

NAG Library Function Document

nag_log_beta (s14cbc)

1 Purpose

nag_log_beta (s14cbc) returns the value of the logarithm of the beta function, $\ln B(a, b)$, via the routine name.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_log_beta (double a, double b, NagError *fail)
```

3 Description

nag_log_beta (s14cbc) calculates values for $\ln B(a, b)$ where B is the beta function given by

$$B(a, b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt$$

or equivalently

$$B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

and $\Gamma(x)$ is the gamma function. Note that the beta function is symmetric, so that $B(a, b) = B(b, a)$.

In order to efficiently obtain accurate results several methods are used depending on the parameters a and b .

Let $a_0 = \min(a, b)$ and $b_0 = \max(a, b)$. Then:

for $a_0 \geq 8$,

$$\ln B = 0.5 \ln(2\pi) - 0.5 \ln(b_0) + \Delta(a_0) + \Delta(b_0) - \Delta(a_0 + b_0) - u - v;$$

where

$$\Delta(a_0) = \ln \Gamma(a_0) - (a_0 - 0.5) \ln a_0 + a_0 - 0.5 \ln(2\pi),$$

$$u = -(a_0 - 0.5) \ln \left[\frac{a_0}{a_0 + b_0} \right] \quad \text{and}$$

$$v = b_0 \ln \left(1 + \frac{a_0}{b_0} \right).$$

for $a_0 < 1$,

for $b_0 \geq 8$,

$$\ln B = \ln \Gamma(a_0) + \ln \frac{\Gamma(b_0)}{\Gamma(a_0 + b_0)};$$

for $b_0 < 8$,

$$\ln B = \ln \Gamma(a_0) + \ln \Gamma(b_0) - \ln \Gamma(a_0 + b_0);$$

for $2 < a_0 < 8$, a_0 is reduced to the interval $[1, 2]$ by $B(a, b) = \frac{a_0-1}{a_0+b_0-1} B(a_0-1, b_0)$;

for $1 \leq a_0 \leq 2$,
 for $b_0 \geq 8$,

$$\ln B = \ln \Gamma(a_0) + \ln \frac{\Gamma(b_0)}{\Gamma(a_0 + b_0)};$$

for $2 < b_0 < 8$, b_0 is reduced to the interval $[1, 2]$;
 for $b_0 \leq 2$,

$$\ln B = \ln \Gamma(a_0) + \ln \Gamma(b_0) - \ln \Gamma(a_0 + b_0).$$

nag_log_beta (s14cbc) is derived from BETALN in DiDonato and Morris (1992).

4 References

DiDonato A R and Morris A H (1992) Algorithm 708: Significant digit computation of the incomplete beta function ratios *ACM Trans. Math. Software* **18** 360–373

5 Arguments

- 1: **a** – double *Input*
On entry: the parameter a of the function.
Constraint: **a** > 0.0.
- 2: **b** – double *Input*
On entry: the parameter b of the function.
Constraint: **b** > 0.0.
- 3: **fail** – NagError * *Input/Output*
 The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL

On entry, **a** = $\langle value \rangle$.
 Constraint: **a** > 0.0.
 On entry, **b** = $\langle value \rangle$.
 Constraint: **b** > 0.0.

7 Accuracy

nag_log_beta (s14cbc) should produce full relative accuracy for all input arguments.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values of the parameters a and b from a file, evaluates the function and prints the results.

10.1 Program Text

```

/* nag_log_beta (s14cbc) Example Program.
 *
 * Copyright 2011 Numerical Algorithms Group.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Integer  exit_status = 0;
    double   a, b, lb;
    NagError fail;

    INIT_FAIL(fail);

    /* Skip heading in data file */
    scanf("%*[\n]");
    printf("nag_log_beta (s14cbc) Example Program Results\n");
    printf("  a      b      ln(beta(a,b))\n");
    while (scanf("%lf %lf", &a, &b) != EOF)
    {
        /* nag_log_beta (s14cbc).
         * Log Beta function ln(beta(a,b))
         */
        lb = nag_log_beta(a, b, &fail);
        if (fail.code != NE_NOERROR)
        {
            printf("Error from nag_log_beta (s14cbc).\n%s\n",
                fail.message);
            exit_status = 1;
            goto END;
        }
        printf("%5.2f %5.2f %12.4e\n", a, b, lb);
    }

    END:
    return exit_status;
}

```

10.2 Program Data

```

nag_log_beta (s14cbc) Example Program Data
0.2  1.0
0.6  1.0
1.0  0.2
1.0  1.0
2.0  2.0
5.0  5.0
6.0  2.0
6.0  3.0

```

10.3 Program Results

```
nag_log_beta (s14cbc) Example Program Results
  a      b      ln(beta(a,b))
 0.20  1.00    1.6094e+00
 0.60  1.00    5.1083e-01
 1.00  0.20    1.6094e+00
 1.00  1.00    0.0000e+00
 2.00  2.00   -1.7918e+00
 5.00  5.00   -6.4457e+00
 6.00  2.00   -3.7377e+00
 6.00  3.00   -5.1240e+00
```
