NAG Library Function Document

nag_prod_limit_surviv_fn (g12aac)

1 Purpose

nag_prod_limit_surviv_fn (g12aac) computes the Kaplan–Meier, (or product-limit), estimates of survival probabilities for a sample of failure times.

2 Specification

```
#include <nag.h>
#include <nagg12.h>
void nag_prod_limit_surviv_fn (Integer n, const double t[],
     const Integer ic[], const Integer freq[], Integer *nd, double tp[],
     double p[], double psig[], NagError *fail)
```

3 Description

A survivor function, $S(t)$, is the probability of surviving to at least time t with $S(t) = 1 - F(t)$, where $F(t)$ is the cumulative distribution function of the failure times. The Kaplan–Meier or product limit estimator provides an estimate of $S(t)$, $\hat{S}(t)$, from sample of failure times which may be progressively right-censored.

Let t_i, $i = 1, 2, \ldots, n_d$, be the ordered distinct failure times for the sample of observed failure/censored times, and let the number of observations in the sample that have not failed by time t_i be n_i. If a failure and a loss (censored observation) occur at the same time t_i, then the failure is treated as if it had occurred slightly before time t_i and the loss as if it had occurred slightly after t_i.

The Kaplan–Meier estimate of the survival probabilities is a step function which in the interval t_i to t_{i+1} is given by

$$\hat{S}(t) = \prod_{j=1}^{i} \left(\frac{n_j - d_j}{n_j} \right)$$

where d_j is the number of failures occurring at time t_j.

nag_prod_limit_surviv_fn (g12aac) computes the Kaplan–Meier estimates and the corresponding estimates of the variances, $\hat{\text{var}}\left(\hat{S}(t)\right)$, using Greenwood's formula,

$$\hat{\text{var}}\left(\hat{S}(t)\right) = \hat{S}(t)^2 \sum_{j=1}^{i} \frac{d_j}{n_j(n_j - d_j)}.$$

4 References

Gross A J and Clark V A (1975) *Survival Distributions: Reliability Applications in the Biomedical Sciences* Wiley

Kalbfleisch J D and Prentice R L (1980) *The Statistical Analysis of Failure Time Data* Wiley

5 Arguments

1: **n** – Integer *Input*

On entry: the number of failure and censored times given in **t**.

Constraint: $\mathbf{n} \geq 2$.

2: **t**[**n**] – const double *Input*

On entry: the failure and censored times; these need not be ordered.

3: **ic**[**n**] – const Integer *Input*

On entry: **ic**$[i-1]$ contains the censoring code of the ith observation, for $i = 1, 2, \ldots, \mathbf{n}$.

ic$[i-1] = 0$
The ith observation is a failure time.

ic$[i-1] = 1$
The ith observation is right-censored.

Constraint: **ic**$[i-1] = 0$ or 1, for $i = 1, 2, \ldots, \mathbf{n}$.

4: **freq**[**n**] – const Integer *Input*

On entry: indicates whether frequencies are provided for each failure and censored time point. If frequencies are provided then **freq** must be dimensioned at least **n**. If the failure and censored times are to be considered as single observations, i.e., a frequency of 1 is to be assumed then **freq** must be set to **NULL**.

Constraint: either **freq** = (Integer$*$)0 or **freq**$[i-1] \geq 0$, for $i = 1, 2, \ldots, \mathbf{n}$.

5: **nd** – Integer * *Output*

On exit: the number of distinct failure times, n_d.

6: **tp**[**n**] – double *Output*

On exit: **tp**$[i-1]$ contains the ith ordered distinct failure time, t_i, for $i = 1, 2, \ldots, n_d$.

7: **p**[**n**] – double *Output*

On exit: **p**$[i-1]$ contains the Kaplan–Meier estimate of the survival probability, $\hat{S}(t)$, for time **tp**$[i-1]$, for $i = 1, 2, \ldots, n_d$.

8: **psig**[**n**] – double *Output*

On exit: **psig**$[i-1]$ contains an estimate of the standard deviation of **p**$[i-1]$, for $i = 1, 2, \ldots, n_d$.

9: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_INT_ARG_LT

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 2.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_INVALID_CENSOR_CODE

On entry, **ic**[⟨*value*⟩] = ⟨*value*⟩. The censor code for an observation must be either 0 or 1.

NE_INVALID_FREQ

On entry, **freq**[$\langle value \rangle$] = $\langle value \rangle$. The value of frequency for an observation must be ≥ 0.

7 Accuracy

The computations are believed to be stable.

8 Parallelism and Performance

Not applicable.

9 Further Comments

If there are no censored observations, $\hat{S}(t)$, reduces to the ordinary binomial estimate of the probability of survival at time t.

10 Example

The remission times for a set of 21 leukaemia patients at 18 distinct time points are read in and the Kaplan–Meier estimate computed and printed. For further details see page 242 of Gross and Clark (1975).

10.1 Program Text

```
/* nag_prod_limit_surviv_fn (g12aac) Example Program.
 *
 * Copyright 1996 Numerical Algorithms Group.
 *
 * Mark 4, 1996.
 * Mark 8 revised, 2004.
 *
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg12.h>

int main(void)
{

  Integer  exit_status = 0, i, *ic = 0, *ifreq = 0, n, nd;
  NagError fail;
  double   *p = 0, *psig = 0, *t = 0, *tp = 0;

  INIT_FAIL(fail);

  printf("nag_prod_limit_surviv_fn (g12aac) Example Program Results\n");

  /* Skip heading in data file */
  scanf("%*[^\n] ");
  scanf("%ld ", &n);
  if (n >= 2)
    {
      if (!(psig = NAG_ALLOC(n, double)) ||
          !(p = NAG_ALLOC(n, double)) ||
          !(t = NAG_ALLOC(n, double)) ||
          !(tp = NAG_ALLOC(n, double)) ||
          !(ifreq = NAG_ALLOC(n, Integer)) ||
          !(ic = NAG_ALLOC(n, Integer)))
        {
          printf("Allocation failure\n");
          exit_status = -1;
          goto END;
```

```
        }
    }
  else
    {
      printf("Invalid n.\n");
      exit_status = 1;
      return exit_status;
    }

  for (i = 0; i < n; ++i)
    scanf("%lf %ld %ld ", &t[i], &ic[i], &ifreq[i]);

  /* nag_prod_limit_surviv_fn (g12aac).
   * Computes Kaplan-Meier (product-limit) estimates of
   * survival probabilities
   */
  nag_prod_limit_surviv_fn(n, t, ic, ifreq, &nd, tp, p, psig, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_prod_limit_surviv_fn (g12aac).\n%s\n",
              fail.message);
      exit_status = 1;
      goto END;
    }

  printf("\n   Time   Survival    Standard\n");
  printf("          probability  deviation\n\n");
  for (i = 0; i < nd; ++i)
    printf(" %6.1f%10.3f  %10.3f\n", tp[i], p[i], psig[i]);

 END:
  NAG_FREE(psig);
  NAG_FREE(p);
  NAG_FREE(t);
  NAG_FREE(tp);
  NAG_FREE(ifreq);
  NAG_FREE(ic);
  return exit_status;
}
```

10.2 Program Data

```
nag_prod_limit_surviv_fn (g12aac) Example Program Data
18
6.0 1 1 6.0 0 3 7.0 0 1 9.0 1 1 10.0 0 1 10.0 1 1
11.0 1 1 13.0 0 1 16.0 0 1 17.0 1 1 19.0 1 1 20.0 1 1
22.0 0 1 23.0 0 1 25.0 1 1 32.0 1 2 34.0 1 1 35.0 1 1
```

10.3 Program Results

```
nag_prod_limit_surviv_fn (g12aac) Example Program Results

   Time   Survival    Standard
          probability  deviation

    6.0     0.857       0.076
    7.0     0.807       0.087
   10.0     0.753       0.096
   13.0     0.690       0.107
   16.0     0.627       0.114
   22.0     0.538       0.128
   23.0     0.448       0.135
```