NAG Library Function Document

nag_friedman_test (g08aec)

1 Purpose

nag_friedman_test (g08aec) performs the Friedman two-way analysis of variance by ranks on k related samples of size n.

2 Specification

```
#include <nag.h>
#include <nagg08.h>
```

```
void nag_friedman_test (Integer k, Integer n, const double x[], Integer tdx,
     double *fr, double *p, NagError *fail)
```

3 Description

The Friedman test investigates the score differences between k matched samples of size n, the scores in the ith sample being denoted by:

$$x_{i1}, x_{i2}, \ldots, x_{in}.$$

(Thus the sample scores may be regarded as a two-way table with k rows and n columns.) The hypothesis under test, H_0, often called the null hypothesis, is that the samples come from the same population, and this is to be tested against the alternative hypothesis H_1 that they come from different populations.

The test is based on the observed distribution of score rankings between the matched observations in different samples.

The test proceeds as follows:

(a) The scores in each column are ranked, r_{ij} denoting the rank within column j of the observation in row i. Average ranks are assigned to tied scores.

(b) The ranks are summed over each row to give rank sums $t_i = \sum_{j=1}^{n} r_{ij}$, for $i = 1, 2, \ldots, k$.

(c) The Friedman test statistic FR is computed, where

$$FR = \frac{12}{nk(k+1)} \sum_{i=1}^{k} \left\{ t_i - \frac{1}{2} n(k+1) \right\}^2.$$

nag_friedman_test (g08aec) returns the value of FR, and also an approximation, p, to the significance of this value. (FR approximately follows a χ^2_{k-1} distribution, so large values of FR imply rejection of H_0). H_0 is rejected by a test of chosen size α if $p < \alpha$. The approximation p is acceptable unless $k = 4$ and $n < 5$, or $k = 3$ and $n < 10$, or $k = 2$ and $n < 20$; for $k = 3$ or 4, tables should be consulted (e.g., **n** of Siegel (1956)); for $k = 2$ the Sign test (see nag_sign_test (g08aac)) or Wilcoxon test (see nag_wilcoxon_test (g08agc)) is in any case more appropriate.

4 References

Siegel S (1956) *Non-parametric Statistics for the Behavioral Sciences* McGraw–Hill

5 Arguments

1: **k** – Integer *Input*

On entry: k, the number of samples.

Constraint: **k** ≥ 2.

2: **n** – Integer *Input*

On entry: the size of each sample, n.

Constraint: **n** ≥ 1.

3: **x**[**k** × **tdx**] – const double *Input*

On entry: **x**$[(i-1) \times$ **tdx** $+ j - 1]$ must be set to the value, x_{ij}, of observation j in sample i, for $i = 1, 2, \ldots, k$ and $j = 1, 2, \ldots, n$.

4: **tdx** – Integer *Input*

On entry: the stride separating matrix column elements in the array **x**.

Constraint: **tdx** $\geq$ **n**.

5: **fr** – double * *Output*

On exit: the value of the Friedman test statistic, FR.

6: **p** – double * *Output*

On exit: the approximate significance, p, of the Friedman test statistic.

7: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_2_INT_ARG_LT

On entry, **tdx** $= \langle value \rangle$ while **n** $= \langle value \rangle$. These arguments must satisfy **tdx** $\geq$ **n**.

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_INT_ARG_LE

On entry, **k** $= \langle value \rangle$.
Constraint: **k** ≥ 2.

NE_INT_ARG_LT

On entry, **n** $= \langle value \rangle$.
Constraint: **n** ≥ 1.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

7 Accuracy

For estimates of the accuracy of the significance p, see nag_prob_chi_sq (g01ecc). The χ^2 approximation is acceptable unless $k = 4$ and $n < 5$, or $k = 3$ and $n < 10$, or $k = 2$ and $n < 20$.

8 Parallelism and Performance

Not applicable.

9 Further Comments

The time taken by nag_friedman_test (g08aec) is approximately proportional to the product nk.

If $k = 2$, the Sign test (see nag_sign_test (g08aac)) or Wilcoxon test (see nag_wilcoxon_test (g08agc)) is more appropriate.

10 Example

This example is taken from page 169 of Siegel (1956). The data relate to training scores of three matched samples of 18 rats, trained under three different patterns of reinforcement.

10.1 Program Text

```
/* nag_friedman_test (g08aec) Example Program.
 *
 * Copyright 2000 Numerical Algorithms Group.
 *
 * Mark 6, 2000.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg08.h>

int main(void)
{
  Integer  exit_status = 0, i, ix, j, k, n;
  NagError fail;
  double   fr, sig, *x = 0;

#define X(I, J) x[((I) -1)*n +(J) -1]

  INIT_FAIL(fail);

  printf("nag_friedman_test (g08aec) Example Program Results\n");

  /* Skip heading in data file */
  scanf("%*[^\n]");

  n = 18;
  k = 3;
  ix = k;
  if (!(x = NAG_ALLOC(ix*n, double)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }
  for (i = 1; i <= ix; ++i)
    for (j = 1; j <= n; ++j)
      scanf("%lf", &X(i, j));

  printf("\nFriedman test\n");
  printf("\nData values\n");
  printf("\n   Group  Group  Group\n");
  printf("     1      2      3\n");
  for (j = 1; j <= 18; ++j)
    {
      for (i = 1; i <= 3; ++i)
        printf("%7.1f", X(i, j));
      printf("\n");
    }
  /* nag_friedman_test (g08aec).
   * Friedman two-way analysis of variance on k matched
   * samples
   */
  nag_friedman_test(k, n, x, n, &fr, &sig, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_friedman_test (g08aec).\n%s\n",
              fail.message);
      exit_status = 1;
      goto END;
    }
  printf("\n");
  printf("%s%6.3f\n", "Test statistic       ", fr);
  printf("%s%6ld\n", "Degrees of freedom   ", k-1);
  printf("%s%6.3f\n", "Significance         ", sig);
 END:
  NAG_FREE(x);
  return exit_status;
}
```

10.2 Program Data

```
nag_friedman_test (g08aec) Example Program Data
  1  2  1  1  3  2  3  1  3  3  2  2  3  2  2.5  3  3  2
  3  3  3  2  1  3  2  3  1  1  3  3  2  3  2.5  2  2  3
  2  1  2  3  2  1  1  2  2  2  1  1  1  1  1  1  1  1
```

10.3 Program Results

```
nag_friedman_test (g08aec) Example Program Results

Friedman test

Data values

   Group  Group  Group
     1      2      3
    1.0    3.0    2.0
    2.0    3.0    1.0
    1.0    3.0    2.0
    1.0    2.0    3.0
    3.0    1.0    2.0
    2.0    3.0    1.0
    3.0    2.0    1.0
    1.0    3.0    2.0
    3.0    1.0    2.0
    3.0    1.0    2.0
    2.0    3.0    1.0
    2.0    3.0    1.0
    3.0    2.0    1.0
    2.0    3.0    1.0
    2.5    2.5    1.0
    3.0    2.0    1.0
    3.0    2.0    1.0
    2.0    3.0    1.0

Test statistic        8.583
Degrees of freedom        2
Significance          0.014
```