NAG Library Function Document

nag_sparse_herm_matvec (f11xsc)

1 Purpose

nag_sparse_herm_matvec (f11xsc) computes a matrix-vector product involving a complex sparse Hermitian matrix stored in symmetric coordinate storage format.

2 Specification

```
#include <nag.h>
#include <nagf11.h>
```

```
void nag_sparse_herm_matvec (Integer n, Integer nnz, const Complex a[],
     const Integer irow[], const Integer icol[],
     Nag_SparseSym_CheckData check, const Complex x[], Complex y[],
     NagError *fail)
```

3 Description

nag_sparse_herm_matvec (f11xsc) computes the matrix-vector product

$$y = Ax$$

where A is an n by n complex Hermitian sparse matrix, of arbitrary sparsity pattern, stored in symmetric coordinate storage (SCS) format (see Section 2.1.2 in the f11 Chapter Introduction). The array **a** stores all the nonzero elements in the lower triangular part of A, while arrays **irow** and **icol** store the corresponding row and column indices respectively.

4 References

None.

5 Arguments

1: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 1$.

2: **nnz** – Integer *Input*

On entry: the number of nonzero elements in the lower triangular part of the matrix A.

Constraint: $1 \leq \mathbf{nnz} \leq \mathbf{n} \times (\mathbf{n} + 1)/2$.

3: **a**[**nnz**] – const Complex *Input*

On entry: the nonzero elements in the lower triangular part of the matrix A, ordered by increasing row index, and by increasing column index within each row. Multiple entries for the same row and column indices are not permitted. The function nag_sparse_herm_sort (f11zpc) may be used to order the elements in this way.

4: **irow**[**nnz**] – const Integer *Input*

5: **icol**[**nnz**] – const Integer *Input*

On entry: the row and column indices of the nonzero elements supplied in array **a**.

Constraints:

irow and **icol** must satisfy the following constraints (which may be imposed by a call to nag_sparse_herm_sort (f11zpc)):

$1 \leq \textbf{irow}[i] \leq \textbf{n}$ and $1 \leq \textbf{icol}[i] \leq \textbf{irow}[i]$, for $i = 0, 1, \ldots, \textbf{nnz} - 1$;
$\textbf{irow}[i-1] < \textbf{irow}[i]$ or $\textbf{irow}[i-1] = \textbf{irow}[i]$ and $\textbf{icol}[i-1] < \textbf{icol}[i]$, for $i = 1, 2, \ldots, \textbf{nnz} - 1$.

6: **check** – Nag_SparseSym_CheckData *Input*

On entry: specifies whether or not the SCS representation of the matrix A, values of **n**, **nnz**, **irow** and **icol** should be checked.

check = Nag_SparseSym_Check
Checks are carried out on the values of **n**, **nnz**, **irow** and **icol**.

check = Nag_SparseSym_NoCheck
None of these checks are carried out.

Constraint: **check** = Nag_SparseSym_Check or Nag_SparseSym_NoCheck.

7: **x**[**n**] – const Complex *Input*

On entry: the vector x.

8: **y**[**n**] – Complex *Output*

On exit: the vector y.

9: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_INT

On entry, $\textbf{n} = \langle value \rangle$.
Constraint: $\textbf{n} \geq 1$.

On entry, $\textbf{nnz} = \langle value \rangle$.
Constraint: $\textbf{nnz} \geq 1$.

NE_INT_2

On entry, $\textbf{nnz} = \langle value \rangle$ and $\textbf{n} = \langle value \rangle$.
Constraint: $\textbf{nnz} \leq \textbf{n} \times (\textbf{n} + 1)/2$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_INVALID_SCS

On entry, $I = \langle value \rangle$, $\textbf{icol}[I-1] = \langle value \rangle$ and $\textbf{irow}[I-1] = \langle value \rangle$.
Constraint: $\textbf{icol}[I-1] \geq 1$ and $\textbf{icol}[I-1] \leq \textbf{irow}[I-1]$.

On entry, $i = \langle value \rangle$, $\textbf{irow}[i-1] = \langle value \rangle$ and $\textbf{n} = \langle value \rangle$.
Constraint: $\textbf{irow}[i-1] \geq 1$ and $\textbf{irow}[i-1] \leq \textbf{n}$.

NE_NOT_STRICTLY_INCREASING

On entry, $\mathbf{a}[i-1]$ is out of order: $i = \langle \mathit{value} \rangle$.

On entry, the location $(\mathbf{irow}[I-1], \mathbf{icol}[I-1])$ is a duplicate: $I = \langle \mathit{value} \rangle$. Consider calling nag_sparse_herm_sort (f11zpc) to reorder and sum or remove duplicates.

7 Accuracy

The computed vector y satisfies the error bound

$$\|y - Ax\|_\infty \leq c(n)\epsilon\|A\|_\infty\|x\|_\infty,$$

where $c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

8 Parallelism and Performance

nag_sparse_herm_matvec (f11xsc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag_sparse_herm_matvec (f11xsc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

9.1 Timing

The time taken for a call to nag_sparse_herm_matvec (f11xsc) is proportional to **nnz**.

10 Example

This example reads in a complex sparse Hermitian positive definite matrix A and a vector x. It then calls nag_sparse_herm_matvec (f11xsc) to compute the matrix-vector product $y = Ax$.

10.1 Program Text

```
/* nag_sparse_herm_matvec (f11xsc) Example Program.
 *
 * Copyright 2011, Numerical Algorithms Group.
 *
 * Mark 23, 2011.
 */

#include <nag.h>
#include <nag_stdlib.h>
#include <nagf11.h>

int main(void)
{
  /* Scalars */
  Integer                exit_status = 0;
  Integer                i, j, n, nnz;
  /* Arrays */
  char                   nag_enum_arg[40];
  Integer                *irow = 0, *icol = 0;
  Complex                *a = 0, *x = 0, *y = 0;
  /* NAG types */
  NagError               fail;
  Nag_SparseSym_CheckData check;

  INIT_FAIL(fail);
  printf("nag_sparse_herm_matvec (f11xsc) Example Program Results\n");

  /* Skip heading in data file */
  scanf("%*[^\n]");

  /* Read order of matrix and number of non-zero entries */
  scanf("%ld%*[^\n]", &n);
  scanf("%ld%*[^\n]", &nnz);

  /* Allocate memory */
  if ( !(a = NAG_ALLOC(nnz, Complex)) ||
       !(x = NAG_ALLOC(n, Complex)) ||
       !(y = NAG_ALLOC(n, Complex)) ||
       !(icol = NAG_ALLOC(nnz, Integer)) ||
       !(irow = NAG_ALLOC(nnz, Integer)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Read the matrix A */
  for (i = 0; i < nnz; i++)
    scanf(" ( %lf , %lf ) %ld%ld%*[^\n]", &a[i].re, &a[i].im,
          &irow[i], &icol[i] );

  /* Read the vector x */
  for (j = 0; j < n; j++) scanf(" ( %lf , %lf ) ", &x[j].re, &x[j].im);
  scanf("%*[^\n]");

  /* Calculate matrix-vector product */
  /* Nag_SparseSym_Check */
  scanf("%39s%*[^\n]", nag_enum_arg);
  check = (Nag_SparseSym_CheckData) nag_enum_name_to_value (nag_enum_arg);

  /* nag_sparse_herm_matvec (f11xsc)
   * Complex sparse Hermitian matrix vector multiply.
   */
  nag_sparse_herm_matvec(n, nnz, a, irow, icol, check, x, y, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_sparse_herm_matvec (f11xsc)\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

  /* Output results */
  printf(" Matrix-vector product\n");
  for (j = 0; j < n; j++)
    printf(" (%13.4e, %13.4e)\n", y[j].re, y[j].im);

 END:
  NAG_FREE(a);
  NAG_FREE(x);
  NAG_FREE(y);
  NAG_FREE(icol);
  NAG_FREE(irow);

  return exit_status;
}
```

10.2 Program Data

```
nag_sparse_herm_matvec (f11xsc) Example Program Data
  9                   : n
 23                   : nnz
( 6., 0.)   1    1
(-1., 1.)   2    1
( 6., 0.)   2    2
( 0., 1.)   3    2
( 5., 0.)   3    3
( 5., 0.)   4    4
( 2.,-2.)   5    1
( 4., 0.)   5    5
( 1., 1.)   6    3
( 2., 0.)   6    4
( 6., 0.)   6    6
(-4., 3.)   7    2
( 0., 1.)   7    5
(-1., 0.)   7    6
( 6., 0.)   7    7
(-1.,-1.)   8    4
( 0.,-1.)   8    6
( 9., 0.)   8    8
( 1., 3.)   9    1
( 1., 2.)   9    5
(-1., 0.)   9    6
( 1., 4.)   9    8
( 9., 0.)   9    9     : (a, irow, icol)[i], i=0,...,nnz-1
( 1., 9.)
( 2.,-8.)
( 3., 7.)
( 4.,-6.)
( 5., 5.)
( 6.,-4.)
( 7., 3.)
( 8.,-2.)
( 9., 1.)             : x[i], i=0,...,n-1
 Nag_SparseSym_Check   : check
```

10.3 Program Results

```
nag_sparse_herm_matvec (f11xsc) Example Program Results
 Matrix-vector product
 (   8.0000e+00,    5.4000e+01)
 (  -1.0000e+01,   -9.2000e+01)
 (   2.5000e+01,    2.7000e+01)
 (   2.6000e+01,   -2.8000e+01)
 (   5.4000e+01,    1.2000e+01)
 (   2.6000e+01,   -2.2000e+01)
 (   4.7000e+01,    6.5000e+01)
 (   7.1000e+01,   -5.7000e+01)
 (   6.0000e+01,    7.0000e+01)
```