NAG Library Function Document

nag_sinh (s10abc)

1 Purpose

nag_sinh (s10abc) returns the value of the hyperbolic sine, $\sinh x$.

2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_sinh (double x, NagError *fail)
```

3 Description

nag_sinh (s10abc) calculates an approximate value for the hyperbolic sine of its argument, $\sinh x$.

For $|x| \leq 1$ it uses the Chebyshev expansion

$$\sinh x = x \times y(t) = x\sum_{r=0} a_r T_r(t)$$

where $t = 2x^2 - 1$.

For $1 < |x| \leq E_1$, $\quad \sinh x = \frac{1}{2}(e^x - e^{-x})$

where E_1 is a machine-dependent constant, details of which are given in the Users' Note for your implementation.

For $|x| > E_1$, the function fails owing to the danger of setting overflow in calculating e^x. The result returned for such calls is $\sinh(\text{sign}\, xE_1)$, i.e., it returns the result for the nearest valid argument.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: **x** – double *Input*

On entry: the argument x of the function.

2: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL_ARG_GT

On entry, $\mathbf{x} = \langle value \rangle$.
Constraint: $|\mathbf{x}| \leq E_1$.
The function has been called with an argument too large in absolute magnitude. There is a danger of overflow. The result returned is the value of $\sinh x$ at the closest argument for which a valid call could be made.

7 Accuracy

If δ and ϵ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq |x \coth x \times \delta|.$$

That is the relative error in the argument, x, is amplified by a factor, approximately $x \coth x$. The equality should hold if δ is greater than the ***machine precision*** (δ is a result of data errors etc.) but, if δ is simply a result of round-off in the machine representation of x, then it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the error amplification factor can be seen in the following graph:

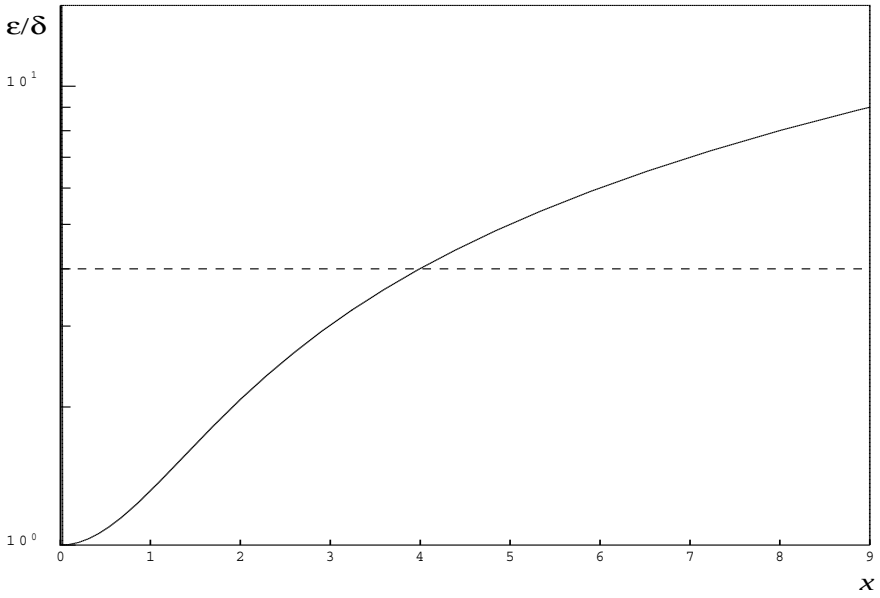

Figure 1

It should be noted that for $|x| \geq 2$

$$\epsilon \sim x\delta = \Delta$$

where Δ is the absolute error in the argument.

8 Parallelism and Performance

Not applicable.

9 Further Comments

None.

10 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

10.1 Program Text

```
/* nag_sinh (s10abc) Example Program.
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>


int main(void)
{
  Integer  exit_status = 0;
  double   x, y;
  NagError fail;

  INIT_FAIL(fail);

  /* Skip heading in data file */
  scanf("%*[^\n]");
  printf("nag_sinh (s10abc) Example Program Results\n");
  printf("     x           y\n");
  while (scanf("%lf", &x) != EOF)
    {
      /* nag_sinh (s10abc).
       * Hyperbolic sine, sinh x
       */
      y = nag_sinh(x, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_sinh (s10abc).\n%s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }
      printf("%12.3e%12.3e\n", x, y);
    }


 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_sinh (s10abc) Example Program Data
           -10.0
            -0.5
             0.0
             0.5
            25.0
```

10.3 Program Results

```
nag_sinh (s10abc) Example Program Results
     x           y
  -1.000e+01  -1.101e+04
  -5.000e-01  -5.211e-01
   0.000e+00   0.000e+00
   5.000e-01   5.211e-01
   2.500e+01   3.600e+10
```