NAG Library Function Document

nag_prob_non_central_f_dist (g01gdc)

1 Purpose

nag_prob_non_central_f_dist (g01gdc) returns the probability associated with the lower tail of the noncentral F or variance-ratio distribution.

2 Specification

```
#include <nag.h>
#include <nagg01.h>
```

```
double nag_prob_non_central_f_dist (double f, double df1, double df2,
       double lambda, double tol, Integer max_iter, NagError *fail)
```

3 Description

The lower tail probability of the noncentral F-distribution with ν_1 and ν_2 degrees of freedom and noncentrality parameter λ, $P(F \leq f : \nu_1, \nu_2;\lambda)$, is defined by

$$P(F \leq f : \nu_1, \nu_2;\lambda) = \int_0^x p(F : \nu_1, \nu_2;\lambda)\, dF,$$

where

$$P(F : \nu_1, \nu_2;\lambda) = \sum_{j=0}^{\infty} e^{-\lambda/2} \frac{(\lambda/2)^j}{j!} \times \frac{(\nu_1 + 2j)^{(\nu_1+2j)/2} \nu_2^{\nu_2/2}}{B((\nu_1 + 2j)/2, \nu_2/2)}$$

$$\times u^{(\nu_1+2j-2)/2} [\nu_2 + (\nu_1 + 2j)u]^{-(\nu_1+2j+\nu_2)/2}$$

and $B(\cdot,\cdot)$ is the beta function.

The probability is computed by means of a transformation to a noncentral beta distribution:

$$P(F \leq f : \nu_1, \nu_2;\lambda) = P_\beta(X \leq x : a, b;\lambda),$$

where $x = \dfrac{\nu_1 f}{\nu_1 f + \nu_2}$ and $P_\beta(X \leq x : a, b;\lambda)$ is the lower tail probability integral of the noncentral beta distribution with parameters a, b, and λ.

If ν_2 is very large, greater than 10^6, then a χ^2 approximation is used.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Arguments

1: **f** – double *Input*

On entry: f, the deviate from the noncentral F-distribution.

Constraint: **f** > 0.0.

2: **df1** – double *Input*

On entry: the degrees of freedom of the numerator variance, ν_1.

Constraint: $0.0 < \textbf{df1} \leq 10^6$.

3: **df2** – double *Input*

On entry: the degrees of freedom of the denominator variance, ν_2.

Constraint: $\textbf{df2} > 0.0$.

4: **lambda** – double *Input*

On entry: λ, the noncentrality parameter.

Constraint: $0.0 \leq \textbf{lambda} \leq -2.0\log(U)$ where U is the safe range parameter as defined by nag_real_safe_small_number (X02AMC).

5: **tol** – double *Input*

On entry: the relative accuracy required by you in the results. If nag_prob_non_central_f_dist (g01gdc) is entered with **tol** greater than or equal to 1.0 or less than $10 \times$ ***machine precision*** (see nag_machine_precision (X02AJC)), then the value of $10 \times$ ***machine precision*** is used instead.

6: **max_iter** – Integer *Input*

On entry: the maximum number of iterations to be used.

Suggested value: 500. See nag_prob_non_central_chi_sq (g01gcc) and nag_prob_non_central_beta_dist (g01gec) for further details.

Constraint: $\textbf{max\_iter} \geq 1$.

7: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_CONV

The solution has failed to converge in $\langle value \rangle$ iterations. Consider increasing **max_iter** or **tol**.

NE_INT_ARG_LT

On entry, $\textbf{max\_iter} = \langle value \rangle$.
Constraint: $\textbf{max\_iter} \geq 1$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_PROB_F

The required probability cannot be computed accurately. This may happen if the result would be very close to zero or one. Alternatively the values of **df1** and **f** may be too large. In the latter case you could try using a normal approximation, see Abramowitz and Stegun (1972).

NE_PROB_F_INIT

The required accuracy was not achieved when calculating the initial value of the central F or χ^2 probability. You should try a larger value of **tol**. If the χ^2 approximation is being used then nag_prob_non_central_f_dist (g01gdc) returns zero otherwise the value returned should be an approximation to the correct value.

NE_REAL_ARG_CONS

On entry, **df1** $= \langle value \rangle$.
Constraint: $0.0 <$ **df1** $\leq 10^6$.

On entry, **df1** $= \langle value \rangle$.
Constraint: **df1** > 0.0.

On entry, **lambda** $= \langle value \rangle$.
Constraint: $0.0 \leq$ **lambda** $\leq -2.0 \times \log(U)$, where U is the safe range parameter as defined by nag_real_safe_small_number (X02AMC).

NE_REAL_ARG_LE

On entry, **df2** $= \langle value \rangle$.
Constraint: **df2** > 0.0.

On entry, **f** $= \langle value \rangle$.
Constraint: **f** > 0.0.

7 Accuracy

The relative accuracy should be as specified by **tol**. For further details see nag_prob_non_central_chi_sq (g01gcc) and nag_prob_non_central_beta_dist (g01gec).

8 Parallelism and Performance

Not applicable.

9 Further Comments

When both ν_1 and ν_2 are large a Normal approximation may be used and when only ν_1 is large a χ^2 approximation may be used. In both cases λ is required to be of the same order as ν_1. See Abramowitz and Stegun (1972) for further details.

10 Example

This example reads values from, and degrees of freedom for, F-distributions, computes the lower tail probabilities and prints all these values until the end of data is reached.

10.1 Program Text

```
/* nag_prob_non_central_f_dist (g01gdc) Example Program.
 *
 * Copyright 2000 Numerical Algorithms Group.
 *
 * NAG C Library
 *
 * Mark 6, 2000.
 */

#include <stdio.h>
#include <nag.h>
#include <nagg01.h>

int main(void)
```

```
{
  Integer  exit_status = 0, max_iter;
  NagError fail;
  double   df1, df2, f, lambda, prob, tol;

  INIT_FAIL(fail);

  printf(
          "nag_prob_non_central_f_dist (g01gdc) Example Program Results\n");

  /* Skip heading in data file */
  scanf("%*[^\n]");

  printf("\n     f       df1     df2     lambda    prob\n\n");
  tol = 5e-6;
  max_iter = 50;
  while ((scanf("%lf %lf %lf %lf %*[^\n]",
               &f, &df1, &df2, &lambda)) != EOF)
    {
      /* nag_prob_non_central_f_dist (g01gdc).
       * Computes probabilities for the non-central F-distribution
       */
      prob = nag_prob_non_central_f_dist(f, df1, df2, lambda, tol, max_iter,
                                          &fail);
      if (fail.code != NE_NOERROR)
        {
          printf(
                  "Error from nag_prob_non_central_f_dist (g01gdc).\n%s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }
      printf("%8.3f %8.3f %8.3f %8.3f %8.4f\n", f, df1, df2, lambda,
              prob);
    }
 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_prob_non_central_f_dist (g01gdc) Example Program Data
  5.5   1.5    25.5   3.0          :f df1 lambda
 39.9   1.0     1.0   2.0          :f df1 lambda
  2.5  20.25    1.0   0.0          :f df1 lambda
```

10.3 Program Results

```
nag_prob_non_central_f_dist (g01gdc) Example Program Results

     f       df1     df2     lambda    prob

   5.500    1.500   25.500    3.000   0.8214
  39.900    1.000    1.000    2.000   0.8160
   2.500   20.250    1.000    0.000   0.5342
```