NAG Library Function Document

nag_elliptic_integral_pi (s21bgc)

1 Purpose

nag_elliptic_integral_pi (s21bgc) returns a value of the classical (Legendre) form of the incomplete elliptic integral of the third kind.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_elliptic_integral_pi (double dn, double phi, double dm,
       NagError *fail)
```

3 Description

nag_elliptic_integral_pi (s21bgc) calculates an approximation to the integral

$$\Pi(n;\phi \mid m) = \int_0^\phi \left(1 - n\sin^2\theta\right)^{-1}\left(1 - m\sin^2\theta\right)^{-\frac{1}{2}} d\theta,$$

where $0 \leq \phi \leq \frac{\pi}{2}$, $m\sin^2\phi \leq 1$, m and $\sin\phi$ may not both equal one, and $n\sin^2\phi \neq 1$.

The integral is computed using the symmetrised elliptic integrals of Carlson (Carlson (1979) and Carlson (1988)). The relevant identity is

$$\Pi(n;\phi \mid m) = \sin\phi R_F(q,r,1) + \frac{1}{3}n\sin^3\phi R_J(q,r,1,s),$$

where $q = \cos^2\phi$, $r = 1 - m\sin^2\phi$, $s = 1 - n\sin^2\phi$, R_F is the Carlson symmetrised incomplete elliptic integral of the first kind (see nag_elliptic_integral_rf (s21bbc)) and R_J is the Carlson symmetrised incomplete elliptic integral of the third kind (see nag_elliptic_integral_rj (s21bdc)).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Carlson B C (1979) Computing elliptic integrals by duplication *Numerische Mathematik* **33** 1–16

Carlson B C (1988) A table of elliptic integrals of the third kind *Math. Comput.* **51** 267–280

5 Arguments

1: **dn** – double *Input*
2: **phi** – double *Input*
3: **dm** – double *Input*

On entry: the arguments n, ϕ and m of the function.

Constraints:

0.0 ≤ **phi** ≤ $\frac{\pi}{2}$;
dm × $\sin^2$(**phi**) ≤ 1.0;
Only one of sin(**phi**) and **dm** may be 1.0;
dn × $\sin^2$(**phi**) ≠ 1.0.

Note that **dm** $\times \sin^2(\textbf{phi}) = 1.0$ is allowable, as long as **dm** $\neq 1.0$.

4: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL

On entry, **phi** $= \langle value \rangle$.
Constraint: $0 \leq$ **phi** $\leq (\pi/2)$.

NE_REAL_2

On entry, **phi** $= \langle value \rangle$ and **dm** $= \langle value \rangle$; the integral is undefined.
Constraint: **dm** $\times \sin^2(\textbf{phi}) \leq 1.0$.

On entry, **phi** $= \langle value \rangle$ and **dn** $= \langle value \rangle$; the integral is infinite.
Constraint: **dn** $\times \sin^2(\textbf{phi}) \neq 1.0$.

NW_INTEGRAL_INFINITE

On entry, $\sin(\textbf{phi}) = 1$ and **dm** $= 1.0$; the integral is infinite.

7 Accuracy

In principle nag_elliptic_integral_pi (s21bgc) is capable of producing full ***machine precision***. However round-off errors in internal arithmetic will result in slight loss of accuracy. This loss should never be excessive as the algorithm does not involve any significant amplification of round-off error. It is reasonable to assume that the result is accurate to within a small multiple of the ***machine precision***.

8 Parallelism and Performance

Not applicable.

9 Further Comments

You should consult the s Chapter Introduction, which shows the relationship between this function and the Carlson definitions of the elliptic integrals. In particular, the relationship between the argument-constraints for both forms becomes clear.

For more information on the algorithms used to compute R_F and R_J, see the function documents for nag_elliptic_integral_rf (s21bbc) and nag_elliptic_integral_rj (s21bdc), respectively.

If you wish to input a value of **phi** outside the range allowed by this function you should refer to Section 17.4 of Abramowitz and Stegun (1972) for useful identities.

10 Example

This example simply generates a small set of nonextreme arguments that are used with the function to produce the table of results.

10.1 Program Text

```
/* nag_elliptic_integral_pi (s21bgc) Example Program.
 *
 * Copyright 2008, Numerical Algorithms Group.
 *
 * Mark 9, 2009.
 */
/* Pre-processor includes */
#include <stdio.h>
#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>
#include <nagx01.h>

int main(void)
{
  /*Integer scalar and array declarations */
  Integer  exit_status = 0;
  Integer  ix;
  /*Double scalar and array declarations */
  double   dm, dn, p, phi, pi;
  NagError fail;

  INIT_FAIL(fail);

  printf("nag_elliptic_integral_pi (s21bgc) Example Program Results\n");
  printf("\n    dn     phi    dm       nag_elliptic_integral_pi\n\n");
  /*
   * nag_pi (x01aac)
   */
  pi = nag_pi;
  for (ix = 1; ix <= 3; ix++)
    {
      phi = ix*pi/6.00e0;
      dm = ix*0.250e0;
      dn = (pow(((-(1.00e0))), (ix+1)))*ix*0.10e0;
      /*
       * nag_elliptic_integral_pi (s21bgc)
       * Elliptic integral of 3rd kind, Legendre form, Pi (n; phi |m)
       */
      p = nag_elliptic_integral_pi(dn, phi, dm, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_elliptic_integral_pi (s21bgc).\n%s\n",
                 fail.message);
          exit_status = 1;
          goto END;
        }
      printf("%7.2f%7.2f%7.2f%12.4f\n", dn, phi, dm, p);
    }

 END:

  return exit_status;
}
```

10.2 Program Data

None.

10.3 Program Results

```
nag_elliptic_integral_pi (s21bgc) Example Program Results

    dn      phi    dm       nag_elliptic_integral_pi

   0.10   0.52   0.25      0.5341
  -0.20   1.05   0.50      1.0778
   0.30   1.57   0.75      2.6568
```