

# NAG Library Function Document

## nag\_best\_subset\_given\_size (h05abc)

### 1 Purpose

Given a set of  $m$  features and a scoring mechanism for any subset of those features, nag\_best\_subset\_given\_size (h05abc) selects the best  $n$  subsets of size  $p$  using a direct communication branch and bound algorithm.

### 2 Specification

```
#include <nag.h>
#include <nagh.h>

void nag_best_subset_given_size (Integer mincr, Integer m, Integer ip,
    Integer nbest, Integer *la, double bscore[], Integer bz[],
    void (*f)(Integer m, Integer drop, Integer lz, const Integer z[],
        Integer la, const Integer a[], double score[], Nag_Comm *comm,
        Integer *info),
    Integer mincnt, double gamma, const double acc[], Nag_Comm *comm,
    NagError *fail)
```

### 3 Description

Given  $\Omega = \{x_i : i \in \mathbb{Z}, 1 \leq i \leq m\}$ , a set of  $m$  unique features and a scoring mechanism  $f(S)$  defined for all  $S \subseteq \Omega$  then nag\_best\_subset\_given\_size (h05abc) is designed to find  $S_{o1} \subseteq \Omega, |S_{o1}| = p$ , an optimal subset of size  $p$ . Here  $|S_{o1}|$  denotes the cardinality of  $S_{o1}$ , the number of elements in the set.

The definition of the optimal subset depends on the properties of the scoring mechanism, if

$$f(S_i) \leq f(S_j), \quad \text{for all } S_j \subseteq \Omega \text{ and } S_i \subseteq S_j \quad (1)$$

then the optimal subset is defined as one of the solutions to

$$\underset{S \subseteq \Omega}{\text{maximize}} f(S) \quad \text{subject to} \quad |S| = p$$

else if

$$f(S_i) \geq f(S_j), \quad \text{for all } S_j \subseteq \Omega \text{ and } S_i \subseteq S_j \quad (2)$$

then the optimal subset is defined as one of the solutions to

$$\underset{S \subseteq \Omega}{\text{minimize}} f(S) \quad \text{subject to} \quad |S| = p.$$

If neither of these properties hold then nag\_best\_subset\_given\_size (h05abc) cannot be used.

As well as returning the optimal subset,  $S_{o1}$ , nag\_best\_subset\_given\_size (h05abc) can return the best  $n$  solutions of size  $p$ . If  $S_{oi}$  denotes the  $i$ th best subset, for  $i = 1, 2, \dots, n-1$ , then the  $(i+1)$ th best subset is defined as the solution to either

$$\underset{S \subseteq \Omega - \{S_{oj} : j \in \mathbb{Z}, 1 \leq j \leq i\}}{\text{maximize}} f(S) \quad \text{subject to} \quad |S| = p$$

or

$$\underset{S \subseteq \Omega - \{S_{oj} : j \in \mathbb{Z}, 1 \leq j \leq i\}}{\text{minimize}} f(S) \quad \text{subject to} \quad |S| = p$$

depending on the properties of  $f$ .

The solutions are found using a branch and bound method, where each node of the tree is a subset of  $\Omega$ . Assuming that (1) holds then a particular node, defined by subset  $S_i$ , can be trimmed from the tree if  $f(S_i) < \hat{f}(S_{on})$  where  $\hat{f}(S_{on})$  is the  $n$ th highest score we have observed so far for a subset of size  $p$ , i.e., our current best guess of the score for the  $n$ th best subset. In addition, because of (1) we can also drop all nodes defined by any subset  $S_j$  where  $S_j \subseteq S_i$ , thus avoiding the need to enumerate the whole tree. Similar short cuts can be taken if (2) holds. A full description of this branch and bound algorithm can be found in Ridout (1988).

Rather than calculate the score at a given node of the tree `nag_best_subset_given_size` (h05abc) utilizes the fast branch and bound algorithm of Somol *et al.* (2004), and attempts to estimate the score where possible. For each feature,  $x_i$ , two values are stored, a count  $c_i$  and  $\hat{\mu}_i$ , an estimate of the contribution of that feature. An initial value of zero is used for both  $c_i$  and  $\hat{\mu}_i$ . At any stage of the algorithm where both  $f(S)$  and  $f(S - \{x_i\})$  have been calculated (as opposed to estimated), the estimated contribution of the feature  $x_i$  is updated to

$$\frac{c_i \hat{\mu}_i + [f(S) - f(S - \{x_i\})]}{c_i + 1}$$

and  $c_i$  is incremented by 1, therefore at each stage  $\hat{\mu}_i$  is the mean contribution of  $x_i$  observed so far and  $c_i$  is the number of observations used to calculate that mean.

As long as  $c_i \geq k$ , for the user-supplied constant  $k$ , then rather than calculating  $f(S - \{x_i\})$  this function estimates it using  $\hat{f}(S - \{x_i\}) = f(S) - \gamma \hat{\mu}_i$  or  $\hat{f}(S) - \gamma \hat{\mu}_i$  if  $f(S)$  has been estimated, where  $\gamma$  is a user-supplied scaling factor. An estimated score is never used to trim a node or returned as the optimal score.

Setting  $k = 0$  in this function will cause the algorithm to always calculate the scores, returning to the branch and bound algorithm of Ridout (1988). In most cases it is preferable to use the fast branch and bound algorithm, by setting  $k > 0$ , unless the score function is iterative in nature, i.e.,  $f(S)$  must have been calculated before  $f(S - \{x_i\})$  can be calculated.

`nag_best_subset_given_size` (h05abc) is a direct communication version of `nag_best_subset_given_size_revcomm` (h05aac).

## 4 References

Narendra P M and Fukunaga K (1977) A branch and bound algorithm for feature subset selection *IEEE Transactions on Computers* **9** 917–922

Ridout M S (1988) Algorithm AS 233: An improved branch and bound algorithm for feature subset selection *Journal of the Royal Statistics Society, Series C (Applied Statistics) (Volume 37)* **1** 139–147

Somol P, Pudil P and Kittler J (2004) Fast branch and bound algorithms for optimal feature selection *IEEE Transactions on Pattern Analysis and Machine Intelligence (Volume 26)* **7** 900–912

## 5 Arguments

1: **mincr** – Integer *Input*

*On entry:* flag indicating whether the scoring function  $f$  is increasing or decreasing.

**mincr** = 1

$f(S_i) \leq f(S_j)$ , i.e., the subsets with the largest score will be selected.

**mincr** = 0

$f(S_i) \geq f(S_j)$ , i.e., the subsets with the smallest score will be selected.

For all  $S_j \subseteq \Omega$  and  $S_i \subseteq S_j$ .

*Constraint:* **mincr** = 0 or 1.

- 2: **m** – Integer *Input*  
*On entry:*  $m$ , the number of features in the full feature set.  
*Constraint:*  $m \geq 2$ .
- 3: **ip** – Integer *Input*  
*On entry:*  $p$ , the number of features in the subset of interest.  
*Constraint:*  $1 \leq ip \leq m$ .
- 4: **nbest** – Integer *Input*  
*On entry:*  $n$ , the maximum number of best subsets required. The actual number of subsets returned is given by **la** on final exit. If on final exit  $la \neq nbest$  then **fail.code** = NE\_TOO\_MANY is returned.  
*Constraint:*  $nbest \geq 1$ .
- 5: **la** – Integer \* *Output*  
*On exit:* the number of best subsets returned.
- 6: **bscore[nbest]** – double *Output*  
*On exit:* holds the score for the **la** best subsets returned in **bz**.
- 7: **bz[(m – ip) × nbest]** – Integer *Output*  
**Note:** where **BZ**( $i, j$ ) appears in this document, it refers to the array element **bz**[( $j - 1$ ) × ( $m - ip$ ) +  $i - 1$ ].  
*On exit:* the  $j$ th best subset is constructed by dropping the features specified in **BZ**( $i, j$ ), for  $i = 1, 2, \dots, m - ip$  and  $j = 1, 2, \dots, la$ , from the set of all features,  $\Omega$ . The score for the  $j$ th best subset is given in **bscore**[ $j - 1$ ].
- 8: **f** – function, supplied by the user *External Function*  
**f** must evaluate the scoring function  $f$ .

The specification of **f** is:

```
void f (Integer m, Integer drop, Integer lz, const Integer z[],
        Integer la, const Integer a[], double score[], Nag_Comm *comm,
        Integer *info)
```

- 1: **m** – Integer *Input*  
*On entry:*  $m = |\Omega|$ , the number of features in the full feature set.
- 2: **drop** – Integer *Input*  
*On entry:* flag indicating whether the intermediate subsets should be constructed by dropping features from the full set (**drop** = 1) or adding features to the empty set (**drop** = 0). See **score** for additional details.
- 3: **lz** – Integer *Input*  
*On entry:* the number of features stored in **z**.
- 4: **z[lz]** – const Integer *Input*  
*On entry:* **z**[ $i - 1$ ], for  $i = 1, 2, \dots, lz$ , contains the list of features which, along with those specified in **a**, define the subsets whose score is required. See **score** for additional details.

5:	<p><b>la</b> – Integer <span style="float: right;"><i>Input</i></span></p> <p><i>On entry:</i> if <b>la</b> &gt; 0, the number of subsets for which a score must be returned.</p> <p>If <b>la</b> = 0, the score for a single subset should be returned. See <b>score</b> for additional details.</p>
6:	<p><b>a[la]</b> – const Integer <span style="float: right;"><i>Input</i></span></p> <p><i>On entry:</i> <b>a</b>[<i>j</i> – 1], for <i>j</i> = 1, 2, ..., <b>la</b>, contains the list of features which, along with those specified in <b>z</b>, define the subsets whose score is required. See <b>score</b> for additional details.</p>
7:	<p><b>score</b>[<b>max</b>(<b>la</b>, 1)] – double <span style="float: right;"><i>Output</i></span></p> <p><i>On exit:</i> the value <math>f(S_j)</math>, for <i>j</i> = 1, 2, ..., <b>la</b>, the score associated with the <i>j</i>th subset. <math>S_j</math> is constructed as follows:</p> <p><b>drop</b> = 1  <math>S_j</math> is constructed by <i>dropping</i> the features specified in the first <b>lz</b> elements of <b>z</b> and the single feature given in <b>a</b>[<i>j</i> – 1] from the full set of features, <math>\Omega</math>. The subset will therefore contain <b>m</b> – <b>lz</b> – 1 features.</p> <p><b>drop</b> = 0  <math>S_j</math> is constructed by <i>adding</i> the features specified in the first <b>lz</b> elements of <b>z</b> and the single feature specified in <b>a</b>[<i>j</i> – 1] to the empty set, <math>\emptyset</math>. The subset will therefore contain <b>lz</b> + 1 features.</p> <p>In both cases the individual features are referenced by the integers 1 to <b>m</b> with 1 indicating the first feature, 2 the second, etc., for some arbitrary ordering of the features, chosen by you prior to calling <code>nag_best_subset_given_size</code> (h05abc). For example, 1 might refer to the first variable in a particular set of data, 2 the second, etc..</p> <p>If <b>la</b> = 0, the score for a single subset should be returned. This subset is constructed by adding or removing only those features specified in the first <b>lz</b> elements of <b>z</b>. If <b>lz</b> = 0, this subset will either be <math>\Omega</math> or <math>\emptyset</math>.</p>
8:	<p><b>comm</b> – Nag_Comm * <span style="float: right;"><i>Communication Structure</i></span></p> <p>Pointer to structure of type Nag_Comm; the following members are relevant to <b>f</b>.</p> <p><b>user</b> – double *  <b>iuser</b> – Integer *  <b>p</b> – Pointer</p> <p>The type Pointer will be <code>void *</code>. Before calling <code>nag_best_subset_given_size</code> (h05abc) you may allocate memory and initialize these pointers with various quantities for use by <b>f</b> when called from <code>nag_best_subset_given_size</code> (h05abc) (see Section 3.2.1.1 in the Essential Introduction).</p>
9:	<p><b>info</b> – Integer * <span style="float: right;"><i>Input/Output</i></span></p> <p><i>On entry:</i> <b>info</b> = 0.</p> <p><i>On exit:</i> set <b>info</b> to a nonzero value if you wish <code>nag_best_subset_given_size</code> (h05abc) to terminate with <b>fail.code</b> = NE_USER_STOP.</p>

- 9: **mincnt** – Integer *Input*
- On entry:* *k*, the minimum number of times the effect of each feature,  $x_i$ , must have been observed before  $f(S - \{x_i\})$  is estimated from  $f(S)$  as opposed to being calculated directly.
- If *k* = 0 then  $f(S - \{x_i\})$  is never estimated. If **mincnt** < 0 then *k* is set to 1.

- 10: **gamma** – double *Input*  
*On entry:*  $\gamma$ , the scaling factor used when estimating scores. If **gamma** < 0 then  $\gamma = 1$  is used.
- 11: **acc[2]** – const double *Input*  
*On entry:* a measure of the accuracy of the scoring function,  $f$ .  
 Letting  $a_i = \epsilon_1 |f(S_i)| + \epsilon_2$ , then when confirming whether the scoring function is strictly increasing or decreasing (as described in **mincr**), or when assessing whether a node defined by subset  $S_i$  can be trimmed, then any values in the range  $f(S_i) \pm a_i$  are treated as being numerically equivalent.  
 If  $0 \leq \mathbf{acc}[0] \leq 1$  then  $\epsilon_1 = \mathbf{acc}[0]$ , otherwise  $\epsilon_1 = 0$ .  
 If  $\mathbf{acc}[1] \geq 0$  then  $\epsilon_2 = \mathbf{acc}[1]$ , otherwise  $\epsilon_2 = 0$ .  
 In most situations setting both  $\epsilon_1$  and  $\epsilon_2$  to zero should be sufficient. Using a nonzero value, when one is not required, can significantly increase the number of subsets that need to be evaluated.
- 12: **comm** – Nag\_Comm \* *Communication Structure*  
 The NAG communication argument (see Section 3.2.1.1 in the Essential Introduction).
- 13: **fail** – NagError \* *Input/Output*  
 The NAG error argument (see Section 3.6 in the Essential Introduction).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

### NE\_BAD\_PARAM

On entry, argument  $\langle value \rangle$  had an illegal value.

### NE\_INT

On entry, **m** =  $\langle value \rangle$ .  
 Constraint: **m**  $\geq 2$ .

On entry, **mincr** =  $\langle value \rangle$ .  
 Constraint: **mincr** = 0 or 1.

On entry, **nbest** =  $\langle value \rangle$ .  
 Constraint: **nbest**  $\geq 1$ .

### NE\_INT\_2

On entry, **ip** =  $\langle value \rangle$  and **m** =  $\langle value \rangle$ .  
 Constraint:  $1 \leq \mathbf{ip} \leq \mathbf{m}$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

### NE\_REAL

On exit from **f**, **score**[ $\langle value \rangle$ ] =  $\langle value \rangle$ , which is inconsistent with the score for the parent node. Score for the parent node is  $\langle value \rangle$ .

**NE\_TOO\_MANY**

On entry, **nbest** =  $\langle value \rangle$ .  
 But only  $\langle value \rangle$  best subsets could be calculated.

**NE\_USER\_STOP**

A nonzero value for **info** has been returned: **info** =  $\langle value \rangle$ .

**7 Accuracy**

The subsets returned by `nag_best_subset_given_size` (h05abc) are guaranteed to be optimal up to the accuracy of the calculated scores.

**8 Parallelism and Performance**

`nag_best_subset_given_size` (h05abc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

**9 Further Comments**

The maximum number of unique subsets of size  $p$  from a set of  $m$  features is  $N = \frac{m!}{(m-p)!p!}$ . The efficiency of the branch and bound algorithm implemented in `nag_best_subset_given_size` (h05abc) comes from evaluating subsets at internal nodes of the tree, that is subsets with more than  $p$  features, and where possible trimming branches of the tree based on the scores at these internal nodes as described in Narendra and Fukunaga (1977). Because of this it is possible, in some circumstances, for more than  $N$  subsets to be evaluated. This will tend to happen when most of the features have a similar effect on the subset score.

If multiple optimal subsets exist with the same score, and **nbest** is too small to return them all, then the choice of which of these optimal subsets is returned is arbitrary.

**10 Example**

This example finds the three linear regression models, with five variables, that have the smallest residual sums of squares when fitted to a supplied dataset. The data used in this example was simulated.

**10.1 Program Text**

```
/* nag_best_subset_given_size (h05abc) Example Program.
 *
 * Copyright 2011, Numerical Algorithms Group.
 *
 * Mark 23, 2011.
 */
/* Pre-processor includes */
#include <stdio.h>
#include <math.h>
#include <string.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg02.h>
#include <nagh05.h>

#define BZ(I,J) bz[(J)*(m - ip) + (I)]

typedef struct {
  Integer n, tdq, tdx;
  double tol;
  double *x, *y, *b, *cov, *h, *p, *q, *res, *se, *wt, *com_ar;
```

```

Integer *sx, cnt;
Nag_IncludeMean mean;
} calc_subset_data;

void read_subset_data(Integer m, calc_subset_data *cs);
void tidy_subset_data(calc_subset_data *cs);
void calc_subset_score(Integer m, Integer drop, Integer lz, const Integer z[],
                      Integer la, const Integer a[], double score[],
                      Nag_Comm *comm, Integer *info);

int main(void)
{
    int exit_status = 0;

    /* Integer scalar and array declarations */
    Integer i, ip, j, la, m, mincnt, mincr, mip, nbest;
    Integer *a = 0, *bz = 0, *ibz = 0, *id = 0;

    /* NAG structures */
    NagError fail;
    Nag_Comm comm;

    /* Other data types */
    calc_subset_data cs;

    /* Double scalar and array declarations */
    double gamma;
    double *bscore = 0;
    double acc[2];

    /* Initialise the error structure */
    INIT_FAIL(fail);

    printf("nag_best_subset_given_size (h05abc) Example Program Results\n\n");

    /* Skip heading in data file */
    scanf("%*[\n] ");
    scanf("%*[\n] ");

    /* Read in the problem size */
    scanf("%ld%ld%ld%*[\n] ", &m, &ip, &nbest);

    /* Read in the control parameters for the subset selection */
    scanf("%ld%ld%lf%lf%*[\n] ", &mincr, &mincnt,
        &gamma, &acc[0], &acc[1]);

    /* Read in the data required for the score function */
    read_subset_data(m, &cs);

    /* Associate the data structure with comm.p */
    comm.p = (void *)

    /* Allocate memory required by the subset selection routine */
    mip = m - ip;
    if (!(a = NAG_ALLOC(MAX(nbest,m), Integer)) ||
        !(bz = NAG_ALLOC(mip*nbest, Integer)) ||
        !(bscore = NAG_ALLOC(MAX(nbest,m),double))) {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    /* Call the forward communication best subset routine
       (nag_best_subset_given_size (h05abc)) */
    nag_best_subset_given_size(mincr, m, ip, nbest, &la, bscore, bz,
        calc_subset_score, mincnt, gamma, acc, &comm,
        &fail);
    if (fail.code != NE_NOERROR) {
        printf("Error from nag_best_subset_given_size (h05abc).\n%s\n",
            fail.message);
        if (fail.code != NE_ARG_TOO_LARGE) {
            exit_status = 1;
        }
    }
}

```

```

        goto END;
    }
}

/* Titles */
printf("    Score          Feature Subset\n");
printf("    -----          -\n");

/* Display the best subsets and corresponding scores.
   nag_best_subset_given_size returns a list
   of features excluded from the best subsets, so this is inverted to give
   the set of features included in each subset */
if (!(id = NAG_ALLOC(m, Integer))) {
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}
for (i = 0; i < la; i++) {
    printf("%12.5e ", bscore[i]);
    for (j = 0; j < m; j++)
        id[j] = 1;
    for (j = 0; j < mip; j++)
        id[BZ(j,i) - 1] = 0;
    for (j = 0; j < m; j++)
        if (id[j]) printf(" %5"NAG_IFMT, j+1);
    printf("\n");
}
printf("\n");
if (fail.code == NE_TOO_MANY) {
    printf("%ld subsets of the requested size do not exist, "
          "only %ld are displayed.\n", nbest, la);
}
printf("%ld subsets evaluated in total\n", cs.cnt);

END:
NAG_FREE(a);
NAG_FREE(bz);
NAG_FREE(ibz);
NAG_FREE(bscore);
NAG_FREE(id);

tidy_subset_data(&cs);

return exit_status;
}

#define CSX(I, J) cs->x[(I) * cs->tdx + J]
void read_subset_data(Integer m, calc_subset_data *cs) {
    /* Read in the data, from stdout, and allocate any arrays that are required
       by the scoring function calc_subset_score */
    Integer i, j;
    cs->sx = 0;
    cs->x = cs->y = cs->b = cs->se = cs->cov = cs->res = cs->h = cs->q =
        cs->p = cs->com_ar = cs->wt = 0;

    /* Skip heading in data file */
    scanf("%*[^\\n] ");

    /* Read in the number of observations for the data used in the linear
       regression */
    scanf("%ld%*[^\\n] ", &cs->n);

    /* Read in the control parameters for the linear regression */
    scanf("%lf%*[^\\n] ", &cs->tol);

    /* Read in the data */
    cs->tdx = m;

    if (!(cs->x = NAG_ALLOC(cs->tdx*cs->n, double)) ||
        !(cs->y = NAG_ALLOC(cs->n, double))) {
        printf("Allocation failure\n");
    }
}

```



```

    tidy_subset_data(cs);
    exit(-1);
}

/* Read in the data for the linear regression */
for (i = 0; i < cs->n; i++) {
    for (j = 0; j < m; j++)
        scanf("%lf", &CSX(i,j));
    scanf("%lf", &cs->y[i]);
    scanf("%*[\n] ");
}

/* No intercept term and no weights */
cs->mean = Nag_MeanZero;

/* Allocate memory required by the regression routine */
cs->tdq = cs->n;
if (!(cs->sx = NAG_ALLOC(m, Integer)) ||
    !(cs->b = NAG_ALLOC(m, double)) ||
    !(cs->se = NAG_ALLOC(m, double)) ||
    !(cs->cov = NAG_ALLOC(m*(m+1)/2, double)) ||
    !(cs->res = NAG_ALLOC(cs->n, double)) ||
    !(cs->h = NAG_ALLOC(cs->n, double)) ||
    !(cs->q = NAG_ALLOC(cs->n*cs->tdq, double)) ||
    !(cs->p = NAG_ALLOC(2*m+m*m, double)) ||
    !(cs->com_ar = NAG_ALLOC(5*(m-1)*m*m, double))) {
    printf("Allocation failure\n");
    tidy_subset_data(cs);
    exit(-1);
}

cs->cnt = 0;
}

void tidy_subset_data(calc_subset_data *cs) {
    /* Tidy up the data structure used by calc_subset_score */
    NAG_FREE(cs->sx);
    NAG_FREE(cs->x);
    NAG_FREE(cs->y);
    NAG_FREE(cs->b);
    NAG_FREE(cs->se);
    NAG_FREE(cs->cov);
    NAG_FREE(cs->res);
    NAG_FREE(cs->h);
    NAG_FREE(cs->q);
    NAG_FREE(cs->p);
    NAG_FREE(cs->com_ar);
    NAG_FREE(cs->wt);
}

void calc_subset_score(Integer m, Integer drop, Integer lz, const Integer z[],
                      Integer la, const Integer a[], double score[],
                      Nag_Comm *comm, Integer *info) {
    /* Calculate the score associated with a particular set of feature subsets.

    This particular example finds the set, of a given size, of explanatory
    variables that best fit a response variable when a linear regression model
    is used. Therefore the feature set is the set of all the explanatory
    variables and the best set of features is defined as set of explanatory
    variables that gives the smallest residual sums of squares. See the
    documentation for g02dac for details on linear regression models.
    */
    calc_subset_data *cs;

    /* Integer scalar and array declarations */
    Integer i, j, inv_drop, ip, irank;

    /* NAG structures */
    NagError fail;

    /* Double scalar and array declarations */

```

```

double rss, df;

/* Other data types */
Nag_Boolean svd;

/* Initialise the error structure */
INIT_FAIL(fail);

/* Point cs to the data structure stored in comm */
cs = (calc_subset_data *) comm->p;

/* Keep track of the number of subsets evaluated */
cs->cnt += MAX(1,la);

/* Set up the initial feature set.
   If drop = 0, this is the Null set (i.e. no features).
   If drop = 1 then this is the full set (i.e. all features) */
for (i = 0; i < m; i++)
    cs->sx[i] = drop;

/* Add (if drop = 0) or remove (if drop = 1) the all the features specified
   in Z (note: z refers to its features with values 1 to M, therefore you must
   subtract 1 when using z as an array reference) */
inv_drop = (drop == 0) ? 1 : 0;
for (i = 0; i < lz; i++)
    cs->sx[z[i] - 1] = inv_drop;

/* Loop over all the elements in a, looping at least once */
for (i = 0; i < MAX(la,1); i++) {
    if (la > 0) {
        if (i > 0) {
            /* Reset the feature altered at the last iteration */
            cs->sx[a[i-1] - 1] = drop;
        }

        /* Add or drop the I'th feature in A */
        cs->sx[a[i] - 1] = inv_drop;
    }

    /* Calculate number of parameters in the regression model */
    for (ip = j = 0; j < m; j++) ip += (cs->sx[j] == 1);

    /* Fit the regression model (g02dac) */
    nag_regsn_mult_linear(cs->mean, cs->n, cs->x, cs->tdx, m, cs->sx, ip, cs->y,
        cs->wt, &rss, &df, cs->b, cs->se, cs->cov, cs->res,
        cs->h, cs->q, cs->tdq, &svd, &irank, cs->p, cs->tol,
        cs->com_ar, &fail);

    /* Return the score (the residual sums of squares) */
    score[i] = rss;
}

/* If g02dac fails, terminate early */
if (fail.code != NE_NOERROR) *info = 1;
}

```

## 10.2 Program Data

nag\_best\_subset\_given\_size (h05abc) Example Program Data

Data required by h05abc

```

14 5 3                :: m,ip,nbest
0 -1 -1.0 -1.0 -1.0  :: mincr, mincnt, gamma, acc[0], acc[1]

```

Data required by the scoring function

```

40                    :: n
1e-6                  :: tol
-1.59   0.19   0.40   0.43  -0.40   0.79   0.06
 0.33   1.60   0.58  -1.12   1.23   1.07  -0.07          -2.44
-0.25   0.61  -0.36   1.16   0.61  -2.05  -0.02
-0.04   0.80  -0.73  -0.63  -0.75  -0.73   1.43          -2.97

```

-2.28	0.46	-0.65	0.33	0.16	-0.21	-1.61	
-0.54	0.48	0.37	-0.95	-2.14	0.48	2.02	7.42
-0.52	1.05	0.64	0.02	-1.12	0.23	0.06	
-1.26	1.40	-0.98	2.47	0.49	-0.02	-0.05	3.00
-0.84	1.86	0.10	0.73	-1.41	0.98	0.20	
-0.89	1.84	2.56	0.60	-0.12	0.71	0.23	8.83
1.12	-0.51	-0.58	0.09	-1.14	2.11	-0.11	
-0.34	-1.04	-0.43	-0.01	-0.38	1.80	0.05	0.03
0.06	0.85	-2.09	0.22	-1.35	-0.36	1.20	
0.41	0.80	-0.28	0.18	0.27	0.92	0.63	2.57
-0.48	-1.02	0.08	-0.06	0.13	-1.18	2.30	
0.03	0.45	0.62	-1.97	0.97	0.93	-0.18	8.31
0.08	-0.31	0.43	-0.38	0.01	1.30	0.66	
0.65	-0.59	0.76	0.04	0.17	-0.76	-0.90	4.55
0.66	1.14	0.40	2.37	1.10	0.17	-0.38	
1.15	-1.00	-0.13	-0.69	-0.62	-0.18	0.00	-23.10
-1.08	-0.21	-1.13	-0.79	-0.76	-1.58	0.38	
-0.03	1.26	-0.51	-0.75	0.86	0.29	0.68	3.38
-0.74	-1.59	-0.58	-1.09	1.18	-1.70	-1.02	
0.36	1.05	1.30	-0.98	-1.36	-1.28	-1.32	-0.13
0.40	-1.58	-1.30	-0.10	-1.34	0.65	-0.56	
0.39	-0.73	-0.32	2.19	-0.49	0.69	0.18	5.47
0.75	-3.09	-0.61	-1.89	0.15	0.77	-0.49	
-0.63	1.20	-0.04	1.02	0.31	0.81	-0.45	13.97
-0.65	1.57	-1.50	-1.45	0.21	0.06	0.24	
2.24	-1.34	0.30	1.39	-0.38	-0.71	0.48	20.94
1.36	1.40	-1.40	-0.90	0.36	-0.21	-0.97	
0.36	-0.26	0.08	0.06	-1.49	0.43	-1.61	-12.87
-1.01	1.50	-0.61	-0.25	-1.01	-0.43	1.90	
-1.33	-0.96	-0.02	0.51	-1.38	-0.78	1.82	28.26
1.34	1.02	3.50	0.10	0.50	0.04	0.61	
-0.57	-2.69	-0.64	-0.34	-0.21	-1.97	-0.19	6.89
0.29	0.67	-0.38	-0.63	-0.24	1.21	-0.09	
0.90	-2.20	1.72	0.29	0.66	0.19	-0.57	5.37
0.67	-0.56	-0.41	1.22	-0.30	0.77	0.82	
0.36	1.18	1.87	-1.48	0.52	1.35	0.13	-1.50
-0.40	-1.10	-0.83	0.71	1.99	-0.24	1.30	
-0.34	-0.70	0.28	0.16	0.27	0.37	-1.79	-23.00
-0.78	0.60	-0.45	-0.26	-0.23	0.89	0.87	
1.01	1.20	0.28	0.79	2.76	0.35	1.31	14.09
-1.29	0.62	-0.59	1.52	0.62	0.21	1.31	
1.09	-0.36	-0.34	-0.03	-0.59	-1.70	-0.03	-11.05
0.40	-1.45	-0.98	2.10	-1.09	-0.53	-0.38	
-1.36	0.13	0.70	-1.51	0.08	-0.62	-0.64	-32.04
0.43	-0.86	0.70	-1.07	-0.76	0.72	-0.14	
-1.58	0.00	0.58	-0.21	1.30	2.02	1.52	23.36
-0.48	0.01	1.30	0.58	-0.54	1.09	0.91	
2.90	1.32	-1.20	-0.59	-0.51	0.20	-1.74	-5.58
-1.32	-1.41	-0.58	-1.29	1.61	-0.35	-0.72	
-1.92	-1.09	0.56	-0.87	-0.71	1.25	0.10	2.48
1.43	0.69	1.34	-0.32	2.84	-1.43	-0.47	
-0.01	0.83	-0.72	-0.78	0.50	-1.22	0.54	-5.30
0.82	0.46	0.15	-0.57	0.93	1.33	-0.23	
-1.07	0.76	0.25	-1.96	0.39	0.24	-0.26	-7.77
-0.91	0.23	-0.19	1.58	-0.27	0.33	-0.60	
-1.39	-0.30	-0.81	-0.95	0.88	-0.09	-0.35	-34.25
0.65	-1.14	1.18	-1.06	-0.68	-0.22	0.21	
0.94	1.08	0.81	-0.33	0.42	-0.90	0.49	26.78
-0.36	-0.50	-0.02	-0.04	0.77	0.62	-1.35	
-0.64	1.20	1.22	0.18	-1.39	-0.81	-0.99	-11.85
-1.82	1.06	0.28	0.14	0.62	-0.80	-1.08	
-2.15	1.37	1.57	-1.48	-0.79	0.28	-0.20	-8.62
1.54	0.50	0.13	-0.68	0.26	-1.13	0.62	
-0.43	0.39	1.14	0.15	1.03	0.46	0.40	12.35
-1.61	-0.61	0.93	-0.37	0.44	-1.45	0.58	
-1.77	0.72	-2.05	-0.03	-1.24	-1.40	-0.06	-1.54
-0.48	0.67	0.04	0.27	-0.84	-0.06	-3.67	
0.09	1.66	-0.30	1.67	1.08	0.00	0.43	-16.59
-1.65	-1.16	-1.17	1.12	0.11	-0.15	0.48	
-1.72	1.08	-0.94	0.49	-0.56	0.95	1.09	-8.69
-0.85	-0.02	1.18	-1.16	0.49	1.56	-0.60	

```

0.32  0.72  -1.20  2.52  1.78  0.16  -0.01          7.82
-0.60 -0.73  -1.23  1.50  0.40  -0.20  -0.65
0.68  1.09  0.40  -1.50  -2.10  0.21  -0.18        -18.56
-0.66 -0.01  -0.01  0.85  -2.04  1.17  -0.56
1.72  -0.18  1.14  -0.96  -0.92  -0.28  1.58          17.21  :: x,y

```

### 10.3 Program Results

nag\_best\_subset\_given\_size (h05abc) Example Program Results

Score	Feature Subset				
-----	-----	-----	-----	-----	-----
1.04753e+03	4	7	8	10	14
1.05987e+03	4	5	7	8	14
1.07016e+03	4	5	7	10	14

45 subsets evaluated in total

---