

# NAG Library Function Document

## nag\_tsa\_spectrum\_bivar\_cov (g13ccc)

### 1 Purpose

nag\_tsa\_spectrum\_bivar\_cov (g13ccc) calculates the smoothed sample cross spectrum of a bivariate time series using one of four lag windows: rectangular, Bartlett, Tukey or Parzen.

### 2 Specification

```
#include <nag.h>
#include <naggl3.h>

void nag_tsa_spectrum_bivar_cov (Integer nxy,
    NagMeanOrTrend mtxy_correction, double pxy, Integer iw, Integer mw,
    Integer ish, Integer ic, Integer nc, double cxy[], double cyx[],
    Integer kc, Integer l, double xg[], double yg[], Complex g[],
    Integer *ng, NagError *fail)
```

### 3 Description

The smoothed sample cross spectrum is a complex valued function of frequency  $\omega$ ,  $f_{xy}(\omega) = cf(\omega) + iqf(\omega)$ , defined by its real part or co-spectrum

$$cf(\omega) = \frac{1}{2\pi} \sum_{k=-M+1}^{M-1} w_k C_{xy}(k+S) \cos(\omega k)$$

and imaginary part or quadrature spectrum

$$qf(\omega) = \frac{1}{2\pi} \sum_{k=-M+1}^{M-1} w_k C_{xy}(k+S) \sin(\omega k)$$

where  $w_k = w_{-k}$ , for  $k = 0, 1, \dots, M-1$ , is the smoothing lag window as defined in the description of nag\_tsa\_spectrum\_univar\_cov (g13cac). The alignment shift  $S$  is recommended to be chosen as the lag  $k$  at which the cross-covariances  $c_{xy}(k)$  peak, so as to minimize bias.

The results are calculated for frequency values

$$\omega_j = \frac{2\pi j}{L}, \quad j = 0, 1, \dots, [L/2],$$

where  $[]$  denotes the integer part.

The cross-covariances  $c_{xy}(k)$  may be supplied by you, or constructed from supplied series  $x_1, x_2, \dots, x_n$ ;  $y_1, y_2, \dots, y_n$  as

$$c_{xy}(k) = \frac{\sum_{t=1}^{n-k} x_t y_{t+k}}{n}, \quad k \geq 0$$

$$c_{xy}(k) = \frac{\sum_{t=1-k}^n x_t y_{t+k}}{n} = c_{yx}(-k), \quad k < 0$$

this convolution being carried out using the finite Fourier transform.

The supplied series may be mean and trend corrected and tapered before calculation of the cross-covariances, in exactly the manner described in `nag_tsa_spectrum_univar_cov` (g13cac) for univariate spectrum estimation. The results are corrected for any bias due to tapering.

The bandwidth associated with the estimates is not returned. It will normally already have been calculated in previous calls of `nag_tsa_spectrum_univar_cov` (g13cac) for estimating the univariate spectra of  $y_t$  and  $x_t$ .

## 4 References

Bloomfield P (1976) *Fourier Analysis of Time Series: An Introduction* Wiley

Jenkins G M and Watts D G (1968) *Spectral Analysis and its Applications* Holden-Day

## 5 Arguments

1: **nxy** – Integer *Input*

*On entry:*  $n$ , the length of the time series  $x$  and  $y$ .

*Constraint:*  $nxy \geq 1$ .

2: **mtxy\_correction** – NagMeanOrTrend *Input*

*On entry:* if cross-covariances are to be calculated by the function (**ic** = 0), **mtxy\_correction** must specify whether the data is to be initially mean or trend corrected.

**mtxy\_correction** = Nag\_NoCorrection

For no correction.

**mtxy\_correction** = Nag\_Mean

For mean correction.

**mtxy\_correction** = Nag\_Trend

For trend correction.

If cross-covariances are supplied (**ic**  $\neq$  0), **mtxy\_correction** should be set to **mtxy\_correction** = Nag\_NoCorrection

*Constraint:* if **ic** = 0, **mtxy\_correction** = Nag\_NoCorrection, Nag\_Mean or Nag\_Trend.

3: **pxy** – double *Input*

*On entry:* if cross-covariances are to be calculated by the function (**ic** = 0), **pxy** must specify the proportion of the data (totalled over both ends) to be initially tapered by the split cosine bell taper. A value of 0.0 implies no tapering.

If cross-covariances are supplied (**ic**  $\neq$  0), **pxy** is not used.

*Constraint:* if **ic** = 0,  $0.0 \leq pxy \leq 1.0$ .

4: **iw** – Integer *Input*

*On entry:* the choice of lag window.

**iw** = 1

Rectangular.

**iw** = 2

Bartlett.

**iw** = 3

Tukey.

**iw** = 4  
Parzen.

Constraint:  $1 \leq \mathbf{iw} \leq 4$ .

5: **mw** – Integer *Input*

*On entry:*  $M$ , the ‘cut-off’ point of the lag window, relative to any alignment shift that has been applied. Windowed cross-covariances at lags  $(-\mathbf{mw} + \mathbf{ish})$  or less, and at lags  $(\mathbf{mw} + \mathbf{ish})$  or greater are zero.

*Constraints:*

$\mathbf{mw} \geq 1$ ;  
 $\mathbf{mw} + |\mathbf{ish}| \leq \mathbf{nxy}$ .

6: **ish** – Integer *Input*

*On entry:*  $S$ , the alignment shift between the  $x$  and  $y$  series. If  $x$  leads  $y$ , the shift is positive.

Constraint:  $-\mathbf{mw} < \mathbf{ish} < \mathbf{mw}$ .

7: **ic** – Integer *Input*

*On entry:* indicates whether cross-covariances are to be calculated in the function or supplied in the call to the function.

**ic** = 0  
Cross-covariances are to be calculated.

**ic**  $\neq$  0  
Cross-covariances are to be supplied.

8: **nc** – Integer *Input*

*On entry:* the number of cross-covariances to be calculated in the function or supplied in the call to the function.

Constraint:  $\mathbf{mw} + |\mathbf{ish}| \leq \mathbf{nc} \leq \mathbf{nxy}$ .

9: **cxv[nc]** – double *Input/Output*

*On entry:* if **ic**  $\neq$  0, **cxv** must contain the **nc** cross-covariances between values in the  $y$  series and earlier values in time in the  $x$  series, for lags from 0 to  $(\mathbf{nc} - 1)$ .

If **ic** = 0, **cxv** need not be set.

*On exit:* if **ic** = 0, **cxv** will contain the **nc** calculated cross-covariances.

If **ic**  $\neq$  0, the contents of **cxv** will be unchanged.

10: **cyx[nc]** – double *Input/Output*

*On entry:* if **ic**  $\neq$  0, **cyx** must contain the **nc** cross-covariances between values in the  $y$  series and later values in time in the  $x$  series, for lags from 0 to  $(\mathbf{nc} - 1)$ .

If **ic** = 0, **cyx** need not be set.

*On exit:* if **ic** = 0, **cyx** will contain the **nc** calculated cross-covariances.

If **ic**  $\neq$  0, the contents of **cyx** will be unchanged.

11: **kc** – Integer *Input*

*On entry:* if **ic** = 0, **kc** must specify the order of the fast Fourier transform (FFT) used to calculate the cross-covariances. **kc** should be a product of small primes such as  $2^m$  where  $m$  is the smallest integer such that  $2^m \geq n + \mathbf{nc}$ .

If  $\mathbf{ic} \neq 0$ , that is if covariances are supplied,  $\mathbf{kc}$  is not used.

*Constraint:*  $\mathbf{kc} \geq \mathbf{nxy} + \mathbf{nc}$ . The largest prime factor of  $\mathbf{kc}$  must not exceed 19, and the total number of prime factors of  $\mathbf{kc}$ , counting repetitions, must not exceed 20. These two restrictions are imposed by the internal FFT algorithm used.

12:  $\mathbf{l}$  – Integer *Input*

*On entry:*  $L$ , the frequency division of the spectral estimates as  $\frac{2\pi}{L}$ . Therefore it is also the order of the FFT used to construct the sample spectrum from the cross-covariances.  $\mathbf{l}$  should be a product of small primes such as  $2^m$  where  $m$  is the smallest integer such that  $2^m \geq 2M - 1$ .

*Constraint:*  $\mathbf{l} \geq 2 \times \mathbf{mw} - 1$ . The largest prime factor of  $\mathbf{l}$  must not exceed 19, and the total number of prime factors of  $\mathbf{l}$ , counting repetitions, must not exceed 20. These two restrictions are imposed by the internal FFT algorithm used.

13:  $\mathbf{xg}[\mathit{dim}]$  – double *Input/Output*

**Note:** the dimension,  $\mathit{dim}$ , of the array  $\mathbf{xg}$  must be at least

$$\begin{aligned} &\max(\mathbf{kc}, \mathbf{l}), \text{ when } \mathbf{ic} = 0; \\ &\mathbf{l}, \text{ when } \mathbf{ic} \neq 0. \end{aligned}$$

*On entry:* if the cross-covariances are to be calculated ( $\mathbf{ic} = 0$ )  $\mathbf{xg}$  must contain the  $\mathbf{nxy}$  data points of the  $x$  series. If covariances are supplied ( $\mathbf{ic} \neq 0$ )  $\mathbf{xg}$  may contain any values.

*On exit:* contains the real parts of the  $\mathbf{ng}$  complex spectral estimates in elements  $\mathbf{xg}[0]$  to  $\mathbf{xg}[\mathbf{ng} - 1]$ , and  $\mathbf{xg}[\mathbf{ng}]$  to  $\mathbf{xg}[\mathit{dim} - 1]$  contain 0.0. The  $y$  series leads the  $x$  series.

14:  $\mathbf{yg}[\mathit{dim}]$  – double *Input/Output*

**Note:** the dimension,  $\mathit{dim}$ , of the array  $\mathbf{yg}$  must be at least

$$\begin{aligned} &\max(\mathbf{kc}, \mathbf{l}), \text{ when } \mathbf{ic} = 0; \\ &\mathbf{l}, \text{ when } \mathbf{ic} \neq 0. \end{aligned}$$

*On entry:* if the cross-covariances are to be calculated ( $\mathbf{ic} = 0$ )  $\mathbf{yg}$  must contain the  $\mathbf{nxy}$  data points of the  $y$  series. If covariances are supplied ( $\mathbf{ic} \neq 0$ )  $\mathbf{yg}$  may contain any values.

*On exit:* contains the imaginary parts of the  $\mathbf{ng}$  complex spectral estimates in elements  $\mathbf{yg}[0]$  to  $\mathbf{yg}[\mathbf{ng} - 1]$ , and  $\mathbf{yg}[\mathbf{ng}]$  to  $\mathbf{yg}[\mathit{dim} - 1]$  contain 0.0. The  $y$  series leads the  $x$  series.

15:  $\mathbf{g}[\mathbf{l}/2 + 1]$  – Complex *Output*

*On exit:* the complex vector that contains the  $\mathbf{ng}$  cross spectral estimates in elements  $\mathbf{g}[0]$  to  $\mathbf{g}[\mathbf{ng} - 1]$ . The  $y$  series leads the  $x$  series.

16:  $\mathbf{ng}$  – Integer \* *Output*

*On exit:* the number,  $[\mathbf{l}/2] + 1$ , of complex spectral estimates.

17: **fail** – NagError \* *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

## 6 Error Indicators and Warnings

### NE\_BAD\_PARAM

On entry, argument  $\langle \mathit{value} \rangle$  had an illegal value.

**NE\_INT**

On entry, **ic** = 0 and **mtxy\_correction**  $\neq$  Nag\_NoCorrection, Nag\_Mean or Nag\_Trend:  
**mtxy\_correction** =  $\langle value \rangle$ .

On entry, **iw** =  $\langle value \rangle$ .  
 Constraint: **iw** = 1, 2, 3 or 4.

On entry, **mw** =  $\langle value \rangle$ .  
 Constraint: **mw**  $\geq$  1.

On entry, **nxy** =  $\langle value \rangle$ .  
 Constraint: **nxy**  $\geq$  1.

**NE\_INT\_2**

On entry, **ish** =  $\langle value \rangle$  and **mw** =  $\langle value \rangle$ .  
 Constraint: **ish**  $\leq$  **mw**.

On entry, **l** =  $\langle value \rangle$  and **mw** =  $\langle value \rangle$ .  
 Constraint: **l**  $\geq$   $2 \times \mathbf{mw} - 1$ .

On entry, **nc** =  $\langle value \rangle$  and **nxy** =  $\langle value \rangle$ .  
 Constraint: **nc**  $\leq$  **nxy**.

**NE\_INT\_3**

On entry, **kc** =  $\langle value \rangle$ , **nxy** =  $\langle value \rangle$  and **nc** =  $\langle value \rangle$ .  
 Constraint: if **ic** = 0, **kc**  $\geq$  **nxy** + **nc**.

On entry, **mw** =  $\langle value \rangle$ , **ish** =  $\langle value \rangle$  and **nxy** =  $\langle value \rangle$ .  
 Constraint: **mw** + **ish**  $\leq$  **nxy**.

On entry, **nc** =  $\langle value \rangle$ , **mw** =  $\langle value \rangle$  and **ish** =  $\langle value \rangle$ .  
 Constraint: **nc**  $\geq$  **mw** + **ish**.

**NE\_INT\_REAL**

On entry, **pxy** =  $\langle value \rangle$ .  
 Constraint: if **ic** = 0, **pxy**  $\leq$  1.0.

On entry, **pxy** =  $\langle value \rangle$ .  
 Constraint: if **ic** = 0, **pxy**  $\geq$  0.0.

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

**NE\_PRIME\_FACTOR**

**kc** has a prime factor exceeding 19, or more than 20 prime factors (counting repetitions):  
**kc** =  $\langle value \rangle$ .

**l** has a prime factor exceeding 19, or more than 20 prime factors (counting repetitions):  
**l** =  $\langle value \rangle$ .

**7 Accuracy**

The FFT is a numerically stable process, and any errors introduced during the computation will normally be insignificant compared with uncertainty in the data.

**8 Parallelism and Performance**

Not applicable.

## 9 Further Comments

nag\_tsa\_spectrum\_bivar\_cov (g13ccc) carries out two FFTs of length  $kc$  to calculate the sample cross-covariances and one FFT of length  $L$  to calculate the sample spectrum. The timing of nag\_tsa\_spectrum\_bivar\_cov (g13ccc) is therefore dependent on the choice of these values. The time taken for an FFT of length  $n$  is approximately proportional to  $n\log(n)$  (but see Section 9 in nag\_sum\_fft\_realherm\_1d (c06pac) for further details).

## 10 Example

This example reads two time series of length 296. It then selects mean correction, a 10% tapering proportion, the Parzen smoothing window and a cut-off point of 35 for the lag window. The alignment shift is set to 3 and 50 cross-covariances are chosen to be calculated. The program then calls nag\_tsa\_spectrum\_bivar\_cov (g13ccc) to calculate the cross spectrum and then prints the cross-covariances and cross spectrum.

### 10.1 Program Text

```

/* nag_tsa_spectrum_bivar_cov (g13ccc) Example Program.
 *
 * Copyright 2002 Numerical Algorithms Group.
 *
 * Mark 7, 2002.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg13.h>

int main(void)
{
    /* Scalars */
    double      pxy;
    Integer      exit_status, i, ic, ii, ish, iw, kc, lf,
                mw, nc, ng, nxy, nxyg;

    /* Arrays */
    double      *cxy = 0, *cyx = 0, *xg = 0, *yg = 0;
    Complex     *g = 0;

    NagMeanOrTrend mtxy;

    NagError      fail;

    INIT_FAIL(fail);

    exit_status = 0;

    printf(
        "nag_tsa_spectrum_bivar_cov (g13ccc) Example Program Results\n");

    /* Skip heading in data file */
    scanf("%*[\n] ");
    scanf("%ld%ld%ld%*[\n] ", &nxy, &nc, &ic);

    if (nxy > 0 && nc > 0)
    {
        /* Set parameters for call to nag_tsa_spectrum_bivar_cov (g13ccc) */
        /* Mean correction and 10 percent taper */
        mtxy = Nag_Mean;
        pxy = 0.1;

        /* Parzen window and zero covariance at lag 35 */
        iw = 4;
        mw = 35;
    }
}

```

```

/* Alignment shift of 3, 50 covariances to be calculated */
ish = 3;
kc = 350;
lf = 80;

if (ic == 0)
    nxyg = MAX(kc, lf);
else
    nxyg = lf;

/* Allocate arrays xg, yg, cxy and cyx */
if (!(xg = NAG_ALLOC(nxyg, double)) ||
    !(yg = NAG_ALLOC(nxyg, double)) ||
    !(cxy = NAG_ALLOC(nc, double)) ||
    !(cyx = NAG_ALLOC(nc, double)) ||
    !(g = NAG_ALLOC((lf/2)+1, Complex)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

if (ic == 0)
{
    for (i = 1; i <= nxy; ++i)
        scanf("%lf", &xg[i-1]);
    scanf("%*[\n] ");
    for (i = 1; i <= nxy; ++i)
        scanf("%lf", &yg[i-1]);
    scanf("%*[\n] ");
}
else
{
    for (i = 1; i <= nc; ++i)
        scanf("%lf", &cxy[i-1]);
    scanf("%*[\n] ");
    for (i = 1; i <= nc; ++i)
        scanf("%lf", &cyx[i-1]);
    scanf("%*[\n] ");
}

/* nag_tsa_spectrum_bivar_cov (g13ccc).
 * Multivariate time series, smoothed sample cross spectrum
 * using rectangular, Bartlett, Tukey or Parzen lag window
 */
nag_tsa_spectrum_bivar_cov(nxy, mtxy, pxy, iw, mw, ish, ic, nc, cxy, cyx,
                          kc, lf, xg, yg, g, &ng, &fail);
if (fail.code != NE_NOERROR)
{
    printf(
        "Error from nag_tsa_spectrum_bivar_cov (g13ccc).\n%s\n",
        fail.message);
    exit_status = 1;
    goto END;
}

printf("\n");
printf("                                Returned cross covariances\n");
printf("\n");
printf(" Lag      XY      YX      Lag      XY      YX      Lag"
       "\n      XY      YX\n");
for (i = 1; i <= nc; i += 3)
{
    for (ii = i; ii <= MIN(i+2, nc); ++ii)
        printf("%4ld%9.4f%9.4f ", ii-1, cxy[ii-1],
              cyx[ii-1]);
    printf("\n");
}

printf("\n");

```

```

printf("                                Returned sample spectrum\n");
printf("\n");
printf("          Real   Imaginary          Real   Imaginary  "
       "          Real   Imaginary\n");
printf(" Lag    part    part    Lag    part    part    Lag"
       "          part    part\n");
for (i = 1; i <= ng; i += 3)
{
  for (ii = i; ii <= MIN(i+2, ng); ++ii)
    printf("%4ld%9.4f%9.4f ", ii-1, g[ii-1].re,
          g[ii-1].im);
  printf("\n");
}
}

END:
NAG_FREE(cxy);
NAG_FREE(cyx);
NAG_FREE(xg);
NAG_FREE(yg);
NAG_FREE(g);

return exit_status;
}

```

## 10.2 Program Data

nag\_tsa\_spectrum\_bivar\_cov (g13ccc) Example Program Data

```

296 50 0
-0.109 0.000 0.178 0.339 0.373 0.441 0.461 0.348
0.127 -0.180 -0.588 -1.055 -1.421 -1.520 -1.302 -0.814
-0.475 -0.193 0.088 0.435 0.771 0.866 0.875 0.891
0.987 1.263 1.775 1.976 1.934 1.866 1.832 1.767
1.608 1.265 0.790 0.360 0.115 0.088 0.331 0.645
0.960 1.409 2.670 2.834 2.812 2.483 1.929 1.485
1.214 1.239 1.608 1.905 2.023 1.815 0.535 0.122
0.009 0.164 0.671 1.019 1.146 1.155 1.112 1.121
1.223 1.257 1.157 0.913 0.620 0.255 -0.280 -1.080
-1.551 -1.799 -1.825 -1.456 -0.944 -0.570 -0.431 -0.577
-0.960 -1.616 -1.875 -1.891 -1.746 -1.474 -1.201 -0.927
-0.524 0.040 0.788 0.943 0.930 1.006 1.137 1.198
1.054 0.595 -0.080 -0.314 -0.288 -0.153 -0.109 -0.187
-0.255 -0.299 -0.007 0.254 0.330 0.102 -0.423 -1.139
-2.275 -2.594 -2.716 -2.510 -1.790 -1.346 -1.081 -0.910
-0.876 -0.885 -0.800 -0.544 -0.416 -0.271 0.000 0.403
0.841 1.285 1.607 1.746 1.683 1.485 0.993 0.648
0.577 0.577 0.632 0.747 0.999 0.993 0.968 0.790
0.399 -0.161 -0.553 -0.603 -0.424 -0.194 -0.049 0.060
0.161 0.301 0.517 0.566 0.560 0.573 0.592 0.671
0.933 1.337 1.460 1.353 0.772 0.218 -0.237 -0.714
-1.099 -1.269 -1.175 -0.676 0.033 0.556 0.643 0.484
0.109 -0.310 -0.697 -1.047 -1.218 -1.183 -0.873 -0.336
0.063 0.084 0.000 0.001 0.209 0.556 0.782 0.858
0.918 0.862 0.416 -0.336 -0.959 -1.813 -2.378 -2.499
-2.473 -2.330 -2.053 -1.739 -1.261 -0.569 -0.137 -0.024
-0.050 -0.135 -0.276 -0.534 -0.871 -1.243 -1.439 -1.422
-1.175 -0.813 -0.634 -0.582 -0.625 -0.713 -0.848 -1.039
-1.346 -1.628 -1.619 -1.149 -0.488 -0.160 -0.007 -0.092
-0.620 -1.086 -1.525 -1.858 -2.029 -2.024 -1.961 -1.952
-1.794 -1.302 -1.030 -0.918 -0.798 -0.867 -1.047 -1.123
-0.876 -0.395 0.185 0.662 0.709 0.605 0.501 0.603
0.943 1.223 1.249 0.824 0.102 0.025 0.382 0.922
1.032 0.866 0.527 0.093 -0.458 -0.748 -0.947 -1.029
-0.928 -0.645 -0.424 -0.276 -0.158 -0.033 0.102 0.251
0.280 0.000 -0.493 -0.759 -0.824 -0.740 -0.528 -0.204
0.034 0.204 0.253 0.195 0.131 0.017 -0.182 -0.262
53.8 53.6 53.5 53.5 53.4 53.1 52.7 52.4 52.2 52.0 52.4 53.0 54.0 54.9 56.0
56.8 56.8 56.4 55.7 55.0 54.3 53.2 52.3 51.6 51.2 50.8 50.5 50.0 49.2 48.4 47.9
47.6 47.5 47.5 47.6 48.1 49.0 50.0 51.1 51.8 51.9 51.7 51.2 50.0 48.3 47.0 45.8
45.6 46.0 46.9 47.8 48.2 48.3 47.9 47.2 47.2 48.1 49.4 50.6 51.5 51.6 51.2 50.5

```



```

50.1 49.8 49.6 49.4 49.3 49.2 49.3 49.7 50.3 51.3 52.8 54.4 56.0 56.9 57.5 57.3
56.6 56.0 55.4 55.4 56.4 57.2 58.0 58.4 58.4 58.1 57.7 57.0 56.0 54.7 53.2 52.1
51.6 51.0 50.5 50.4 51.0 51.8 52.4 53.0 53.4 53.6 53.7 53.8 53.8 53.8 53.3 53.0
52.9 53.4 54.6 56.4 58.0 59.4 60.2 60.0 59.4 58.4 57.6 56.9 56.4 56.0 55.7 55.3
55.0 54.4 53.7 52.8 51.6 50.6 49.4 48.8 48.5 48.7 49.2 49.8 50.4 50.7 50.9 50.7
50.5 50.4 50.2 50.4 51.2 52.3 53.2 53.9 54.1 54.0 53.6 53.2 53.0 52.8 52.3 51.9
51.6 51.6 51.4 51.2 50.7 50.0 49.4 49.3 49.7 50.6 51.8 53.0 54.0 55.3 55.9 55.9
54.6 53.5 52.4 52.1 52.3 53.0 53.8 54.6 55.4 55.9 55.9 55.2 54.4 53.7 53.6 53.6
53.2 52.5 52.0 51.4 51.0 50.9 52.4 53.5 55.6 58.0 59.5 60.0 60.4 60.5 60.2 59.7
59.0 57.6 56.4 55.2 54.5 54.1 54.1 54.4 55.5 56.2 57.0 57.3 57.4 57.0 56.4 55.9
55.5 55.3 55.2 55.4 56.0 56.5 57.1 57.3 56.8 55.6 55.0 54.1 54.3 55.3 56.4 57.2
57.8 58.3 58.6 58.8 58.8 58.6 58.0 57.4 57.0 56.4 56.3 56.4 56.4 56.0 55.2 54.0
53.0 52.0 51.6 51.6 51.1 50.4 50.0 50.0 52.0 54.0 55.1 54.5 52.8 51.4 50.8 51.2
52.0 52.8 53.8 54.5 54.9 54.9 54.8 54.4 53.7 53.3 52.8 52.6 52.6 53.0 54.3 56.0
57.0 58.0 58.6 58.5 58.3 57.8 57.3 57.0

```

### 10.3 Program Results

nag\_tsa\_spectrum\_bivar\_cov (g13ccc) Example Program Results

Returned cross covariances

Lag	XY	YX	Lag	XY	YX	Lag	XY	YX
0	-1.6700	-1.6700	1	-2.0581	-1.3606	2	-2.4859	-1.1383
3	-2.8793	-0.9926	4	-3.1473	-0.9009	5	-3.2239	-0.8382
6	-3.0929	-0.7804	7	-2.7974	-0.7074	8	-2.4145	-0.6147
9	-2.0237	-0.5080	10	-1.6802	-0.4032	11	-1.4065	-0.3159
12	-1.2049	-0.2554	13	-1.0655	-0.2250	14	-0.9726	-0.2238
15	-0.9117	-0.2454	16	-0.8658	-0.2784	17	-0.8180	-0.3081
18	-0.7563	-0.3257	19	-0.6750	-0.3315	20	-0.5754	-0.3321
21	-0.4701	-0.3308	22	-0.3738	-0.3312	23	-0.3023	-0.3332
24	-0.2665	-0.3384	25	-0.2645	-0.3506	26	-0.2847	-0.3727
27	-0.3103	-0.3992	28	-0.3263	-0.4152	29	-0.3271	-0.4044
30	-0.3119	-0.3621	31	-0.2837	-0.2919	32	-0.2568	-0.2054
33	-0.2427	-0.1185	34	-0.2490	-0.0414	35	-0.2774	0.0227
36	-0.3218	0.0697	37	-0.3705	0.1039	38	-0.4083	0.1356
39	-0.4197	0.1805	40	-0.3920	0.2460	41	-0.3241	0.3319
42	-0.2273	0.4325	43	-0.1216	0.5331	44	-0.0245	0.6199
45	0.0528	0.6875	46	0.1074	0.7329	47	0.1448	0.7550
48	0.1713	0.7544	49	0.1943	0.7349			

Returned sample spectrum

Lag	Real part	Imaginary part	Lag	Real part	Imaginary part	Lag	Real part	Imaginary part
0	-6.5500	0.0000	1	-5.4267	-1.9842	2	-3.1323	-2.7307
3	-1.2649	-2.3998	4	-0.2102	-1.7520	5	0.3411	-1.1903
6	0.6063	-0.7420	7	0.6178	-0.3586	8	0.4391	-0.1008
9	0.2422	0.0061	10	0.1233	0.0409	11	0.0574	0.0529
12	0.0174	0.0452	13	-0.0008	0.0289	14	-0.0058	0.0161
15	-0.0051	0.0084	16	-0.0027	0.0040	17	-0.0010	0.0015
18	-0.0006	0.0006	19	-0.0005	0.0003	20	-0.0003	0.0003
21	-0.0003	0.0004	22	-0.0003	0.0003	23	-0.0003	0.0002
24	-0.0004	0.0001	25	-0.0004	-0.0000	26	-0.0003	-0.0001
27	-0.0002	-0.0001	28	-0.0001	0.0001	29	-0.0002	0.0003
30	-0.0003	0.0002	31	-0.0002	0.0001	32	-0.0001	0.0000
33	-0.0000	-0.0000	34	0.0001	-0.0001	35	0.0001	-0.0002
36	0.0001	-0.0001	37	0.0001	-0.0001	38	0.0001	-0.0001
39	0.0001	-0.0001	40	0.0001	0.0000			