NAG Library Function Document

nag_dpoequ (f07ffc)

1 Purpose

nag_dpoequ (f07ffc) computes a diagonal scaling matrix S intended to equilibrate a real n by n symmetric positive definite matrix A and reduce its condition number.

2 Specification

```
#include <nag.h>
#include <nagf07.h>
```

```
void nag_dpoequ (Nag_OrderType order, Integer n, const double a[],
     Integer pda, double s[], double *scond, double *amax, NagError *fail)
```

3 Description

nag_dpoequ (f07ffc) computes a diagonal scaling matrix S chosen so that

$$s_j = 1/\sqrt{a_{jj}}.$$

This means that the matrix B given by

$$B = SAS,$$

has diagonal elements equal to unity. This in turn means that the condition number of B, $\kappa_2(B)$, is within a factor n of the matrix of smallest possible condition number over all possible choices of diagonal scalings (see Corollary 7.6 of Higham (2002)).

4 References

Higham N J (2002) *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: **n** ≥ 0.

3: **a**[dim] – const double *Input*

Note: the dimension, *dim*, of the array **a** must be at least $\max(1, \mathbf{pda} \times \mathbf{n})$.

The (i,j)th element of the matrix A is stored in

$\mathbf{a}[(j-1) \times \mathbf{pda} + i - 1]$ when **order** = Nag_ColMajor;
$\mathbf{a}[(i-1) \times \mathbf{pda} + j - 1]$ when **order** = Nag_RowMajor.

On entry: the matrix A whose scaling factors are to be computed. Only the diagonal elements of the array **a** are referenced.

4: **pda** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **a**.

Constraint: $\mathbf{pda} \geq \max(1, \mathbf{n})$.

5: **s**[**n**] – double *Output*

On exit: if **fail.code** = NE_NOERROR, **s** contains the diagonal elements of the scaling matrix S.

6: **scond** – double * *Output*

On exit: if **fail.code** = NE_NOERROR, **scond** contains the ratio of the smallest value of **s** to the largest value of **s**. If $\mathbf{scond} \geq 0.1$ and **amax** is neither too large nor too small, it is not worth scaling by S.

7: **amax** – double * *Output*

On exit: $\max|a_{ij}|$. If **amax** is very close to overflow or underflow, the matrix A should be scaled.

8: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_BAD_PARAM

On entry, argument $\langle value\rangle$ had an illegal value.

NE_INT

On entry, $\mathbf{n} = \langle value\rangle$.
Constraint: $\mathbf{n} \geq 0$.

On entry, $\mathbf{pda} = \langle value\rangle$.
Constraint: $\mathbf{pda} > 0$.

NE_INT_2

On entry, $\mathbf{pda} = \langle value\rangle$ and $\mathbf{n} = \langle value\rangle$.
Constraint: $\mathbf{pda} \geq \max(1, \mathbf{n})$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_MAT_NOT_POS_DEF

The $\langle value\rangle$th diagonal element of A is not positive (and hence A cannot be positive definite).

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly. See Section 3.6.5 in the Essential Introduction for further information.

7 Accuracy

The computed scale factors will be close to the exact scale factors.

8 Parallelism and Performance

Not applicable.

9 Further Comments

The complex analogue of this function is nag_zpoequ (f07ftc).

10 Example

This example equilibrates the symmetric positive definite matrix A given by

$$A = \begin{pmatrix} 4.16 & -3.12 \times 10^5 & 0.56 & -0.10 \\ -3.12 \times 10^5 & 5.03 \times 10^{10} & -0.83 \times 10^5 & 1.18 \times 10^5 \\ 0.56 & -0.83 \times 10^5 & 0.76 & 0.34 \\ -0.10 & 1.18 \times 10^5 & 0.34 & 1.18 \end{pmatrix}.$$

Details of the scaling factors and the scaled matrix are output.

10.1 Program Text

```
/* nag_dpoequ (f07ffc) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 23, 2011.
 */

#include <stdio.h>
#include <nag.h>
#include <nagx04.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx02.h>

int main(void)
{
  /* Scalars */
  double        amax, big, scond, small;
  Integer       i, j, n, pda;
  Integer       exit_status = 0;
  /* Arrays */
  double        *a = 0, *s = 0;

  /* Nag Types */
  NagError      fail;
  Nag_OrderType order;

#ifdef NAG_COLUMN_MAJOR
#define A(I, J) a[(J-1)*pda + I - 1]
  order = Nag_ColMajor;
#else
#define A(I, J) a[(I-1)*pda + J - 1]
  order = Nag_RowMajor;
#endif
```

```
  INIT_FAIL(fail);

  printf("nag_dpoequ (f07ffc) Example Program Results\n\n");

  /* Skip heading in data file */
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif
#ifdef _WIN32
  scanf_s("%"NAG_IFMT"%*[^\n]", &n);
#else
  scanf("%"NAG_IFMT"%*[^\n]", &n);
#endif
  if (n < 0)
    {
      printf("Invalid n\n");
      exit_status = 1;
      goto END;
    }

  pda = n;
  /* Allocate memory */
  if (!(a = NAG_ALLOC(n * n, double)) ||
      !(s = NAG_ALLOC(n, double)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Read the upper triangular part of the matrix A from data file */
  for (i = 1; i <= n; ++i)
#ifdef _WIN32
    for (j = i; j <= n; ++j) scanf_s("%lf", &A(i, j));
#else
    for (j = i; j <= n; ++j) scanf("%lf", &A(i, j));
#endif
#ifdef _WIN32
  scanf_s("%*[^\n]");
#else
  scanf("%*[^\n]");
#endif

  /* Print the matrix A using nag_gen_real_mat_print (x04cac). */
  fflush(stdout);
  nag_gen_real_mat_print(order, Nag_UpperMatrix, Nag_NonUnitDiag, n, n, a, pda,
                         "Matrix A", 0, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_gen_real_mat_print (x04cac).\n%s\n",
              fail.message);
      exit_status = 1;
      goto END;
    }

  printf("\n");
  /* Compute diagonal scaling factors using nag_dpoequ (f07ffc).*/
  nag_dpoequ(order, n, a, pda, s, &scond, &amax, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_dpoequ (f07ffc).\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

  /* Print scond, amax and the scale factors */
  printf("scond = %10.1e, amax = %10.1e\n\n", scond, amax);
  printf("Diagonal scaling factors\n");
  for (i = 0; i < n; ++i) printf("%11.1e%s", s[i], i%7 == 6?"\n":" ");
```

```
  printf("\n\n");

  /* Compute values close to underflow and overflow using
   * nag_real_safe_small_number (x02amc), nag_machine_precision (x02ajc) and
   * nag_real_base (x02bhc)
   */
  small = nag_real_safe_small_number / (nag_machine_precision * nag_real_base);
  big = 1.0 / small;
  if (scond < 0.1 || amax < small || amax > big)
    {
      /* Scale A */
      for (j = 1; j <= n; ++j)
        for (i = 1; i <= j; ++i) A(i, j) *= s[i-1] * s[j-1];

      /* Print the scaled matrix using nag_gen_real_mat_print (x04cac). */
      fflush(stdout);
      nag_gen_real_mat_print(order, Nag_UpperMatrix, Nag_NonUnitDiag, n, n, a,
                             pda, "Scaled matrix", 0, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_gen_real_mat_print (x04cac).\n%s\n",
                 fail.message);
          exit_status = 1;
          goto END;
        }
    }

 END:
  NAG_FREE(a);
  NAG_FREE(s);

  return exit_status;
}
#undef A
```

10.2 Program Data

```
nag_dpoequ (f07ffc) Example Program Data
  4                                          : n
  4.16      -3.12e+05   0.56      -0.10
             5.03e+10  -0.83e+05   1.18e+05
                        0.76       0.34
                                   1.18      : matrix A
```

10.3 Program Results

```
nag_dpoequ (f07ffc) Example Program Results

 Matrix A
             1            2            3            4
 1    4.1600e+00  -3.1200e+05   5.6000e-01  -1.0000e-01
 2                 5.0300e+10  -8.3000e+04   1.1800e+05
 3                              7.6000e-01   3.4000e-01
 4                                           1.1800e+00

scond =    3.9e-06, amax =    5.0e+10

Diagonal scaling factors
    4.9e-01     4.5e-06     1.1e+00     9.2e-01

 Scaled matrix
           1          2          3          4
 1      1.0000    -0.6821     0.3149    -0.0451
 2                 1.0000    -0.4245     0.4843
 3                            1.0000     0.3590
 4                                       1.0000
```