NAG Library Function Document

nag_cos_integral (s13acc)

1 Purpose

nag_cos_integral (s13acc) returns the value of the cosine integral

$$\mathrm{Ci}(x) = \gamma + \ln x + \int_0^x \frac{\cos u - 1}{u}\,du, \quad x > 0$$

where γ denotes Euler's constant.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_cos_integral (double x, NagError *fail)
```

3 Description

nag_cos_integral (s13acc) calculates an approximate value for $\mathrm{Ci}(x)$.

For $0 < x \leq 16$ it is based on the Chebyshev expansion

$$\mathrm{Ci}(x) = \ln x + \sum_{r=0}^{\prime} a_r T_r(t), t = 2\left(\frac{x}{16}\right)^2 - 1.$$

For $16 < x < x_{\mathrm{hi}}$ where the value of x_{hi} is given in the Users' Note for your implementation,

$$\mathrm{Ci}(x) = \frac{f(x)\sin x}{x} - \frac{g(x)\cos x}{x^2}$$

where $f(x) = \sum_{r=0} f_r T_r(t)$ and $g(x) = \sum_{r=0} g_r T_r(t)$, $t = 2\left(\frac{16}{x}\right)^2 - 1$.

For $x \geq x_{\mathrm{hi}}$, $\mathrm{Ci}(x) = 0$ to within the accuracy possible (see Section 7).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: **x** – double *Input*

On entry: the argument x of the function.

Constraint: **x** > 0.0.

2: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL_ARG_LE

On entry, $\mathbf{x} = \langle value \rangle$.
Constraint: $\mathbf{x} > 0.0$.
The function has been called with an argument less than or equal to zero for which $\mathrm{Ci}(x)$ is not defined.

7 Accuracy

If E and ϵ are the absolute and relative errors in the result and δ is the relative error in the argument then in principle these are related by

$$|E| \simeq |\delta \cos x| \text{and } |\epsilon| \simeq \left|\frac{\delta \cos x}{\mathrm{Ci}(x)}\right|.$$

That is accuracy will be limited by ***machine precision*** near the origin and near the zeros of $\cos x$, but near the zeros of $\mathrm{Ci}(x)$ only absolute accuracy can be maintained.

The behaviour of this amplification is shown in Figure 1.

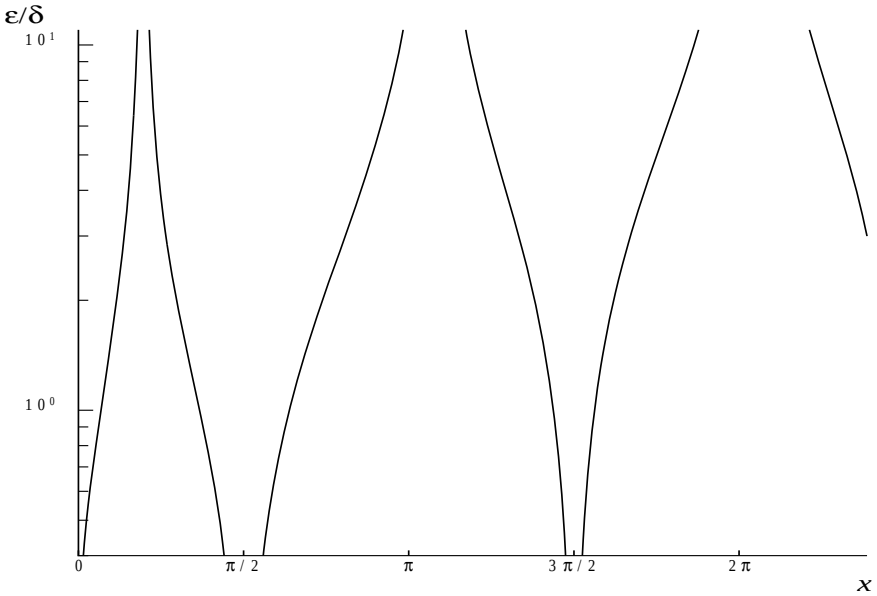

Figure 1

For large values of x, $\mathrm{Ci}(x) \sim \dfrac{\sin x}{x}$ therefore $\epsilon \sim \delta x \cot x$ and since δ is limited by the finite precision of the machine it becomes impossible to return results which have any relative accuracy. That is, when $x \geq 1/\delta$ we have that $|\mathrm{Ci}(x)| \leq 1/x \sim E$ and hence is not significantly different from zero.

Hence x_{hi} is chosen such that for values of $x \geq x_{\mathrm{hi}}$, $\mathrm{Ci}(x)$ in principle would have values less than the ***machine precision*** and so is essentially zero.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
/* nag_cos_integral (s13acc) Example Program.
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  Integer  exit_status = 0;
  double   x, y;
  NagError fail;

  INIT_FAIL(fail);

  /* Skip heading in data file */
  scanf("%*[^\n]");
  printf("nag_cos_integral (s13acc) Example Program Results\n");
  printf("     x           y\n");
  while (scanf("%lf", &x) != EOF)
    {
      /* nag_cos_integral (s13acc).
       * Cosine integral Ci(x)
       */
      y = nag_cos_integral(x, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_cos_integral (s13acc).\n%s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }
      printf("%12.3e%12.3e\n", x, y);
    }

 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_cos_integral (s13acc) Example Program Data
             0.2
             0.4
             0.6
             0.8
             1.0
```

10.3 Program Results

```
nag_cos_integral (s13acc) Example Program Results
     x           y
   2.000e-01  -1.042e+00
   4.000e-01  -3.788e-01
   6.000e-01  -2.227e-02
   8.000e-01   1.983e-01
   1.000e+00   3.374e-01
```