NAG Library Function Document

nag_kelvin_ker (s19acc)

1 Purpose

nag_kelvin_ker (s19acc) returns a value for the Kelvin function $\operatorname{ker} x$.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_kelvin_ker (double x, NagError *fail)
```

3 Description

nag_kelvin_ker (s19acc) evaluates an approximation to the Kelvin function $\operatorname{ker} x$.

Note: for $x < 0$ the function is undefined and at $x = 0$ it is infinite so we need only consider $x > 0$.

The function is based on several Chebyshev expansions:

For $0 < x \leq 1$,

$$\operatorname{ker} x = -f(t)\log(x) + \frac{\pi}{16}x^2 g(t) + y(t)$$

where $f(t)$, $g(t)$ and $y(t)$ are expansions in the variable $t = 2x^4 - 1$.

For $1 < x \leq 3$,

$$\operatorname{ker} x = \exp\left(-\frac{11}{16}x\right)q(t)$$

where $q(t)$ is an expansion in the variable $t = x - 2$.

For $x > 3$,

$$\operatorname{ker} x = \sqrt{\frac{\pi}{2x}}e^{-x/\sqrt{2}}\left[\left(1 + \frac{1}{x}c(t)\right)\cos\beta - \frac{1}{x}d(t)\sin\beta\right]$$

where $\beta = \frac{x}{\sqrt{2}} + \frac{\pi}{8}$, and $c(t)$ and $d(t)$ are expansions in the variable $t = \frac{6}{x} - 1$.

When x is sufficiently close to zero, the result is computed as

$$\operatorname{ker} x = -\gamma - \log\left(\frac{x}{2}\right) + \left(\pi - \frac{3}{8}x^2\right)\frac{x^2}{16}$$

and when x is even closer to zero, simply as $\operatorname{ker} x = -\gamma - \log\left(\frac{x}{2}\right)$.

For large x, $\operatorname{ker} x$ is asymptotically given by $\sqrt{\frac{\pi}{2x}}e^{-x/\sqrt{2}}$ and this becomes so small that it cannot be computed without underflow and the function fails.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: **x** – double *Input*

On entry: the argument x of the function.

Constraint: $\mathbf{x} > 0.0$.

2: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL_ARG_GT

On entry, $\mathbf{x} = \langle value \rangle$. The function returns zero.
Constraint: $\mathbf{x} \leq \langle value \rangle$.
x is too large, the result underflows and the function returns zero.

NE_REAL_ARG_LE

On entry, $\mathbf{x} = \langle value \rangle$.
Constraint: $\mathbf{x} > 0.0$.
The function is undefined and returns zero.

7 Accuracy

Let E be the absolute error in the result, ϵ be the relative error in the result and δ be the relative error in the argument. If δ is somewhat larger than the ***machine precision***, then we have:

$$E \simeq \left| \frac{x}{\sqrt{2}} (\operatorname{ker}_1 x + \operatorname{kei}_1 x) \right| \delta,$$

$$\epsilon \simeq \left| \frac{x}{\sqrt{2}} \frac{\operatorname{ker}_1 x + \operatorname{kei}_1 x}{\operatorname{ker} x} \right| \delta.$$

For very small x, the relative error amplification factor is approximately given by $\frac{1}{|\log(x)|}$, which implies a strong attenuation of relative error. However, ϵ in general cannot be less than the ***machine precision***.

For small x, errors are damped by the function and hence are limited by the ***machine precision***.

For medium and large x, the error behaviour, like the function itself, is oscillatory, and hence only the absolute accuracy for the function can be maintained. For this range of x, the amplitude of the absolute error decays like $\sqrt{\frac{\pi x}{2}} e^{-x/\sqrt{2}}$ which implies a strong attenuation of error. Eventually, $\operatorname{ker} x$, which asymptotically behaves like $\sqrt{\frac{\pi}{2x}} e^{-x/\sqrt{2}}$, becomes so small that it cannot be calculated without causing underflow, and the function returns zero. Note that for large x the errors are dominated by those of the standard math library function exp.

8 Parallelism and Performance

Not applicable.

9 Further Comments

Underflow may occur for a few values of x close to the zeros of $\ker x$, below the limit which causes a failure with **fail.code** = NE_REAL_ARG_GT.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
/* nag_kelvin_ker (s19acc) Example Program.
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  Integer  exit_status = 0;
  double   x, y;
  NagError fail;

  INIT_FAIL(fail);

  /* Skip heading in data file */
  scanf("%*[^\n]");
  printf("nag_kelvin_ker (s19acc) Example Program Results\n");
  printf("     x           y\n");
  while (scanf("%lf", &x) != EOF)
    {
      /* nag_kelvin_ker (s19acc).
       * Kelvin function ker x
       */
      y = nag_kelvin_ker(x, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_kelvin_ker (s19acc).\n%s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }
      printf("%12.3e%12.3e\n", x, y);
    }

 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_kelvin_ker (s19acc) Example Program Data
             0.1
             1.0
             2.5
             5.0
             10.0
             15.0
```

10.3 Program Results

```
nag_kelvin_ker (s19acc) Example Program Results
      x           y
   1.000e-01   2.420e+00
   1.000e+00   2.867e-01
   2.500e+00  -6.969e-02
   5.000e+00  -1.151e-02
   1.000e+01   1.295e-04
   1.500e+01  -1.514e-08
```