NAG Library Function Document

nag_prob_studentized_range (g01emc)

1 Purpose

nag_prob_studentized_range (g01emc) returns the probability associated with the lower tail of the distribution of the Studentized range statistic.

2 Specification

```
#include <nag.h>
#include <nagg01.h>
double nag_prob_studentized_range (double q, double v, Integer ir,
       NagError *fail)
```

3 Description

The externally Studentized range, q, for a sample, $x_1, x_2, \ldots, x_r$, is defined as:

$$q = \frac{\max(x_i) - \min(x_i)}{\hat{\sigma}_e},$$

where $\hat{\sigma}_e$ is an independent estimate of the standard error of the x_i's. The most common use of this statistic is in the testing of means from a balanced design. In this case for a set of group means, $\bar{T}_1, \bar{T}_2, \ldots, \bar{T}_r$, the Studentized range statistic is defined to be the difference between the largest and smallest means, $\bar{T}_{\text{largest}}$ and $\bar{T}_{\text{smallest}}$, divided by the square root of the mean-square experimental error, MS_{error}, over the number of observations in each group, n, i.e.,

$$q = \frac{\bar{T}_{\text{largest}} - \bar{T}_{\text{smallest}}}{\sqrt{MS_{\text{error}}/n}}.$$

The Studentized range statistic can be used as part of a multiple comparisons procedure such as the Newman–Keuls procedure or Duncan's multiple range test (see Montgomery (1984) and Winer (1970)).

For a Studentized range statistic the probability integral, $P(q; v, r)$, for v degrees of freedom and r groups can be written as:

$$P(q; v, r) = C \int_0^\infty x^{v-1} e^{-vx^2/2} \left\{ r \int_{-\infty}^{\infty} \phi(y)[\Phi(y) - \Phi(y - qx)]^{r-1}\, dy \right\} dx,$$

where

$$C = \frac{v^{v/2}}{\Gamma(v/2)2^{v/2-1}}, \quad \phi(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \quad \text{and} \quad \Phi(y) = \int_{-\infty}^{y} \phi(t)\, dt.$$

The above two-dimensional integral is evaluated using numerical quadrature with the upper and lower limits computed to give stated accuracy (see Section 7).

If the degrees of freedom v are greater than 2000 the probability integral can be approximated by its asymptotic form:

$$P(q; r) = r \int_{-\infty}^{\infty} \phi(y)[\Phi(y) - \Phi(y - q)]^{r-1}\, dy.$$

This integral is evaluated using nag_1d_quad_inf_1 (d01smc).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Lund R E and Lund J R (1983) Algorithm AS 190: probabilities and upper quartiles for the studentized range *Appl. Statist.* **32(2)** 204–210

Montgomery D C (1984) *Design and Analysis of Experiments* Wiley

Winer B J (1970) *Statistical Principles in Experimental Design* McGraw–Hill

5 Arguments

1: **q** – double *Input*

On entry: q, the Studentized range statistic.

Constraint: **q** > 0.0.

2: **v** – double *Input*

On entry: v, the number of degrees of freedom for the experimental error.

Constraint: **v** ≥ 1.0.

3: **ir** – Integer *Input*

On entry: r, the number of groups.

Constraint: **ir** ≥ 2.

4: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

If on exit **fail.code** = NE_INT or NE_REAL, then nag_prob_studentized_range (g01emc) returns to 0.0.

6 Error Indicators and Warnings

NE_ACCURACY

Warning – There is some doubt as to whether full accuracy has been achieved.

NE_INT

On entry, **ir** $= \langle value \rangle$.
Constraint: **ir** ≥ 2.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL

On entry, **q** $= \langle value \rangle$.
Constraint: **q** > 0.0.

On entry, **v** $= \langle value \rangle$.
Constraint: **v** ≥ 1.0.

7 Accuracy

The returned value will have absolute accuracy to at least four decimal places (usually five), unless **fail**.**code** = NE_ACCURACY. When **fail**.**code** = NE_ACCURACY it is usual that the returned value will be a good estimate of the true value.

8 Parallelism and Performance

nag_prob_studentized_range (g01emc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

None.

10 Example

The lower tail probabilities for the distribution of the Studentized range statistic are computed and printed for a range of values of q, ν and r.

10.1 Program Text

```
/* nag_prob_studentized_range (g01emc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 * Mark 7b revised, 2004.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{
  /* Scalars */
  Integer  exit_status = 0;
  double   q, v, valp;
  Integer  i, ir;
  NagError fail;

  INIT_FAIL(fail);

  printf(
          "nag_prob_studentized_range (g01emc) Example Program Results\n");

  /* Skip heading in data file */
  scanf("%*[^\n] ");
  printf("\n%s\n\n", "  q       v    ir    Quantile ");
  for (i = 1; i <= 3; ++i)
    {
      scanf("%lf%lf%ld%*[^\n] ", &q, &v, &ir);
      /* nag_prob_studentized_range (g01emc).
       * Computes probability for the Studentized range statistic
       */
      valp = nag_prob_studentized_range(q, v, ir, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf(
                  "Error from nag_prob_studentized_range (g01emc).\n%s\n",
                  fail.message);
```

```
          exit_status = 1;
          goto END;
        }
      printf("%7.4f%2s%4.1f%1s%3ld%1s%10.4f\n", q, "",
              v, "", ir, "", valp);
    }

 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_prob_studentized_range (g01emc) Example Program Data
4.6543  10.0   5
2.8099  60.0  12
4.2636   5.0   4
```

10.3 Program Results

```
nag_prob_studentized_range (g01emc) Example Program Results

  q       v    ir    Quantile

 4.6543  10.0   5     0.9500
 2.8099  60.0  12     0.3000
 4.2636   5.0   4     0.9000
```