

## NAG Library Function Document

### nag\_mann\_whitney (g08amc)

#### 1 Purpose

nag\_mann\_whitney (g08amc) performs the Mann–Whitney  $U$  test on two independent samples of possibly unequal size and calculates the exact probability for the Mann–Whitney rank sum test statistic for the case where there are either ties or no ties in the samples pooled together.

#### 2 Specification

```
#include <nag.h>
#include <nagg08.h>

void nag_mann_whitney (Integer n1, const double x[], Integer n2,
    const double y[], Nag_TailProbability tail, Nag_CompProb exact,
    double *u, double *z, double *p, NagError *fail)
```

#### 3 Description

The Mann–Whitney  $U$  test investigates the difference between two populations defined by the distribution functions  $F(x)$  and  $G(y)$  respectively. The data consist of two independent samples of size  $n_1$  and  $n_2$ , denoted by  $x_1, x_2, \dots, x_{n_1}$  and  $y_1, y_2, \dots, y_{n_2}$ , taken from the two populations.

The hypothesis under test,  $H_0$ , often called the null hypothesis, is that the two distributions are the same, that is  $F(x) = G(x)$ , and this is to be tested against an alternative hypothesis  $H_1$  which is

$H_1 : F(x) \neq G(y)$ ; or

$H_1 : F(x) < G(y)$ , i.e., the  $x$ 's tend to be greater than the  $y$ 's; or

$H_1 : F(x) > G(y)$ , i.e., the  $x$ 's tend to be less than the  $y$ 's,

using a two tailed, upper tailed or lower tailed probability respectively. You select the alternative hypothesis by choosing the appropriate tail probability to be computed (see the description of argument **tail** in Section 5).

Note that when using this test to test for differences in the distributions one is primarily detecting differences in the location of the two distributions. That is to say, if we reject the null hypothesis  $H_0$  in favour of the alternative hypothesis  $H_1: F(x) > G(y)$  we have evidence to suggest that the location, of the distribution defined by  $F(x)$ , is less than the location, of the distribution defined by  $G(y)$ .

The Mann–Whitney  $U$  test differs from the Median test (see nag\_median\_test (g08acc)) in that the ranking of the individual scores within the pooled sample is taken into account, rather than simply the position of a score relative to the median of the pooled sample. It is therefore a more powerful test if score differences are meaningful.

The test procedure involves ranking the pooled sample, average ranks being used for ties. Let  $r_{1i}$  be the rank assigned to  $x_i$ , for  $i = 1, 2, \dots, n_1$  and  $r_{2j}$  the rank assigned to  $y_j$ , for  $j = 1, 2, \dots, n_2$ . Then the test statistic  $U$  is defined as follows;

$$U = \sum_{i=1}^{n_1} r_{1i} - \frac{n_1(n_1 + 1)}{2}$$

$U$  is also the number of times a score in the second sample precedes a score in the first sample (where we only count a half if a score in the second sample actually equals a score in the first sample).

nag\_mann\_whitney (g08amc) returns:

- (a) the test statistic  $U$ ;  
 (b) the approximate Normal test statistic,

$$z = \frac{U - \text{mean}(U) \pm \frac{1}{2}}{\sqrt{\text{var}(U)}}$$

where

$$\text{mean}(U) = \frac{n_1 n_2}{2}$$

and

$$\text{var}(U) = \frac{n_1 n_2 (n_1 + n_2 + 1)}{12} - \frac{n_1 n_2}{(n_1 + n_2)(n_1 + n_2 - 1)} \times TS$$

where

$$TS = \sum_{j=1}^{\tau} \frac{(t_j)(t_j - 1)(t_j + 1)}{12}$$

$\tau$  is the number of groups of ties in the sample and  $t_j$  is the number of ties in the  $j$ th group.

Note that if no ties are present the variance of  $U$  reduces to  $\frac{n_1 n_2}{12}(n_1 + n_2 + 1)$ .

- (c) An indicator as to whether ties were present in the pooled sample or not.  
 (d) The tail probability,  $p$ , corresponding to  $U$ , depending on the choice of **tail**, i.e., the choice of alternative hypothesis,  $H_1$ . An exact probability or a normal approximation may be selected using the **exact** option. For large values of  $n_1$  and  $n_2$  the normal approximation should be adequate while the exact computation may require extensive calculation, particularly in cases where there are ties in the observations. For small samples the exact probability should be used.

The value of  $p$  can be used to perform a significance test on the null hypothesis  $H_0$  against the alternative hypothesis  $H_1$ . Let  $\alpha$  be the size of the significance test (that is,  $\alpha$  is the probability of rejecting  $H_0$  when  $H_0$  is true). If  $p < \alpha$  then the null hypothesis is rejected. Typically  $\alpha$  might be 0.05 or 0.01.

## 4 References

Conover W J (1980) *Practical Nonparametric Statistics* Wiley

Neumann N (1988) Some procedures for calculating the distributions of elementary nonparametric test statistics *Statistical Software Newsletter* **14(3)** 120–126

Siegel S (1956) *Non-parametric Statistics for the Behavioral Sciences* McGraw–Hill

## 5 Arguments

- 1: **n1** – Integer *Input*  
*On entry:* the number of non-tied pairs,  $n_1$ .  
*Constraint:* **n1**  $\geq$  1.
- 2: **x[n1]** – const double *Input*  
*On entry:* the first vector of observations.  $x_1, x_2, \dots, x_{n_1}$ .
- 3: **n2** – Integer *Input*  
*On entry:* the size of the second sample,  $n_2$ .  
*Constraint:* **n2**  $\geq$  1.

- 4: **y[n2]** – const double *Input*  
*On entry:* the second vector of observations.  $y_1, y_2, \dots, y_{n_2}$ .
- 5: **tail** – Nag\_TailProbability *Input*  
*On entry:* indicates the choice of tail probability, and hence the alternative hypothesis.  
**tail** = Nag\_TwoTail  
 A two tailed probability is calculated and the alternative hypothesis is  $H_1 : F(x) \neq G(y)$ .  
**tail** = Nag\_UpperTail  
 An upper tailed probability is calculated and the alternative hypothesis  $H_1 : F(x) < G(y)$ ,  
 i.e., the  $x$ 's tend to be greater than the  $y$ 's.  
**tail** = Nag\_LowerTail  
 A lower tailed probability is calculated and the alternative hypothesis  $H_1 : F(x) > G(y)$ ,  
 i.e., the  $x$ 's tend to be less than the  $y$ 's.  
*Constraint:* **tail** = Nag\_TwoTail, Nag\_UpperTail or Nag\_LowerTail.
- 6: **exact** – Nag\_CompProb *Input*  
*On entry:* indicates if exact probability,  $p$ , is to be computed.  
**exact** = Nag\_CompProbExact  
 Compute the exact probability.  
**exact** = Nag\_CompProbApprox  
 Compute the approximate probability.  
*Constraint:* **exact** = Nag\_CompProbExact or Nag\_CompProbApprox.
- 7: **u** – double \* *Output*  
*On exit:* the Mann–Whitney rank sum statistic,  $U$ .
- 8: **z** – double \* *Output*  
*On exit:* the approximate Normal test statistic,  $z$ , as described in Section 3.
- 9: **p** – double \* *Output*  
*On exit:* the exact tail probability,  $p$ , as specified by the argument **tail**.
- 10: **fail** – NagError \* *Input/Output*  
 The NAG error argument (see Section 3.6 in the Essential Introduction).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

### NE\_BAD\_PARAM

On entry, argument **exact** had an illegal value.

On entry, argument **tail** had an illegal value.

### NE\_G08AH\_SAMP\_IDEN

The pooled samples are all the same, that is the variance of  $U = 0.0$ .

**NE\_INT\_ARG\_LT**

On entry, **n1** must not be less than 1: **n1** =  $\langle value \rangle$ .

On entry, **n2** must not be less than 1: **n2** =  $\langle value \rangle$ .

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

**NE\_MANN\_WHITNEY\_STAT**

The computed Mann–Whitney statistic,  $U = \langle value \rangle$ . If the exact tail probability is to be computed, then  $U \geq 0$ .

**7 Accuracy**

The approximate tail probability,  $p$ , returned by `nag_mann_whitney` (g08amc) is a good approximation to the exact probability for cases where  $\max(n_1, n_2) \geq 30$  and  $(n_1 + n_2) \geq 40$ . The relative error of the approximation should be less than 10 percent, for most cases falling in this range.

**8 Parallelism and Performance**

Not applicable.

**9 Further Comments**

The time taken by `nag_mann_whitney` (g08amc) increases with  $n_1$  and  $n_2$ .

**10 Example**

The example program performs the Mann–Whitney test on two independent samples of sizes 16 and 23 respectively. This is used to test the null hypothesis that the distributions of the two populations from which the samples were taken are the same against the alternative hypothesis that the distributions are different. The test statistic, the approximate Normal statistic and the approximate two-tail probability are printed. An exact tail probability is also calculated and printed depending on whether ties were found in the pooled sample or not.

**10.1 Program Text**

```

/* nag_mann_whitney (g08amc) Example Program.
 *
 * Copyright 2000 Numerical Algorithms Group.
 *
 * Mark 6a revised, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg08.h>

int main(void)
{
    Integer  exit_status = 0, i, n1, n2;
    NagError fail;
    double   p, u, *x = 0, *y = 0, z;

    INIT_FAIL(fail);

    printf("nag_mann_whitney (g08amc) Example Program Results\n\n");

```

```

/* Skip heading in data file */
scanf("%*[^\\n]");
scanf("%ld %ld ", &n1, &n2);
printf("%s%5ld\\n", "Sample size of group 1 = ", n1);
printf("%s%5ld\\n", "Sample size of group 2 = ", n2);
if (!(x = NAG_ALLOC(n1, double))
    || !(y = NAG_ALLOC(n2, double)))
{
    printf("Allocation failure\\n");
    exit_status = -1;
    goto END;
}
printf("\\n");
for (i = 1; i <= n1; ++i)
    scanf("%lf", &x[i - 1]);
printf("%s\\n", "Mann-Whitney U test");
printf("\\n");
printf("%s\\n", "Data values");
printf("\\n");
printf("%s", "    Group 1 ");
for (i = 1; i <= n1; ++i)
    printf("%5.1f%s", x[i - 1], i%8?" ":"\\n");
for (i = 1; i <= n2; ++i)
    scanf("%lf", &y[i - 1]);
printf("\\n");
printf("%s", "    Group 2 ");
for (i = 1; i <= n2; ++i)
    printf("%5.1f%s", y[i - 1], i%8?" ":"\\n");

/* nag_mann_whitney (g08amc).
 * Performs the Mann-Whitney U test on two independent
 * samples
 */
nag_mann_whitney(n1, x, n2, y, Nag_LowerTail, Nag_CompProbApprox,
                &u, &z, &p, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_mann_whitney (g08amc).\\n%s\\n",
          fail.message);
    exit_status = 1;
    goto END;
}
printf("\\n\\n");
printf("%s%8.4f\\n", "Test statistic           = ", u);
printf("%s%8.4f\\n", "Normal Statistic        = ", z);
printf("%s%8.4f\\n", "Approximate tail probability = ", p);
/* nag_mann_whitney (g08amc), see above. */
nag_mann_whitney(n1, x, n2, y, Nag_LowerTail, Nag_CompProbExact,
                &u, &z, &p, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_mann_whitney (g08amc).\\n%s\\n",
          fail.message);
    exit_status = 1;
    goto END;
}
printf("%s%8.4f\\n", "Exact tail probability       = ", p);
END:
NAG_FREE(x);
NAG_FREE(y);
return exit_status;
}

```

## 10.2 Program Data

```
nag_mann_whitney (g08amc) Example Program Data
16 23
13.0 6.0 12.0 7.0 12.0 7.0 10.0 7.0
10.0 7.0 16.0 7.0 10.0 8.0 9.0 8.0
17.0 6.0 10.0 8.0 15.0 8.0 15.0 10.0 15.0 10.0 14.0 10.0
14.0 11.0 14.0 11.0 13.0 12.0 13.0 12.0 13.0 12.0 12.0
```

## 10.3 Program Results

```
nag_mann_whitney (g08amc) Example Program Results
```

```
Sample size of group 1 = 16
Sample size of group 2 = 23
```

```
Mann-Whitney U test
```

```
Data values
```

```
Group 1 13.0 6.0 12.0 7.0 12.0 7.0 10.0 7.0
        10.0 7.0 16.0 7.0 10.0 8.0 9.0 8.0

Group 2 17.0 6.0 10.0 8.0 15.0 8.0 15.0 10.0
        15.0 10.0 14.0 10.0 14.0 11.0 14.0 11.0
        13.0 12.0 13.0 12.0 13.0 12.0 12.0
```

```
Test statistic           = 86.0000
Normal Statistic        = -2.8039
Approximate tail probability = 0.0025
Exact tail probability   = 0.0020
```

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