NAG Library Function Document

nag_dsyev (f08fac)

1 Purpose

nag_dsyev (f08fac) computes all the eigenvalues and, optionally, all the eigenvectors of a real n by n symmetric matrix A.

2 Specification

```
#include <nag.h>
#include <nagf08.h>
```

```
void nag_dsyev (Nag_OrderType order, Nag_JobType job, Nag_UploType uplo,
     Integer n, double a[], Integer pda, double w[], NagError *fail)
```

3 Description

The symmetric matrix A is first reduced to tridiagonal form, using orthogonal similarity transformations, and then the QR algorithm is applied to the tridiagonal matrix to compute the eigenvalues and (optionally) the eigenvectors.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **job** – Nag_JobType *Input*

On entry: indicates whether eigenvectors are computed.

job = Nag_EigVals
Only eigenvalues are computed.

job = Nag_DoBoth
Eigenvalues and eigenvectors are computed.

Constraint: **job** = Nag_EigVals or Nag_DoBoth.

3: **uplo** – Nag_UploType *Input*

On entry: if **uplo** = Nag_Upper, the upper triangular part of A is stored.

If **uplo** $=$ Nag_Lower, the lower triangular part of A is stored.

Constraint: **uplo** $=$ Nag_Upper or Nag_Lower.

4: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5: **a**[*dim*] – double *Input/Output*

Note: the dimension, *dim*, of the array **a** must be at least $\max(1, \mathbf{pda} \times \mathbf{n})$.

On entry: the n by n symmetric matrix A.

If **order** $=$ Nag_ColMajor, A_{ij} is stored in $\mathbf{a}[(j-1) \times \mathbf{pda} + i - 1]$.

If **order** $=$ Nag_RowMajor, A_{ij} is stored in $\mathbf{a}[(i-1) \times \mathbf{pda} + j - 1]$.

If **uplo** $=$ Nag_Upper, the upper triangular part of A must be stored and the elements of the array below the diagonal are not referenced.

If **uplo** $=$ Nag_Lower, the lower triangular part of A must be stored and the elements of the array above the diagonal are not referenced.

On exit: if **job** $=$ Nag_DoBoth, then **a** contains the orthonormal eigenvectors of the matrix A.

If **job** $=$ Nag_EigVals, then on exit the lower triangle (if **uplo** $=$ Nag_Lower) or the upper triangle (if **uplo** $=$ Nag_Upper) of **a**, including the diagonal, is overwritten.

6: **pda** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **a**.

Constraint: $\mathbf{pda} \geq \max(1, \mathbf{n})$.

7: **w**[**n**] – double *Output*

On exit: the eigenvalues in ascending order.

8: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_CONVERGENCE

The algorithm failed to converge; ⟨*value*⟩ off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

NE_INT

On entry, $\mathbf{n} = \langle value \rangle$.
Constraint: $\mathbf{n} \geq 0$.

On entry, **pda** $= \langle value \rangle$.
Constraint: **pda** > 0.

NE_INT_2

On entry, **pda** $= \langle value \rangle$ and **n** $= \langle value \rangle$.
Constraint: **pda** $\geq \max(1, \mathbf{n})$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. See Section 4.7 of Anderson *et al.* (1999) for further details.

8 Parallelism and Performance

nag_dsyev (f08fac) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag_dsyev (f08fac) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the X06 Chapter Introduction for information on how to control and interrogate the OpenMP environment used within this function. Please also consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The total number of floating-point operations is proportional to n^3.

The complex analogue of this function is nag_zheev (f08fnc).

10 Example

This example finds all the eigenvalues and eigenvectors of the symmetric matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 3 & 4 \\ 3 & 3 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix},$$

together with approximate error bounds for the computed eigenvalues and eigenvectors.

10.1 Program Text

```
/* nag_dsyev (f08fac) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 23, 2011.
 */

#include <math.h>
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf08.h>
#include <nagx02.h>
#include <nagx04.h>

int main(void)
{
  /* Scalars */
  double        eerrbd, eps;
  Integer       i, j, n, pda;
  Integer       exit_status = 0;
  /* Arrays */
  double        *a = 0, *rcondz = 0, *w = 0, *zerrbd = 0;
  /* Nag Types */
  Nag_OrderType order;
  NagError      fail;

#ifdef NAG_COLUMN_MAJOR
#define A(I, J) a[(J - 1) * pda + I - 1]
  order = Nag_ColMajor;
#else
#define A(I, J) a[(I - 1) * pda + J - 1]
  order = Nag_RowMajor;
#endif

  INIT_FAIL(fail);

  printf("nag_dsyev (f08fac) Example Program Results\n\n");

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif
#ifdef _WIN32
  scanf_s("%"NAG_IFMT"%*[^\n]", &n);
#else
  scanf("%"NAG_IFMT"%*[^\n]", &n);
#endif

  /* Allocate memory */
  if (!(a = NAG_ALLOC(n*n, double)) ||
      !(rcondz = NAG_ALLOC(n, double)) ||
      !(w = NAG_ALLOC(n, double)) ||
      !(zerrbd = NAG_ALLOC(n, double)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }
  pda = n;

  /* Read the upper triangular part of the matrix A from data file */
  for (i = 1; i <= n; ++i)
    for (j = i; j <= n; ++j)
#ifdef _WIN32
      scanf_s("%lf", &A(i, j));
#else
```

```
      scanf("%lf", &A(i, j));
#endif
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif

  /* nag_dsyev (f08fac).
   * Solve the symmetric eigenvalue problem.
   */
  nag_dsyev(order, Nag_DoBoth, Nag_Upper, n, a, pda, w, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_dsyev (f08fac).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

  /* Normalize the eigenvectors */
  for(j=1; j<=n; j++)
    for(i=n; i>=1; i--)
      A(i, j) = A(i, j) / A(1,j);

  /* Print solution */
  printf("Eigenvalues\n");
  for (j = 0; j < n; ++j)
    printf("%8.4f%s", w[j], (j+1)%8 == 0?"\n":" ");
  printf("\n\n");

  /* nag_gen_real_mat_print (x04cac).
   * Print eigenvectors.
   */
  fflush(stdout);
  nag_gen_real_mat_print(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, n,
                         a, pda, "Eigenvectors", 0, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_gen_real_mat_print (x04cac).\n%s\n",
             fail.message);
      exit_status = 1;
      goto END;
    }

  /* Get the machine precision, eps, using nag_machine_precision (X02AJC)
   * and compute the approximate error bound for the computed eigenvalues.
   * Note that for the 2-norm, ||A|| = max {|w[i]|, i=0..n-1}, and since
   * the eigenvalues are in ascending order ||A|| = max( |w[0]|, |w[n-1]|).
   */
  eps = nag_machine_precision;
  eerrbd = eps * MAX(fabs(w[0]), fabs(w[n-1]));

  /* nag_ddisna (f08flc).
   * Estimate reciprocal condition numbers for the eigenvectors.
   */
  nag_ddisna(Nag_EigVecs, n, n, w, rcondz, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_ddisna (f08flc).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

  /* Compute the error estimates for the eigenvectors */
  for (i = 0; i < n; ++i)
    zerrbd[i] = eerrbd / rcondz[i];

  /* Print the approximate error bounds for the eigenvalues and vectors */
  printf("\nError estimate for the eigenvalues\n");
  printf("%11.1e\n\n", eerrbd);
  printf("Error estimates for the eigenvectors\n");
```

```
  for (i = 0; i < n; ++i)
    printf("%11.1e%s", zerrbd[i], (i+1)%6 == 0?"\n":" ");

 END:
  NAG_FREE(a);
  NAG_FREE(rcondz);
  NAG_FREE(w);
  NAG_FREE(zerrbd);

  return exit_status;
}

#undef A
```

10.2 Program Data

```
nag_dsyev (f08fac) Example Program Data

  4                 :Value of n

  1.0  2.0  3.0  4.0
       2.0  3.0  4.0
            3.0  4.0
                 4.0 :End of matrix A
```

10.3 Program Results

```
nag_dsyev (f08fac) Example Program Results

Eigenvalues
 -2.0531  -0.5146  -0.2943  12.8621

 Eigenvectors
           1          2          3          4
 1      1.0000     1.0000     1.0000     1.0000
 2      0.5129    -0.9431    -2.3976     1.0777
 3     -0.2240    -1.0537     2.3508     1.2393
 4     -0.8518     0.8831    -0.8879     1.4972

Error estimate for the eigenvalues
    1.4e-15

Error estimates for the eigenvectors
    9.3e-16     6.5e-15     6.5e-15     1.1e-16
```