

# NAG Library Function Document

## nag\_shifted\_log (s01bac)

### 1 Purpose

nag\_shifted\_log (s01bac) returns a value of the shifted logarithmic function,  $\ln(1+x)$ .

### 2 Specification

```
#include <nag.h>
#include <nags.h>
double nag_shifted_log (double x, NagError *fail)
```

### 3 Description

nag\_shifted\_log (s01bac) computes values of  $\ln(1+x)$ , retaining full relative precision even when  $|x|$  is small. The function is based on the Chebyshev expansion

$$\ln \frac{1+p^2+2p\bar{x}}{1+p^2-2p\bar{x}} = 4 \sum_{k=0}^{\infty} \frac{p^{2k+1}}{2k+1} T_{2k+1}(\bar{x}).$$

Setting  $\bar{x} = \frac{x(1+p^2)}{2p(x+2)}$ , and choosing  $p = \frac{q-1}{q+1}$ ,  $q = \sqrt[4]{2}$  the expansion is valid in the domain  $x \in \left[ \frac{1}{\sqrt{2}} - 1, \sqrt{2} - 1 \right]$ .

Outside this domain,  $\ln(1+x)$  is computed by the standard logarithmic function.

### 4 References

Lyusternik L A, Chervonenkis O A and Yanpolskii A R (1965) *Handbook for Computing Elementary Functions* p. 57 Pergamon Press

### 5 Arguments

- 1: **x** – double *Input*  
*On entry:* the argument  $x$  of the function.  
*Constraint:*  $x > -1.0$ .
- 2: **fail** – NagError \* *Input/Output*  
The NAG error argument (see Section 3.6 in the Essential Introduction).

### 6 Error Indicators and Warnings

#### NE\_REAL\_ARG\_LE

On entry,  $x = \langle \text{value} \rangle$ .  
Constraint:  $x > -1.0$ .

## 7 Accuracy

The returned result should be accurate almost to *machine precision*, with a limit of about 20 significant figures due to the precision of internal constants. Note however that if  $x$  lies very close to  $-1.0$  and is not exact (for example if  $x$  is the result of some previous computation and has been rounded), then precision will be lost in the computation of  $1 + x$ , and hence  $\ln(1 + x)$ , in `nag_shifted_log` (`s01bac`).

## 8 Parallelism and Performance

Not applicable.

## 9 Further Comments

Empirical tests show that the time taken for a call of `nag_shifted_log` (`s01bac`) usually lies between about 1.25 and 2.5 times the time for a call to the standard logarithm function.

## 10 Example

The example program reads values of the argument  $x$  from a file, evaluates the function at each value of  $x$  and prints the results.

### 10.1 Program Text

```

/* nag_shifted_log (s01bac) Example Program.
 *
 * Copyright 2002 Numerical Algorithms Group.
 *
 * Mark 7, 2002.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

int main(void)
{
    Integer  exit_status = 0;
    double   x, y;
    NagError fail;

    INIT_FAIL(fail);

    /* Skip heading in data file */
    scanf("%*[\n]");
    printf("nag_shifted_log (s01bac) Example Program Results\n");
    printf("      x              y\n");
    while (scanf("%lf", &x) != EOF)
    {
        /* nag_shifted_log (s01bac).
         * ln(1+x)
         */
        y = nag_shifted_log(x, &fail);
        if (fail.code != NE_NOERROR)
        {
            printf("Error from nag_shifted_log (s01bac).\n%s\n",
                   fail.message);
            exit_status = 1;
            goto END;
        }
        printf("%13.4e %13.4e\n", x, y);
    }

    END:
    return exit_status;
}

```

}

## **10.2 Program Data**

```
nag_shifted_log (s01bac) Example Program Data
  2.50e+0
  1.25e-1
 -9.06e-1
  1.29e-3
 -7.83e-6
  1.00e-9
```

## **10.3 Program Results**

```
nag_shifted_log (s01bac) Example Program Results
      x              y
  2.5000e+00   1.2528e+00
  1.2500e-01   1.1778e-01
 -9.0600e-01  -2.3645e+00
  1.2900e-03   1.2892e-03
 -7.8300e-06  -7.8300e-06
  1.0000e-09   1.0000e-09
```

---