dsptrd (f08gec) Example Program Results

Eigenvalues
 -5.0034  -1.9987   0.2013   8.0008

 Eigenvectors
           1          2          3          4
 1      1.0000     1.0000     1.0000     1.0000
 2     -0.6148    -3.4333     0.4489     0.6668
 3     -0.8378     1.7553    -1.3572     0.8248
 4      1.0219    -1.6052    -1.8213     0.0988
```