NAG Library Function Document

nag_prob_chi_sq (g01ecc)

1 Purpose

nag_prob_chi_sq (g01ecc) returns the lower or upper tail probability for the χ^2-distribution with real degrees of freedom.

2 Specification

```
#include <nag.h>
#include <nagg01.h>
double nag_prob_chi_sq (Nag_TailProbability tail, double x, double df,
       NagError *fail)
```

3 Description

The lower tail probability for the χ^2-distribution with ν degrees of freedom, $P(X \leq x : \nu)$ is defined by:

$$P(X \leq x : \nu) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} \int_{0.0}^{x} X^{\nu/2-1} e^{-X/2}\, dX, \quad x \geq 0, \nu > 0.$$

To calculate $P(X \leq x : \nu)$ a transformation of a gamma distribution is employed, i.e., a χ^2-distribution with ν degrees of freedom is equal to a gamma distribution with scale parameter 2 and shape parameter $\nu/2$.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

5 Arguments

1: **tail** – Nag_TailProbability *Input*

On entry: indicates whether the upper or lower tail probability is required.

tail = Nag_LowerTail
The lower tail probability is returned, i.e., $P(X \leq x : \nu)$.

tail = Nag_UpperTail
The upper tail probability is returned, i.e., $P(X \geq x : \nu)$.

Constraint: **tail** = Nag_LowerTail or Nag_UpperTail.

2: **x** – double *Input*

On entry: x, the value of the χ^2 variate with ν degrees of freedom.

Constraint: **x** ≥ 0.0.

3: **df** – double *Input*

On entry: ν, the degrees of freedom of the χ^2-distribution.

Constraint: **df** > 0.0.

4: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALG_NOT_CONV

The series used to calculate the gamma probabilities has failed to converge. The result returned should represent an approximation to the solution.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL_ARG_LE

On entry, **df** $=$ ⟨*value*⟩.
Constraint: **df** > 0.0.

NE_REAL_ARG_LT

On entry, **x** $=$ ⟨*value*⟩.
Constraint: **x** ≥ 0.0.

7 Accuracy

A relative accuracy of five significant figures is obtained in most cases.

8 Parallelism and Performance

Not applicable.

9 Further Comments

For higher accuracy the transformation described in Section 3 may be used with a direct call to nag_incomplete_gamma (s14bac).

10 Example

Values from various χ^2-distributions are read, the lower tail probabilities calculated, and all these values printed out, until the end of data is reached.

10.1 Program Text

```
/* nag_prob_chi_sq (g01ecc) Example Program.
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 1, 1990.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>
```

```
int main(void)
{
  Integer  exit_status = 0;
  double   df, prob, x;
  NagError fail;

  INIT_FAIL(fail);

  /* Skip heading in data file */
  scanf("%*[^\n]");
  printf("nag_prob_chi_sq (g01ecc) Example Program Results\n");
  printf("  x      df     prob\n\n");
  while (scanf("%lf %lf", &x, &df) != EOF)
    {
      /* nag_prob_chi_sq (g01ecc).
       * Probabilities for chi^2 distribution
       */
      prob = nag_prob_chi_sq(Nag_LowerTail, x, df, &fail);
      if (fail.code != NE_NOERROR)
        {
          printf("Error from nag_prob_chi_sq (g01ecc).\n%s\n",
                  fail.message);
          exit_status = 1;
          goto END;
        }
      printf("%6.3f%8.3f%8.4f\n", x, df, prob);
    }

 END:
  return exit_status;
}
```

10.2 Program Data

```
nag_prob_chi_sq (g01ecc) Example Program Data
  8.26   20.0
  6.2     7.5
 55.76   45.0
```

10.3 Program Results

```
nag_prob_chi_sq (g01ecc) Example Program Results
  x      df     prob

 8.260  20.000  0.0100
 6.200   7.500  0.4279
55.760  45.000  0.8694
```