# NAG Library Function Document

# nag_tsa_varma_diagnostic (g13dsc)

## 1 Purpose

nag_tsa_varma_diagnostic (g13dsc) is a diagnostic checking function suitable for use after fitting a vector ARMA model to a multivariate time series using nag_tsa_varma_estimate (g13ddc). The residual cross-correlation matrices are returned along with an estimate of their asymptotic standard errors and correlations. Also, nag_tsa_varma_diagnostic (g13dsc) calculates the modified Li–McLeod portmanteau statistic and its significance level for testing model adequacy.

## 2 Specification

```
#include <nag.h>
#include <nagg13.h>
```

```
void nag_tsa_varma_diagnostic (Integer k, Integer n, const double v[],
     Integer kmax, Integer ip, Integer iq, Integer m, const double par[],
     const Nag_Boolean parhld[], double qq[], Integer ishow,
     const char *outfile, double r0[], double r[], double rcm[],
     Integer pdrcm, double *chi, Integer *idf, double *siglev,
     NagError *fail)
```

## 3 Description

Let $W_t = (w_{1t}, w_{2t}, \ldots, w_{kt})^{\mathrm{T}}$, for $t = 1, 2, \ldots, n$, denote a vector of $k$ time series which is assumed to follow a multivariate ARMA model of the form

$$\begin{aligned} W_t - \mu = \; & \phi_1(W_{t-1} - \mu) + \phi_2(W_{t-2} - \mu) + \cdots + \phi_p\big(W_{t-p} - \mu\big) \\ & + \epsilon_t - \theta_1\epsilon_{t-1} - \theta_2\epsilon_{t-2} - \cdots - \theta_q\epsilon_{t-q}, \end{aligned} \qquad (1)$$

where $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t}, \ldots, \epsilon_{kt})^{\mathrm{T}}$, for $t = 1, 2, \ldots, n$, is a vector of $k$ residual series assumed to be Normally distributed with zero mean and positive definite covariance matrix $\Sigma$. The components of $\epsilon_t$ are assumed to be uncorrelated at non-simultaneous lags. The $\phi_i$ and $\theta_j$ are $k$ by $k$ matrices of parameters. $\{\phi_i\}$, for $i = 1, 2, \ldots, p$, are called the autoregressive (AR) parameter matrices, and $\{\theta_i\}$, for $i = 1, 2, \ldots, q$, the moving average (MA) parameter matrices. The parameters in the model are thus the $p$ ($k$ by $k$) $\phi$-matrices, the $q$ ($k$ by $k$) $\theta$-matrices, the mean vector $\mu$ and the residual error covariance matrix $\Sigma$. Let

$$A(\phi) = \begin{bmatrix} \phi_1 & I & 0 & . & . & . & 0 \\ \phi_2 & 0 & I & 0 & . & . & 0 \\ . & & & . & & & \\ . & & & & . & & \\ . & & & & & . & \\ \phi_{p-1} & 0 & . & . & . & 0 & I \\ \phi_p & 0 & . & . & . & 0 & 0 \end{bmatrix}_{pk \times pk} \quad \text{and} \quad B(\theta) = \begin{bmatrix} \theta_1 & I & 0 & . & . & . & 0 \\ \theta_2 & 0 & I & 0 & . & . & 0 \\ . & & & . & & & \\ . & & & & . & & \\ . & & & & & . & \\ \theta_{q-1} & 0 & . & . & . & & I \\ \theta_q & 0 & . & . & . & . & 0 \end{bmatrix}_{qk \times qk}$$

where $I$ denotes the $k$ by $k$ identity matrix.

The ARMA model (1) is said to be stationary if the eigenvalues of $A(\phi)$ lie inside the unit circle, and invertible if the eigenvalues of $B(\theta)$ lie inside the unit circle. The ARMA model is assumed to be both stationary and invertible. Note that some of the elements of the $\phi$- and/or $\theta$-matrices may have been fixed at pre-specified values (for example by calling nag_tsa_varma_estimate (g13ddc)).

The estimated residual cross-correlation matrix at lag $l$ is defined to the $k$ by $k$ matrix $\hat{R}_l$ whose $(i, j)$th element is computed as

$$\hat{r}_{ij}(l) = \frac{\sum_{t=l+1}^{n}\left(\hat{\epsilon}_{it-l} - \bar{\epsilon}_i\right)\left(\hat{\epsilon}_{jt} - \bar{\epsilon}_j\right)}{\sqrt{\sum_{t=1}^{n}\left(\hat{\epsilon}_{it} - \bar{\epsilon}_i\right)^2 \sum_{t=1}^{n}\left(\hat{\epsilon}_{jt} - \bar{\epsilon}_j\right)^2}}, \quad l = 0, 1, \ldots, i \text{ and } j = 1, 2, \ldots, k,$$

where $\hat{\epsilon}_{it}$ denotes an estimate of the $t$th residual for the $i$th series $\epsilon_{it}$ and $\bar{\epsilon}_i = \sum_{t=1}^{n} \hat{\epsilon}_{it}/n$. (Note that $\hat{R}_l$ is an estimate of $E\left(\epsilon_{t-l}\epsilon_t^{\mathrm{T}}\right)$, where $E$ is the expected value.)

A modified portmanteau statistic, $Q^*_{(m)}$, is calculated from the formula (see Li and McLeod (1981))

$$Q^*_{(m)} = \frac{k^2 m(m+1)}{2n} + n\sum_{l=1}^{m} \hat{r}(l)^{\mathrm{T}}\left(\hat{R}_0^{-1} \otimes \hat{R}_0^{-1}\right)\hat{r}(l),$$

where $\otimes$ denotes Kronecker product, $\hat{R}_0$ is the estimated residual cross-correlation matrix at lag zero and $\hat{r}(l) = \mathrm{vec}\left(\hat{R}_l^{\mathrm{T}}\right)$, where vec of a $k$ by $k$ matrix is a vector with the $(i,j)$th element in position $(i-1)k+j$. $m$ denotes the number of residual cross-correlation matrices computed. (Advice on the choice of $m$ is given in Section 9.2.) Let $l_C$ denote the total number of 'free' parameters in the ARMA model excluding the mean, $\mu$, and the residual error covariance matrix $\Sigma$. Then, under the hypothesis of model adequacy, $Q^*_{(m)}$, has an asymptotic $\chi^2$-distribution on $mk^2 - l_C$ degrees of freedom.

Let $\underline{\hat{r}} = \left(\mathrm{vec}\left(R_1^{\mathrm{T}}\right), \mathrm{vec}\left(R_2^{\mathrm{T}}\right), \ldots, \mathrm{vec}\left(R_m^{\mathrm{T}}\right)\right)$ then the covariance matrix of $\underline{\hat{r}}$ is given by

$$\mathrm{Var}(\underline{\hat{r}}) = \left[Y - X\left(X^{\mathrm{T}}GG^{\mathrm{T}}X\right)^{-1}X^{\mathrm{T}}\right]/n,$$

where $Y = I_m \otimes (\Delta \otimes \Delta)$ and $G = I_m(GG^{\mathrm{T}})$. $\Delta$ is the dispersion matrix $\Sigma$ in correlation form and $G$ a nonsingular $k$ by $k$ matrix such that $GG^{\mathrm{T}} = \Delta^{-1}$ and $G\Delta G^{\mathrm{T}} = I_k$. The construction of the matrix $X$ is discussed in Li and McLeod (1981). (Note that the mean, $\mu$, plays no part in calculating $\mathrm{Var}(\underline{\hat{r}})$ and therefore is not required as input to nag_tsa_varma_diagnostic (g13dsc).)

## 4 References

Li W K and McLeod A I (1981) Distribution of the residual autocorrelations in multivariate ARMA time series models *J. Roy. Statist. Soc. Ser. B* **43** 231–239

## 5 Arguments

The output quantities **k**, **n**, **v**, **kmax**, **ip**, **iq**, **par**, **parhld** and **qq** from nag_tsa_varma_estimate (g13ddc) are suitable for input to nag_tsa_varma_diagnostic (g13dsc).

1: **k** – Integer *Input*

*On entry*: $k$, the number of residual time series.

*Constraint*: $\mathbf{k} \geq 1$.

2: **n** – Integer *Input*

*On entry*: $n$, the number of observations in each residual series.

3: **v**[**kmax** × **n**] – const double *Input*

*On entry*: **v**[**kmax** × $(t-1) + i - 1$] must contain an estimate of the $i$th component of $\epsilon_t$, for $i = 1, 2, \ldots, k$ and $t = 1, 2, \ldots, n$.

*Constraints*:

no two rows of **v** may be identical;
in each row there must be at least two distinct elements.

4: **kmax** – Integer *Input*

*On entry*: the first dimension of the arrays **V**, **QQ** and **R0**.

*Constraint*: $\mathbf{kmax} \geq \mathbf{k}$.

5: **ip** – Integer *Input*

*On entry*: $p$, the number of AR parameter matrices.

*Constraint*: $\mathbf{ip} \geq 0$.

6: **iq** – Integer *Input*

*On entry*: $q$, the number of MA parameter matrices.

*Constraint*: $\mathbf{iq} \geq 0$.

**Note**: $\mathbf{ip} = \mathbf{iq} = 0$ is **not permitted**.

7: **m** – Integer *Input*

*On entry*: the value of $m$, the number of residual cross-correlation matrices to be computed. See Section 9.2 for advice on the choice of **m**.

*Constraint*: $\mathbf{ip} + \mathbf{iq} < \mathbf{m} < \mathbf{n}$.

8: **par**$[(\mathbf{ip} + \mathbf{iq}) \times \mathbf{k} \times \mathbf{k}]$ – const double *Input*

*On entry*: the parameter estimates read in row by row in the order $\phi_1, \phi_2, \ldots, \phi_p$, $\theta_1, \theta_2, \ldots, \theta_q$.

Thus,

if $\mathbf{ip} > 0$, $\mathbf{par}[(l-1) \times k \times k + (i-1) \times k + j - 1]$ must be set equal to an estimate of the $(i,j)$th element of $\phi_l$, for $l = 1, 2, \ldots, p$ and $i = 1, 2, \ldots, k$;

if $\mathbf{iq} \geq 0$, $\mathbf{par}[p \times k \times k + (l-1) \times k \times k + (i-1) \times k + j - 1]$ must be set equal to an estimate of the $(i,j)$th element of $\theta_l$, for $l = 1, 2, \ldots, q$ and $i = 1, 2, \ldots, k$.

The first $p \times k \times k$ elements of **par** must satisfy the stationarity condition and the next $q \times k \times k$ elements of **par** must satisfy the invertibility condition.

9: **parhld**$[(\mathbf{ip} + \mathbf{iq}) \times \mathbf{k} \times \mathbf{k}]$ – const Nag_Boolean *Input*

*On entry*: $\mathbf{parhld}[i-1]$ must be set to Nag_TRUE if $\mathbf{par}[i-1]$ has been held constant at a pre-specified value and Nag_FALSE if $\mathbf{par}[i-1]$ is a free parameter, for $i = 1, 2, \ldots, (p+q) \times k \times k$.

10: **qq**$[\mathbf{kmax} \times \mathbf{k}]$ – double *Input/Output*

*On entry*: $\mathbf{qq}[\mathbf{kmax} \times (j-1) + i - 1]$ is an efficient estimate of the $(i,j)$th element of $\Sigma$. The lower triangle only is needed.

*Constraint*: **qq** must be positive definite.

*On exit*: if **fail.code** = NE_G13D_AR, NE_G13D_DIAG, NE_G13D_FACT, NE_G13D_ITER, NE_G13D_MA, NE_G13D_RES, NE_G13D_ZERO_VAR or NE_NOT_POS_DEF, then the upper triangle is set equal to the lower triangle.

11: **ishow** – Integer *Input*

*On entry*: must be nonzero if the residual cross-correlation matrices $\{\hat{r}_{ij}(l)\}$ and their standard errors $\{\mathrm{se}(\hat{r}_{ij}(l))\}$, the modified portmanteau statistic with its significance and a summary table are to be printed. The summary table indicates which elements of the residual correlation matrices are significant at the 5% level in either a positive or negative direction; i.e., if $\hat{r}_{ij}(l) > 1.96 \times \mathrm{se}(\hat{r}_{ij}(l))$ then a '+' is printed, if $\hat{r}_{ij}(l) < -1.96 \times \mathrm{se}(\hat{r}_{ij}(l))$ then a '−' is printed, otherwise a fullstop (.) is printed. The summary table is only printed if $k \leq 6$ on entry.

The residual cross-correlation matrices, their standard errors and the modified portmanteau statistic with its significance are available also as output variables in **r**, **rcm**, **chi**, **idf** and **siglev**.

12: **outfile** – const char * *Input*

*On entry*: the name of a file to which diagnostic output will be directed. If **outfile** is **NULL** the diagnostic output will be directed to standard output.

13: **r0**[**kmax** × **k**] – double *Output*

*On exit*: if $i \neq j$, then **r0**[**kmax** $\times (j-1)+i-1$] contains an estimate of the $(i,j)$th element of the residual cross-correlation matrix at lag zero, $\hat{R}_0$. When $i = j$, **r0**[**kmax** $\times (j-1)+i-1$] contains the standard deviation of the $i$th residual series. If **fail.code** = NE_G13D_RES or NE_G13D_ZERO_VAR on exit then the first **k** rows and columns of **r0** are set to zero.

14: **r**[*dim*] – double *Output*

**Note**: the dimension, *dim*, of the array **r** must be at least **kmax** × **kmax** × **m**.

Where **R**$(l,i,j)$ appears in this document, it refers to the array element **r**[$(j-1) \times$ **kmax** × **kmax** + $(i-1) \times$ **kmax** + $l-1$].

*On exit*: **R**$(l,i,j)$ is an estimate of the $(i,j)$th element of the residual cross-correlation matrix at lag $l$, for $i = 1,2,\dots,k$, $j = 1,2,\dots,k$ and $l = 1,2,\dots,m$. If **fail.code** = NE_G13D_RES or NE_G13D_ZERO_VAR on exit then all elements of **r** are set to zero.

15: **rcm**[**pdrcm** × **m** × **k** × **k**] – double *Output*

*On exit*: the estimated standard errors and correlations of the elements in the array **r**. The correlation between **R**$(l,i,j)$ and **R**$(l_2,i_2,j_2)$ is returned as **rcm**[**pdrcm** $\times t+s$] where $s = (l-1) \times k \times k + (j-1) \times k + i$ and $t = (l_2-1) \times k \times k + (j_2-1) \times k + i_2$ except that if $s = t$, then **rcm**[**pdrcm** $\times t+s$] contains the standard error of **R**$(l,i,j)$. If on exit, **fail.code** = NE_G13D_DIAG or NE_G13D_FACT, then all off-diagonal elements of **RCM** are set to zero and all diagonal elements are set to $1/\sqrt{n}$.

16: **pdrcm** – Integer *Input*

*On entry*: the first dimension of the array **RCM**.

*Constraint*: **pdrcm** ≥ **m** × **k** × **k**.

17: **chi** – double * *Output*

*On exit*: the value of the modified portmanteau statistic, $Q^*_{(m)}$. If **fail.code** = NE_G13D_RES or NE_G13D_ZERO_VAR on exit then **chi** is returned as zero.

18: **idf** – Integer * *Output*

*On exit*: the number of degrees of freedom of **chi**.

19: **siglev** – double * *Output*

*On exit*: the significance level of **chi** based on **idf** degrees of freedom. If **fail.code** = NE_G13D_RES or NE_G13D_ZERO_VAR on exit, **siglev** is returned as one.

20: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

## 6 Error Indicators and Warnings

**NE_ALLOC_FAIL**

Dynamic memory allocation failed.

**NE_BAD_PARAM**

On entry, argument ⟨*value*⟩ had an illegal value.

**NE_G13D_AR**

On entry, the AR parameter estimates are outside the stationarity region.

**NE_G13D_ARMA**

On entry, **ip** $= 0$ and **iq** $= 0$.

**NE_G13D_DIAG**

The matrix **rcm** could not be computed because one of its diagonal elements was found to be non-positive.

**NE_G13D_FACT**

On entry, the AR operator has a factor in common with the MA operator.

**NE_G13D_ITER**

Excessive iterations needed to find zeros of determinental polynomials.

**NE_G13D_MA**

On entry, the MA parameter matrices are outside the invertibility region.

**NE_G13D_RES**

On entry, at least two of the residual series are identical.

**NE_G13D_ZERO_VAR**

On entry, at least one of the residual series in the array **v** has near-zero variance.

**NE_INT**

On entry, **ip** $= \langle value \rangle$.
Constraint: **ip** $\geq 0$.

On entry, **iq** $= \langle value \rangle$.
Constraint: **iq** $\geq 0$.

On entry, **k** $= \langle value \rangle$.
Constraint: **k** $\geq 1$.

**NE_INT_2**

On entry, **kmax** $= \langle value \rangle$ and **k** $= \langle value \rangle$.
Constraint: **kmax** $\geq$ **k**.

On entry, **m** $= \langle value \rangle$ and **n** $= \langle value \rangle$.
Constraint: **m** $<$ **n**.

**NE_INT_3**

On entry, **m** $= \langle value \rangle$, **ip** $= \langle value \rangle$ and **iq** $= \langle value \rangle$.
Constraint: **m** $>$ **ip** $+$ **iq**.

On entry, **pdrcm** = ⟨*value*⟩, **m** = ⟨*value*⟩ and **k** = ⟨*value*⟩.
Constraint: **pdrcm** $\geq$ **m** $\times$ **k** $\times$ **k**.

**NE_INTERNAL_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

**NE_NOT_CLOSE_FILE**

Cannot close file ⟨*value*⟩.

**NE_NOT_POS_DEF**

On entry, the covariance matrix **qq** is not positive definite.

**NE_NOT_WRITE_FILE**

Cannot open file ⟨*value*⟩ for writing.

# 7 Accuracy

The computations are believed to be stable.

# 8 Parallelism and Performance

nag_tsa_varma_diagnostic (g13dsc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag_tsa_varma_diagnostic (g13dsc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

# 9 Further Comments

## 9.1 Timing

The time taken by nag_tsa_varma_diagnostic (g13dsc) depends upon the number of residual cross-correlation matrices to be computed, $m$, and the number of time series, $k$.

## 9.2 Choice of *m*

The number of residual cross-correlation matrices to be computed, $m$, should be chosen to ensure that when the ARMA model (1) is written as either an infinite order autoregressive process, i.e.,

$$W_t - \mu = \sum_{j=1}^{\infty} \pi_j \left( W_{t-j} - \mu \right) + \epsilon_t$$

or as an infinite order moving average process, i.e.,

$$W_t - \mu = \sum_{j=1}^{\infty} \psi_j \epsilon_{t-j} + \epsilon_t$$

then the two sequences of $k$ by $k$ matrices $\{\pi_1, \pi_2, \ldots\}$ and $\{\psi_1, \psi_2, \ldots\}$ are such that $\pi_j$ and $\psi_j$ are approximately zero for $j > m$. An overestimate of $m$ is therefore preferable to an under-estimate of $m$. In many instances the choice $m = 10$ will suffice. In practice, to be on the safe side, you should try setting $m = 20$.

### 9.3 Checking a 'White Noise' Model

If you have fitted the 'white noise' model

$$W_t - \mu = \epsilon_t$$

then nag_tsa_varma_diagnostic (g13dsc) should be entered with $p = 1$, $q = 0$, and the first $k^2$ elements of **par** and **parhld** set to zero and Nag_TRUE respectively.

### 9.4 Approximate Standard Errors

When **fail.code** = NE_G13D_DIAG or NE_G13D_FACT all the standard errors in **rcm** are set to $1/\sqrt{n}$. This is the asymptotic standard error of $\hat{r}_{ij}(l)$ when all the autoregressive and moving average parameters are assumed to be known rather than estimated.

### 9.5 Alternative Tests

$\hat{R}_0$ is useful in testing for instantaneous causality. If you wish to carry out a likelihood ratio test then the covariance matrix at lag zero $\left(\hat{C}_0\right)$ can be used. It can be recovered from $\hat{R}_0$ by setting

$$\begin{aligned}\hat{C}_0(i,j) &= \hat{R}_0(i,j) \times \hat{R}_0(i,i) \times \hat{R}_0(j,j), &&\text{for } i \neq j\\ &= \hat{R}_0(i,j) \times \hat{R}_0(i,j), &&\text{for } i = j\end{aligned}$$

## 10 Example

This example fits a bivariate AR(1) model to two series each of length 48. $\mu$ has been estimated but $\phi_1(2,1)$ has been constrained to be zero. Ten residual cross-correlation matrices are to be computed.

### 10.1 Program Text

```
/* nag_tsa_varma_diagnostic (g13dsc) Example Program.
 *
 * Copyright 2005 Numerical Algorithms Group.
 *
 * Mark 8, 2004.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg13.h>

int main(void)
{
  /* Scalars */
  double          cgetol, chi, rlogl, siglev;
  Integer         exit_status = 0, i, icm, idf, kmax, ip, iprint, iq, pdrcm,
                  ishow;
  Integer         j, k, m, maxcal, n, niter, npar;
  Nag_Boolean     exact;
  Nag_IncludeMean mean;
  /* Arrays */
  char            nag_enum_arg[40];
  double          *cm = 0, *g = 0, *par = 0, *qq = 0, *r0 = 0, *r = 0;
  double          *rcm = 0, *v = 0, *w = 0;
  Integer         *iw = 0;
  /* Nag types */
  Nag_Boolean     *parhld = 0;
  NagError        fail;

#define W(I, J)  w[(J - 1) * kmax + I - 1]
#define QQ(I, J) qq[(J - 1) * kmax + I - 1]

  INIT_FAIL(fail);
```

```
printf("nag_tsa_varma_diagnostic (g13dsc) Example Program Results\n");
fflush(stdout);

/* Skip heading in data file */
scanf("%*[^\n] ");
scanf("%ld%ld%*[^\n] ", &k, &n);

if (k > 0 && n >= 3)
  {
    kmax = k;
    /* Allocate memory */
    if (!(qq = NAG_ALLOC(k * kmax, double)) ||
        !(r0 = NAG_ALLOC(k * kmax, double)) ||
        !(v = NAG_ALLOC(n * kmax, double)) ||
        !(w = NAG_ALLOC(n * kmax, double)))
      {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
      }
  }
else
  {
    printf("Invalid parameter values\n");
    exit_status = -1;
    goto END;
  }

for (i = 1; i <= k; ++i)
  {
    for (j = 1; j <= n; ++j)
      {
        scanf("%lf", &W(i, j));
      }
  }
scanf("%*[^\n]");
scanf("%ld%ld %39s %ld%*[^\n] ", &ip, &iq,
      nag_enum_arg, &m);
/* nag_enum_name_to_value (x04nac).
 * Converts NAG enum member name to value
 */
mean = (Nag_IncludeMean) nag_enum_name_to_value(nag_enum_arg);
if (ip >= 0 && iq >= 0)
  {
    npar = (ip + iq) * k * k;
    if (mean == Nag_MeanInclude)
      {
        npar += k;
      }
    icm = npar;
    pdrcm = m * k * k;
  }
else
  {
    printf("Invalid parameter values\n");
    exit_status = -1;
    goto END;
  }

/* Allocate memory */
if (!(cm = NAG_ALLOC(npar * icm, double)) ||
    !(g = NAG_ALLOC(npar, double)) ||
    !(par = NAG_ALLOC(npar, double)) ||
    !(r = NAG_ALLOC(k * k * m, double)) ||
    !(rcm = NAG_ALLOC(m*k*k * pdrcm, double)) ||
    !(parhld = NAG_ALLOC(npar, Nag_Boolean)))
  {
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
```

```
    }

  for (i = 1; i <= npar; ++i)
    {
      par[i-1] = 0.0;
      parhld[i-1] = Nag_FALSE;
    }

  for (j = 1; j <= k; ++j)
    {
      for (i = j; i <= k; ++i)
        {
          QQ(i, j) = 0.0;
        }
    }
  parhld[2] = Nag_TRUE;
  exact = Nag_TRUE;
  /* ** Set iprint > 0 to obtain intermediate output ** */
  iprint = -1;
  cgetol = 1.0e-4;
  maxcal = npar * 40 * (npar + 5);
  ishow = 2;

  /* nag_tsa_varma_estimate (g13ddc).
   * Multivariate time series, estimation of VARMA model
   */
  fflush(stdout);
  nag_tsa_varma_estimate(k, n, ip, iq, mean, par, npar, qq, kmax, w, parhld,
                         exact, iprint, cgetol, maxcal, ishow, 0, &niter,
                         &rlogl, v, g, cm, icm, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("\n nag_tsa_varma_estimate (g13ddc) message: %s\n\n",
              fail.message);
      exit_status = 1;
      goto END;
    }

  if (fail.code == NE_NOERROR || fail.code == NE_G13D_MAXCAL ||
      fail.code == NE_MAX_LOGLIK || fail.code == NE_G13D_BOUND ||
      fail.code == NE_G13D_DERIV ||
      fail.code == NE_HESS_NOT_POS_DEF)
    {
      printf("\nOutput from nag_tsa_varma_diagnostic (g13dsc)\n");
      fflush(stdout);
      ishow = 1;
      /* nag_tsa_varma_diagnostic (g13dsc).
       * Multivariate time series, diagnostic checking of
       * residuals, following nag_tsa_varma_estimate (g13ddc)
       */
      nag_tsa_varma_diagnostic(k, n, v, k, ip, iq, m, par, parhld, qq, ishow,
                               0, r0, r, rcm, pdrcm, &chi, &idf, &siglev,
                               &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("nag_tsa_varma_diagnostic (g13dsc) message: %s\n\n",
                  fail.message);
          exit_status = 1;
        }
    }

 END:
  NAG_FREE(cm);
  NAG_FREE(g);
  NAG_FREE(par);
  NAG_FREE(qq);
  NAG_FREE(r0);
  NAG_FREE(r);
  NAG_FREE(rcm);
  NAG_FREE(v);
  NAG_FREE(w);
```

```
  NAG_FREE(iw);
  NAG_FREE(parhld);

  return exit_status;
}
```

## 10.2 Program Data

```
nag_tsa_varma_diagnostic (g13dsc) Example Program Data
2 48  : k, no. of time series, n, no. of obs in each time series
-1.490 -1.620  5.200  6.230  6.210  5.860
 4.090  3.180  2.620  1.490  1.170  0.850
-0.350  0.240  2.440  2.580  2.040  0.400
 2.260  3.340  5.090  5.000  4.780  4.110
 3.450  1.650  1.290  4.090  6.320  7.500
 3.890  1.580  5.210  5.250  4.930  7.380
 5.870  5.810  9.680  9.070  7.290  7.840
 7.550  7.320  7.970  7.760  7.000  8.350
 7.340  6.350  6.960  8.540  6.620  4.970
 4.550  4.810  4.750  4.760 10.880 10.010
11.620 10.360  6.400  6.240  7.930  4.040
 3.730  5.600  5.350  6.810  8.270  7.680
 6.650  6.080 10.250  9.140 17.750 13.300
 9.630  6.800  4.080  5.060  4.940  6.650
 7.940 10.760 11.890  5.850  9.010  7.500
10.020 10.380  8.150  8.370 10.730 12.140 : End of time series
1 0 Nag_MeanInclude 10 : ip, iq, mean and m
```

## 10.3 Program Results

```
nag_tsa_varma_diagnostic (g13dsc) Example Program Results

 VALUE OF LOG LIKELIHOOD FUNCTION ON EXIT = -0.20280E+03

 MAXIMUM LIKELIHOOD ESTIMATES OF AR PARAMETER MATRICES
 -----------------------------------------------------

 PHI(1)    ROW-WISE :     0.802   0.065
                       ( 0.091)( 0.102)

                          0.000   0.575
                       ( 0.000)( 0.121)

 MAXIMUM LIKELIHOOD ESTIMATE OF PROCESS MEAN
 -------------------------------------------

                          4.271   7.825
                       ( 1.219)( 0.776)

 MAXIMUM LIKELIHOOD ESTIMATE OF SIGMA MATRIX
 -------------------------------------------

                          2.964

                          0.637   5.380

           RESIDUAL SERIES NUMBER  1
           -------------------------

   T     1      2      3      4      5      6      7      8
 V(T)  -3.33  -1.24   5.75   1.27   0.32   0.11  -1.27  -0.73

   T     9     10     11     12     13     14     15     16
 V(T)  -0.58  -1.26  -0.67  -1.13  -2.02  -0.57   1.24  -0.13

   T    17     18     19     20     21     22     23     24
 V(T)  -0.77  -2.09   1.34   0.95   1.71   0.23  -0.01  -0.60

   T    25     26     27     28     29     30     31     32
 V(T)  -0.68  -1.89  -0.77   2.05   2.11   0.94  -3.32  -2.50
```

```
  T    33     34     35     36     37     38     39     40
V(T)   3.16   0.47   0.05   2.77  -0.82   0.25   3.99   0.20

  T    41     42     43     44     45     46     47     48
V(T)  -0.70   1.07   0.44   0.28   1.09   0.50  -0.10   1.70

           RESIDUAL SERIES NUMBER  2
           -------------------------

  T     1      2      3      4      5      6      7      8
V(T)  -0.19  -1.20  -0.02   1.21  -1.62  -2.16  -1.63  -1.13

  T     9     10     11     12     13     14     15     16
V(T)  -1.34  -1.30   4.82   0.43   2.54   0.35  -2.88  -0.77

  T    17     18     19     20     21     22     23     24
V(T)   1.02  -3.85  -1.92   0.13  -1.20   0.41   1.03  -0.40

  T    25     26     27     28     29     30     31     32
V(T)  -1.09  -1.07   3.43  -0.08   9.17  -0.23  -1.34  -2.06

  T    33     34     35     36     37     38     39     40
V(T)  -3.16  -0.61  -1.30   0.48   0.79   2.87   2.38  -4.31

  T    41     42     43     44     45     46     47     48
V(T)   2.32  -1.01   2.38   1.29  -1.14   0.36   2.59   2.64
```

```
Output from nag_tsa_varma_diagnostic (g13dsc)

 RESIDUAL CROSS-CORRELATION MATRICES
 -----------------------------------

 LAG     1          :    0.130   0.112
                       ( 0.119)( 0.143)
                         0.094   0.043
                       ( 0.069)( 0.102)

 LAG     2          :   -0.312   0.021
                       ( 0.128)( 0.144)
                        -0.162   0.098
                       ( 0.125)( 0.132)

 LAG     3          :    0.004  -0.176
                       ( 0.134)( 0.144)
                        -0.168  -0.091
                       ( 0.139)( 0.140)

 LAG     4          :   -0.090  -0.120
                       ( 0.137)( 0.144)
                         0.099  -0.232
                       ( 0.142)( 0.143)

 LAG     5          :    0.041   0.093
                       ( 0.140)( 0.144)
                        -0.009  -0.089
                       ( 0.144)( 0.144)

 LAG     6          :    0.234  -0.008
                       ( 0.141)( 0.144)
                         0.069  -0.103
                       ( 0.144)( 0.144)

 LAG     7          :   -0.076   0.007
                       ( 0.142)( 0.144)
                         0.168   0.000
                       ( 0.144)( 0.144)

 LAG     8          :   -0.074   0.559
                       ( 0.143)( 0.144)
                         0.008  -0.101
```

```
                       ( 0.144)( 0.144)

 LAG     9          :    0.091   0.193
                       ( 0.144)( 0.144)
                         0.055   0.170
                       ( 0.144)( 0.144)

 LAG    10          :   -0.060   0.061
                       ( 0.144)( 0.144)
                         0.191   0.089
                       ( 0.144)( 0.144)

 SUMMARY TABLE
 -------------

 LAGS   1 -  10

 ***************************
 *            *            *
 * .-........ * .......+.. *
 *            *            *
 ***************************
 *            *            *
 * .......... * .......... *
 *            *            *
 ***************************

 LI-MCLEOD PORTMANTEAU STATISTIC =     49.234
              SIGNIFICANCE LEVEL =      0.086
 (BASED ON  37 DEGREES OF FREEDOM)
```