NAG Library Function Document

nag_scaled_log_gamma (s14ahc)

1 Purpose

nag_scaled_log_gamma (s14ahc) returns the value of $\ln G(x)$, the scaled logarithm of the gamma function $\Gamma(x)$.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_scaled_log_gamma (double x, NagError *fail)
```

3 Description

nag_scaled_log_gamma (s14ahc) calculates an approximate value for $\ln G(x)$, where $G(x) = \Gamma(x+1)/\left(\frac{x}{e}\right)^x$. This is a variant of the $\ln \Gamma(x)$ function (see also nag_log_gamma (s14abc)), which avoids rounding problems for very large arguments by computing $\ln \Gamma(x)$ with the Stirling approximation factored out.

For $0 < x < 15$, $\ln G(x) = \ln \Gamma(x+1) - x \ln x + x$;

and for $15 \leq x$, $\ln G(x) = \frac{1}{2}\ln x + \ln\left(\sqrt{2\pi}\right) + \frac{1}{x}R(1/x^2)$, where R is a suitable Remez approximation.

For $x \leq 0.0$, the value $\ln G(x)$ is undefined; nag_scaled_log_gamma (s14ahc) returns zero and exits with **fail.code** = NE_REAL_ARG_LE.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: **x** – double *Input*

On entry: the argument x of the function.

Constraint: **x** > 0.0.

2: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL_ARG_LE

On entry, **x** = ⟨*value*⟩.
Constraint: **x** > 0.0.

7 Accuracy

nag_scaled_log_gamma (s14ahc) has been designed to produce full relative accuracy for all input arguments. Empirical results obtained by comparing with multiprecision software confirm this.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
/* nag_scaled_log_gamma (s14ahc) Example Program.
 *
 * Copyright 2009, Numerical Algorithms Group.
 *
 * Mark 9, 2009.
 */
/* Pre-processor includes */
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  /*Integer scalar and array declarations */
  Integer  exit_status = 0;
  /*Double scalar and array declarations */
  double   x, y;
  NagError fail;

  INIT_FAIL(fail);

  printf("nag_scaled_log_gamma (s14ahc) Example Program Results\n");
  /* Skip heading in data file*/
  scanf("%*[^\n] ");
  printf("\n%s\n\n", "       x          y");
  while (scanf("%lf%*[^\n] ", &x) != EOF)
    {
      /*
       * nag_scaled_log_gamma (s14ahc)
       * Scaled logarithm of Gamma function, G(x)
       */
      y = nag_scaled_log_gamma(x, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_scaled_log_gamma (s14ahc) %s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }
      printf("%14.5e%14.5e\n", x, y);
    }

 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_scaled_log_gamma (s14ahc) Example Program Data
             1.0
             1.25
             1.5
             1.75
             2.0
             5.0
             10.0
             20.0
             1000.0
```

10.3 Program Results

```
nag_scaled_log_gamma (s14ahc) Example Program Results

       x          y

  1.00000e+00  1.00000e+00
  1.25000e+00  1.09594e+00
  1.50000e+00  1.17649e+00
  1.75000e+00  1.24589e+00
  2.00000e+00  1.30685e+00
  5.00000e+00  1.74030e+00
  1.00000e+01  2.07856e+00
  2.00000e+01  2.42097e+00
  1.00000e+03  4.37290e+00
```