NAG Library Function Document

nag_elliptic_integral_E (s21bfc)

1 Purpose

nag_elliptic_integral_E (s21bfc) returns a value of the classical (Legendre) form of the incomplete elliptic integral of the second kind.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_elliptic_integral_E (double phi, double dm, NagError *fail)
```

3 Description

nag_elliptic_integral_E (s21bfc) calculates an approximation to the integral

$$E(\phi \mid m) = \int_0^{\phi} \left(1 - m\sin^2\theta\right)^{\frac{1}{2}} d\theta,$$

where $0 \leq \phi \leq \frac{\pi}{2}$ and $m\sin^2\phi \leq 1$.

The integral is computed using the symmetrised elliptic integrals of Carlson (Carlson (1979) and Carlson (1988)). The relevant identity is

$$E(\phi \mid m) = \sin\phi R_F(q, r, 1) - \frac{1}{3} m \sin^3\phi R_D(q, r, 1),$$

where $q = \cos^2\phi$, $r = 1 - m\sin^2\phi$, R_F is the Carlson symmetrised incomplete elliptic integral of the first kind (see nag_elliptic_integral_rf (s21bbc)) and R_D is the Carlson symmetrised incomplete elliptic integral of the second kind (see nag_elliptic_integral_rd (s21bcc)).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Carlson B C (1979) Computing elliptic integrals by duplication *Numerische Mathematik* **33** 1–16

Carlson B C (1988) A table of elliptic integrals of the third kind *Math. Comput.* **51** 267–280

5 Arguments

1: **phi** – double *Input*

2: **dm** – double *Input*

On entry: the arguments ϕ and m of the function.

Constraints:

$0.0 \leq$ **phi** $\leq \frac{\pi}{2}$;
dm $\times \sin^2($**phi**$) \leq 1.0$.

3: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL

On entry, **phi** $= \langle value \rangle$.
Constraint: $0 \leq$ **phi** $\leq \frac{\pi}{2}$.

NE_REAL_2

On entry, **phi** $= \langle value \rangle$ and **dm** $= \langle value \rangle$; the integral is undefined.
Constraint: **dm** $\times \sin^2($**phi**$) \leq 1.0$.

7 Accuracy

In principle nag_elliptic_integral_E (s21bfc) is capable of producing full ***machine precision***. However round-off errors in internal arithmetic will result in slight loss of accuracy. This loss should never be excessive as the algorithm does not involve any significant amplification of round-off error. It is reasonable to assume that the result is accurate to within a small multiple of the ***machine precision***.

8 Parallelism and Performance

Not applicable.

9 Further Comments

You should consult the s Chapter Introduction, which shows the relationship between this function and the Carlson definitions of the elliptic integrals. In particular, the relationship between the argument-constraints for both forms becomes clear.

For more information on the algorithms used to compute R_F and R_D, see the function documents for nag_elliptic_integral_rf (s21bbc) and nag_elliptic_integral_rd (s21bcc), respectively.

If you wish to input a value of **phi** outside the range allowed by this function you should refer to Section 17.4 of Abramowitz and Stegun (1972) for useful identities. For example, $E(-\phi|m) = -E(\phi|m)$. A parameter $m > 1$ can be replaced by one less than unity using $E(\phi|m) = \sqrt{m}E\left(\phi\sqrt{m}|\frac{1}{m}\right) - (m-1)\phi$.

10 Example

This example simply generates a small set of nonextreme arguments that are used with the function to produce the table of results.

10.1 Program Text

```
/* nag_elliptic_integral_E (s21bfc) Example Program.
 *
 * Copyright 2008, Numerical Algorithms Group.
 *
 * Mark 9, 2009.
 */
/* Pre-processor includes */
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>
#include <nagx01.h>

int main(void)
```

```
{
  /*Integer scalar and array declarations */
  Integer  exit_status = 0;
  Integer  ix;
  /*Double scalar and array declarations */
  double   dm, e, phi, pi;
  NagError fail;

  INIT_FAIL(fail);

  printf("nag_elliptic_integral_E (s21bfc) Example Program Results\n");
  printf("\n    phi    dm       nag_elliptic_integral_E\n\n");
  pi = nag_pi;
  for (ix = 1; ix <= 3; ix++)
    {
      phi = ix*pi/6.00e0;
      dm = ix*0.250e0;
      /*
       * nag_elliptic_integral_E (s21bfc)
       * Elliptic integral of 2nd kind, Legendre form, E( phi |m)
       */
      e = nag_elliptic_integral_E(phi, dm, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_elliptic_integral_E (s21bfc).\n%s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }
      printf("%7.2f%7.2f%12.4f\n", phi, dm, e);
    }

 END:

  return exit_status;
}
```

10.2 Program Data

None.

10.3 Program Results

```
nag_elliptic_integral_E (s21bfc) Example Program Results

    phi    dm       nag_elliptic_integral_E

   0.52   0.25      0.5179
   1.05   0.50      0.9650
   1.57   0.75      1.2111
```