NAG Library Function Document

nag_exp_integral (s13aac)

1 Purpose

nag_exp_integral (s13aac) returns the value of the exponential integral $E_1(x)$.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_exp_integral (double x, NagError *fail)
```

3 Description

nag_exp_integral (s13aac) calculates an approximate value for

$$E_1(x) = -\mathrm{Ei}(-x) = \int_x^\infty \frac{e^{-u}}{u} du.$$

using Chebyshev expansions, where x is real. For $x < 0$, the real part of the principal value of the integral is taken. The value $E_1(0)$ is infinite, and so, when $x = 0$, nag_exp_integral (s13aac) exits with an error and returns the largest representable machine number.

For $0 < x \leq 4$,

$$E_1(x) = y(t) - \ln x = \sum_r a_r T_r(t) - \ln x,$$

where $t = \frac{1}{2}x - 1$.

For $x > 4$,

$$E_1(x) = \frac{e^{-x}}{x} y(t) = \frac{e^{-x}}{x} \sum_r a_r T_r(t),$$

where $t = -1.0 + \frac{14.5}{(x+3.25)} = \frac{11.25-x}{3.25+x}$.

In both cases, $-1 \leq t \leq +1$.

For $x < 0$, the approximation is based on expansions proposed by Cody and Thatcher Jr. (1969). Precautions are taken to maintain good relative accuracy in the vicinity of $x_0 \approx -0.372507\ldots$, which corresponds to a simple zero of $\mathrm{Ei}(-x)$.

nag_exp_integral (s13aac) guards against producing underflows and overflows by using the argument x_{hi}; see the Users' Note for your implementation for the value of x_{hi}. To guard against overflow, if $x < -x_{\mathrm{hi}}$ the function terminates and returns the negative of the largest representable machine number. To guard against underflow, if $x > x_{\mathrm{hi}}$ the result is set directly to zero.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Cody W J and Thatcher Jr. H C (1969) Rational Chebyshev approximations for the exponential integral $\mathrm{Ei}(x)$ *Math. Comp.* **23** 289–303

5 Arguments

1: **x** – double *Input*

On entry: the argument x of the function.

Constraint: $-x_{\mathrm{hi}} \leq \mathbf{x} < 0.0$ or $\mathbf{x} > 0.0$.

2: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL_ARG_LE

On entry, $\mathbf{x} = 0.0$ and the function is infinite.

The evaluation has been abandoned due to the likelihood of overflow. The argument $\mathbf{x} < -x_{\mathrm{hi}}$.

7 Accuracy

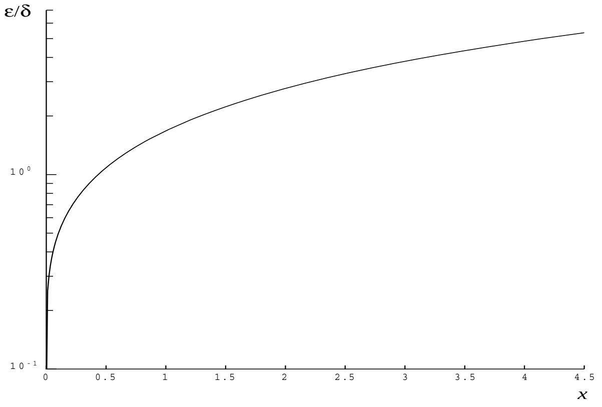
Unless stated otherwise, it is assumed that $x > 0$.

If δ and ϵ are the relative errors in argument and result respectively, then in principle,

$$|\epsilon| \simeq \left|\frac{e^{-x}}{E_1(x)} \times \delta\right|$$

so the relative error in the argument is amplified in the result by at least a factor $e^{-x}/E_1(x)$. The equality should hold if δ is greater than the ***machine precision*** (δ due to data errors etc.) but if δ is simply a result of round-off in the machine representation, it is possible that an extra figure may be lost in internal calculation and round-off.

The behaviour of this amplification factor is shown in the following graph:



It should be noted that, for absolutely small x, the amplification factor tends to zero and eventually the error in the result will be limited by ***machine precision***.

For absolutely large x,

$$\epsilon \sim x\delta = \Delta,$$

the absolute error in the argument.

For $x < 0$, empirical tests have shown that the maximum relative error is a loss of approximately 1 decimal place.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

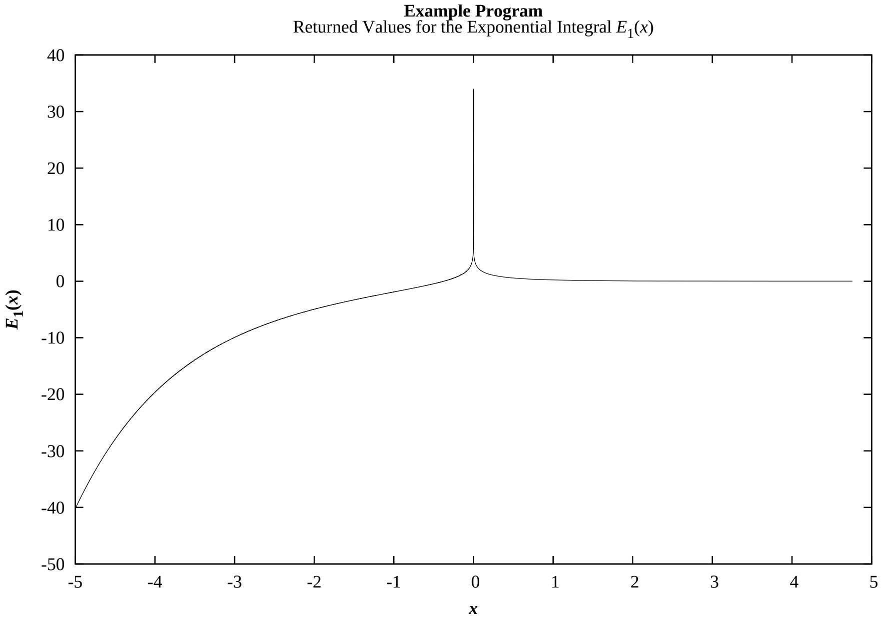
```
/* nag_exp_integral (s13aac) Example Program.
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
  Integer  exit_status = 0;
  double   x, y;
  NagError fail;

  INIT_FAIL(fail);

  /* Skip heading in data file */
  scanf("%*[^\n]");
  printf("nag_exp_integral (s13aac) Example Program Results\n");
  printf("    x            y\n");
  while (scanf("%lf", &x) != EOF)
    {
      /* nag_exp_integral (s13aac).
       * Exponential integral E_1(x)
       */
      y = nag_exp_integral(x, &fail);
      printf("%12.3e%12.3e\n", x, y);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_exp_integral (s13aac).\n%s\n",
                  fail.message);
          if (fail.code != NE_REAL_ARG_LE)
            exit_status = 1;
        }
    }


  return exit_status;
}
```

10.2 Program Data

```
nag_exp_integral (s13aac) Example Program Data
             2.0
             9.0
            -1.0
         -1000.0
```

10.3 Program Results

```
nag_exp_integral (s13aac) Example Program Results
     x          y
   2.000e+00   4.890e-02
   9.000e+00   1.245e-05
  -1.000e+00  -1.895e+00
  -1.000e+03 -1.798e+308
Error from nag_exp_integral (s13aac).
NE_REAL_ARG_LE:
  The evaluation has been abandoned due to the likelihood of overflow.
  The argument x < -x_hi.
```


Example Program
Returned Values for the Exponential Integral $E_1(x)$
$E_1(x)$
x