NAG Library Function Document

nag_1d_cheb_intg (e02ajc)

1 Purpose

nag_1d_cheb_intg (e02ajc) determines the coefficients in the Chebyshev series representation of the indefinite integral of a polynomial given in Chebyshev series form.

2 Specification

```
#include <nag.h>
#include <nage02.h>
```

```
void nag_1d_cheb_intg (Integer n, double xmin, double xmax, const double a[],
     Integer ia1, double qatm1, double aintc[], Integer iaint1,
     NagError *fail)
```

3 Description

nag_1d_cheb_intg (e02ajc) forms the polynomial which is the indefinite integral of a given polynomial. Both the original polynomial and its integral are represented in Chebyshev series form. If supplied with the coefficients a_i, for $i = 0, 1, \ldots, n$, of a polynomial $p(x)$ of degree n, where

$$p(x) = \tfrac{1}{2}a_0 + a_1 T_1(\bar{x}) + \cdots + a_n T_n(\bar{x}),$$

the function returns the coefficients a'_i, for $i = 0, 1, \ldots, n+1$, of the polynomial $q(x)$ of degree $n+1$, where

$$q(x) = \tfrac{1}{2}a'_0 + a'_1 T_1(\bar{x}) + \cdots + a'_{n+1} T_{n+1}(\bar{x}),$$

and

$$q(x) = \int p(x)dx.$$

Here $T_j(\bar{x})$ denotes the Chebyshev polynomial of the first kind of degree j with argument $\bar{x}$. It is assumed that the normalized variable $\bar{x}$ in the interval $[-1, +1]$ was obtained from your original variable x in the interval $[x_{\text{min}}, x_{\text{max}}]$ by the linear transformation

$$\bar{x} = \frac{2x - (x_{\text{max}} + x_{\text{min}})}{x_{\text{max}} - x_{\text{min}}}$$

and that you require the integral to be with respect to the variable x. If the integral with respect to $\bar{x}$ is required, set $x_{\text{max}} = 1$ and $x_{\text{min}} = -1$.

Values of the integral can subsequently be computed, from the coefficients obtained, by using nag_1d_cheb_eval2 (e02akc).

The method employed is that of Chebyshev series (see Chapter 8 of Modern Computing Methods (1961)), modified for integrating with respect to x. Initially taking $a_{n+1} = a_{n+2} = 0$, the function forms successively

$$a'_i = \frac{a_{i-1} - a_{i+1}}{2i} \times \frac{x_{\text{max}} - x_{\text{min}}}{2}, \quad i = n+1, n, \ldots, 1.$$

The constant coefficient a'_0 is chosen so that $q(x)$ is equal to a specified value, **qatm1**, at the lower end point of the interval on which it is defined, i.e., $\bar{x} = -1$, which corresponds to $x = x_{\text{min}}$.

4 References

Modern Computing Methods (1961) Chebyshev-series *NPL Notes on Applied Science* **16** (2nd Edition) HMSO

5 Arguments

1: **n** – Integer *Input*

On entry: n, the degree of the given polynomial $p(x)$.

Constraint: $\mathbf{n} \geq 0$.

2: **xmin** – double *Input*

3: **xmax** – double *Input*

On entry: the lower and upper end points respectively of the interval $[x_{\mathrm{min}}, x_{\mathrm{max}}]$. The Chebyshev series representation is in terms of the normalized variable $\bar{x}$, where

$$\bar{x} = \frac{2x - (x_{\mathrm{max}} + x_{\mathrm{min}})}{x_{\mathrm{max}} - x_{\mathrm{min}}}.$$

Constraint: **xmax** $>$ **xmin**.

4: **a**[*dim*] – const double *Input*

Note: the dimension, *dim*, of the array **a** must be at least $(1 + (\mathbf{n} + 1 - 1) \times \mathbf{ia1})$.

On entry: the Chebyshev coefficients of the polynomial $p(x)$. Specifically, element $i \times \mathbf{ia1}$ of **a** must contain the coefficient a_i, for $i = 0, 1, \ldots, n$. Only these $n + 1$ elements will be accessed.

5: **ia1** – Integer *Input*

On entry: the index increment of **a**. Most frequently the Chebyshev coefficients are stored in adjacent elements of **a**, and **ia1** must be set to 1. However, if for example, they are stored in $\mathbf{a}[0], \mathbf{a}[3], \mathbf{a}[6], \ldots$, then the value of **ia1** must be 3. See also Section 9.

Constraint: $\mathbf{ia1} \geq 1$.

6: **qatm1** – double *Input*

On entry: the value that the integrated polynomial is required to have at the lower end point of its interval of definition, i.e., at $\bar{x} = -1$ which corresponds to $x = x_{\mathrm{min}}$. Thus, **qatm1** is a constant of integration and will normally be set to zero by you.

7: **aintc**[*dim*] – double *Output*

Note: the dimension, *dim*, of the array **aintc** must be at least $(1 + (\mathbf{n} + 1) \times \mathbf{iaint1})$.

On exit: the Chebyshev coefficients of the integral $q(x)$. (The integration is with respect to the variable x, and the constant coefficient is chosen so that $q(x_{\mathrm{min}})$ equals **qatm1**). Specifically, element $i \times \mathbf{iaint1}$ of **aintc** contains the coefficient a_i', for $i = 0, 1, \ldots, n + 1$.

8: **iaint1** – Integer *Input*

On entry: the index increment of **aintc**. Most frequently the Chebyshev coefficients are required in adjacent elements of **aintc**, and **iaint1** must be set to 1. However, if, for example, they are to be stored in $\mathbf{aintc}[0], \mathbf{aintc}[3], \mathbf{aintc}[6], \ldots$, then the value of **iaint1** must be 3. See also Section 9.

Constraint: $\mathbf{iaint1} \geq 1$.

9: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.
See Section 3.2.1.2 in the Essential Introduction for further information.

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_INT

On entry, **ia1** $= \langle value \rangle$.
Constraint: **ia1** ≥ 1.

On entry, **iaint1** $= \langle value \rangle$.
Constraint: **iaint1** ≥ 1.

On entry, $\mathbf{n} + 1 = \langle value \rangle$.
Constraint: $\mathbf{n} + 1 \geq 1$.

On entry, $\mathbf{n} = \langle value \rangle$.
Constraint: $\mathbf{n} \geq 0$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

An unexpected error has been triggered by this function. Please contact NAG.
See Section 3.6.6 in the Essential Introduction for further information.

NE_NO_LICENCE

Your licence key may have expired or may not have been installed correctly.
See Section 3.6.5 in the Essential Introduction for further information.

NE_REAL_2

On entry, **xmax** $= \langle value \rangle$ and **xmin** $= \langle value \rangle$.
Constraint: **xmax** $>$ **xmin**.

7 Accuracy

In general there is a gain in precision in numerical integration, in this case associated with the division by $2i$ in the formula quoted in Section 3.

8 Parallelism and Performance

Not applicable.

9 Further Comments

The time taken is approximately proportional to $n + 1$.

The increments **ia1**, **iaint1** are included as arguments to give a degree of flexibility which, for example, allows a polynomial in two variables to be integrated with respect to either variable without rearranging the coefficients.

10 Example

Suppose a polynomial has been computed in Chebyshev series form to fit data over the interval $[-0.5, 2.5]$. The following program evaluates the integral of the polynomial from 0.0 to 2.0. (For the purpose of this example, **xmin**, **xmax** and the Chebyshev coefficients are simply supplied . Normally a program would read in or generate data and compute the fitted polynomial).

10.1 Program Text

```
/* nag_1d_cheb_intg (e02ajc) Example Program.
 *
 * Copyright 2014 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nage02.h>

int main(void)
{
  /* Initialized data */
  const double xmin = -0.5;
  const double xmax = 2.5;
  const double a[7] =
  { 2.53213, 1.13032, 0.2715, 0.04434, 0.00547, 5.4e-4, 4e-5 };

  /* Scalars */
  double       ra, rb, result, xa, xb, zero;
  Integer      exit_status, n, one;
  NagError     fail;

  /* Arrays */
  double       *aint = 0;

  INIT_FAIL(fail);

  exit_status = 0;
  printf("nag_1d_cheb_intg (e02ajc) Example Program Results\n");

  n = 6;
  zero = 0.0;
  one = 1;

  /* Allocate memory */
  if (!(aint = NAG_ALLOC(n + 2, double)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* nag_1d_cheb_intg (e02ajc).
   * Integral of fitted polynomial in Chebyshev series form
   */
  nag_1d_cheb_intg(n, xmin, xmax, a, one, zero, aint, one, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_1d_cheb_intg (e02ajc).\n%s\n",
              fail.message);
      exit_status = 1;
      goto END;
    }

  xa = 0.0;
  xb = 2.0;
  /* nag_1d_cheb_eval2 (e02akc).
```

```
   * Evaluation of fitted polynomial in one variable from
   * Chebyshev series form
   */
  nag_1d_cheb_eval2(n+1, xmin, xmax, aint, one, xa, &ra, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_1d_cheb_eval2 (e02akc).\n%s\n",
              fail.message);
      exit_status = 1;
      goto END;
    }

  /* nag_1d_cheb_eval2 (e02akc), see above. */
  nag_1d_cheb_eval2(n+1, xmin, xmax, aint, one, xb, &rb, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_1d_cheb_eval2 (e02akc).\n%s\n",
              fail.message);
      exit_status = 1;
      goto END;
    }

  result = rb - ra;
  printf("\n");
  printf("Value of definite integral is %10.4f\n", result);
 END:
  NAG_FREE(aint);

  return exit_status;
}
```

10.2 Program Data

None.

10.3 Program Results

```
nag_1d_cheb_intg (e02ajc) Example Program Results

Value of definite integral is     2.1515
```