NAG Library Function Document

nag_sparse_nsym_precon_ilu_solve (f11dbc)

1 Purpose

nag_sparse_nsym_precon_ilu_solve (f11dbc) solves a system of linear equations involving the incomplete LU preconditioning matrix generated by nag_sparse_nsym_fac (f11dac).

2 Specification

```
#include <nag.h>
#include <nagf11.h>
```

```
void nag_sparse_nsym_precon_ilu_solve (Nag_TransType trans, Integer n,
     const double a[], Integer la, const Integer irow[],
     const Integer icol[], const Integer ipivp[], const Integer ipivq[],
     const Integer istr[], const Integer idiag[],
     Nag_SparseNsym_CheckData check, const double y[], double x[],
     NagError *fail)
```

3 Description

nag_sparse_nsym_precon_ilu_solve (f11dbc) solves a system of linear equations

$$Mx = y, \quad \text{or} \quad M^{\mathrm{T}}x = y,$$

according to the value of the argument **trans**, where the matrix $M = PLDUQ$, corresponds to an incomplete LU decomposition of a sparse matrix stored in coordinate storage (CS) format (see Section 2.1.1 in the f11 Chapter Introduction), as generated by nag_sparse_nsym_fac (f11dac).

In the above decomposition L is a lower triangular sparse matrix with unit diagonal elements, D is a diagonal matrix, U is an upper triangular sparse matrix with unit diagonal elements and, P and Q are permutation matrices. L, D and U are supplied to nag_sparse_nsym_precon_ilu_solve (f11dbc) through the matrix

$$C = L + D^{-1} + U - 2I$$

which is an **n** by **n** sparse matrix, stored in CS format, as returned by nag_sparse_nsym_fac (f11dac). The permutation matrices P and Q are returned from nag_sparse_nsym_fac (f11dac) via the arrays **ipivp** and **ipivq**.

It is envisaged that a common use of nag_sparse_nsym_precon_ilu_solve (f11dbc) will be to carry out the preconditioning step required in the application of nag_sparse_nsym_basic_solver (f11bec) to sparse linear systems. nag_sparse_nsym_precon_ilu_solve (f11dbc) is used for this purpose by the Black Box function nag_sparse_nsym_fac_sol (f11dcc).

nag_sparse_nsym_precon_ilu_solve (f11dbc) may also be used in combination with nag_sparse_nsym_fac (f11dac) to solve a sparse system of linear equations directly (see Section 9.5 in nag_sparse_nsym_fac (f11dac)). This use of nag_sparse_nsym_precon_ilu_solve (f11dbc) is demonstrated in Section 10.

4 References

None.

5 Arguments

1: **trans** – Nag_TransType *Input*

On entry: specifies whether or not the matrix M is transposed.

trans = Nag_NoTrans
$Mx = y$ is solved.

trans = Nag_Trans
$M^{\mathrm{T}}x = y$ is solved.

Constraint: **trans** = Nag_NoTrans or Nag_Trans.

2: **n** – Integer *Input*

On entry: n, the order of the matrix M. This **must** be the same value as was supplied in the preceding call to nag_sparse_nsym_fac (f11dac).

Constraint: **n** ≥ 1.

3: **a**[**la**] – const double *Input*

On entry: the values returned in the array **a** by a previous call to nag_sparse_nsym_fac (f11dac).

4: **la** – Integer *Input*

On entry: the dimension of the arrays **a**, **irow** and **icol**. This **must** be the same value returned by the preceding call to nag_sparse_nsym_fac (f11dac).

5: **irow**[**la**] – const Integer *Input*
6: **icol**[**la**] – const Integer *Input*
7: **ipivp**[**n**] – const Integer *Input*
8: **ipivq**[**n**] – const Integer *Input*
9: **istr**[**n** + **1**] – const Integer *Input*
10: **idiag**[**n**] – const Integer *Input*

On entry: the values returned in arrays **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** by a previous call to nag_sparse_nsym_fac (f11dac).

11: **check** – Nag_SparseNsym_CheckData *Input*

On entry: specifies whether or not the CS representation of the matrix M should be checked.

check = Nag_SparseNsym_Check
Checks are carried on the values of **n**, **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag**.

check = Nag_SparseNsym_NoCheck
None of these checks are carried out.

See also Section 9.2.

Constraint: **check** = Nag_SparseNsym_Check or Nag_SparseNsym_NoCheck.

12: **y**[**n**] – const double *Input*

On entry: the right-hand side vector y.

13: **x**[**n**] – double *Output*

On exit: the solution vector x.

14: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **n** $=$ ⟨*value*⟩.
Constraint: **n** ≥ 1.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_INVALID_CS

On entry, $i =$ ⟨*value*⟩, **icol**$[i-1] =$ ⟨*value*⟩, and **n** $=$ ⟨*value*⟩.
Constraint: **icol**$[i-1] \geq 1$ and **icol**$[i-1] \leq$ **n**.
Check that **a**, **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** have not been corrupted between calls to nag_sparse_nsym_precon_ilu_solve (f11dbc) and nag_sparse_nsym_fac (f11dac).

On entry, $i =$ ⟨*value*⟩, **irow**$[i-1] =$ ⟨*value*⟩, **n** $=$ ⟨*value*⟩.
Constraint: **irow**$[i-1] \geq 1$ and **irow**$[i-1] \leq$ **n**.
Check that **a**, **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** have not been corrupted between calls to nag_sparse_nsym_precon_ilu_solve (f11dbc) and nag_sparse_nsym_fac (f11dac).

NE_INVALID_CS_PRECOND

On entry, **idiag**$[i-1]$ appears to be incorrect: $i =$ ⟨*value*⟩.
Check that **a**, **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** have not been corrupted between calls to nag_sparse_nsym_precon_ilu_solve (f11dbc) and nag_sparse_nsym_fac (f11dac).

On entry, **istr** appears to be invalid.
Check that **a**, **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** have not been corrupted between calls to nag_sparse_nsym_precon_ilu_solve (f11dbc) and nag_sparse_nsym_fac (f11dac).

On entry, **istr**$[i-1]$ is inconsistent with **irow**: $i =$ ⟨*value*⟩.
Check that **a**, **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** have not been corrupted between calls to nag_sparse_nsym_precon_ilu_solve (f11dbc) and nag_sparse_nsym_fac (f11dac).

NE_INVALID_ROWCOL_PIVOT

On entry, $i =$ ⟨*value*⟩, **ipivp**$[i-1] =$ ⟨*value*⟩, **n** $=$ ⟨*value*⟩.
Constraint: **ipivp**$[i-1] \geq 1$ and **ipivp**$[i-1] \leq$ **n**.
Check that **a**, **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** have not been corrupted between calls to nag_sparse_nsym_precon_ilu_solve (f11dbc) and nag_sparse_nsym_fac (f11dac).

On entry, $i =$ ⟨*value*⟩, **ipivq**$[i-1] =$ ⟨*value*⟩, **n** $=$ ⟨*value*⟩.
Constraint: **ipivq**$[i-1] \geq 1$ and **ipivq**$[i-1] \leq$ **n**.
Check that **a**, **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** have not been corrupted between calls to nag_sparse_nsym_precon_ilu_solve (f11dbc) and nag_sparse_nsym_fac (f11dac).

On entry, **ipivp**$[i-1]$ is a repeated value: $i =$ ⟨*value*⟩.
Check that **a**, **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** have not been corrupted between calls to nag_sparse_nsym_precon_ilu_solve (f11dbc) and nag_sparse_nsym_fac (f11dac).

On entry, **ipivq**$[i-1]$ is a repeated value: $i =$ ⟨*value*⟩.
Check that **a**, **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** have not been corrupted between calls to nag_sparse_nsym_precon_ilu_solve (f11dbc) and nag_sparse_nsym_fac (f11dac).

NE_NOT_STRICTLY_INCREASING

On entry, **a**$[i-1]$ is out of order: $i = \langle value \rangle$.
Check that **a**, **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** have not been corrupted between calls to nag_sparse_nsym_precon_ilu_solve (f11dbc) and nag_sparse_nsym_fac (f11dac).

On entry, the location $(\textbf{irow}[i-1], \textbf{icol}[i-1])$ is a duplicate: $i = \langle value \rangle$.
Check that **a**, **irow**, **icol**, **ipivp**, **ipivq**, **istr** and **idiag** have not been corrupted between calls to nag_sparse_nsym_precon_ilu_solve (f11dbc) and nag_sparse_nsym_fac (f11dac).

7 Accuracy

If **trans** = Nag_NoTrans the computed solution x is the exact solution of a perturbed system of equations $(M + \delta M)x = y$, where

$$|\delta M| \leq c(n)\epsilon P|L||D||U|Q,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***. An equivalent result holds when **trans** = Nag_Trans.

8 Parallelism and Performance

Not applicable.

9 Further Comments

9.1 Timing

The time taken for a call to nag_sparse_nsym_precon_ilu_solve (f11dbc) is proportional to the value of **nnzc** returned from nag_sparse_nsym_fac (f11dac).

9.2 Use of check

It is expected that a common use of nag_sparse_nsym_precon_ilu_solve (f11dbc) will be to carry out the preconditioning step required in the application of nag_sparse_nsym_basic_solver (f11bec) to sparse linear systems. In this situation nag_sparse_nsym_precon_ilu_solve (f11dbc) is likely to be called many times with the same matrix M. In the interests of both reliability and efficiency, you are recommended to set **check** = Nag_SparseNsym_Check for the first of such calls, and for all subsequent calls set **check** = Nag_SparseNsym_NoCheck.

10 Example

This example reads in a sparse nonsymmetric matrix A and a vector y. It then calls nag_sparse_nsym_fac (f11dac), with **lfill** $= -1$ and **dtol** $= 0.0$, to compute the **complete** LU decomposition

$$A = PLDUQ.$$

Finally it calls nag_sparse_nsym_precon_ilu_solve (f11dbc) to solve the system

$$PLDUQx = y.$$

10.1 Program Text

```
/* nag_sparse_nsym_precon_ilu_solve (f11dbc) Example Program.
 *
 * Copyright 2011, Numerical Algorithms Group.
 *
 * Mark 23, 2011.
 */
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf11.h>
int main(void)
```

```
{
  /* Scalars */
  Integer                   exit_status = 0;
  double                    dtol;
  Integer                   i, la, lfill, n, nnz, nnzc, npivm;
  /* Arrays */
  double                    *a = 0, *x = 0, *y = 0;
  Integer                   *icol = 0, *idiag = 0, *ipivp = 0, *ipivq = 0,
                            *irow = 0, *istr = 0;
  /* NAG types */
  Nag_SparseNsym_Piv        pstrat;
  Nag_SparseNsym_Fact       milu;
  Nag_SparseNsym_CheckData  check;
  Nag_TransType             trans;
  NagError                  fail;

  INIT_FAIL(fail);

  printf("nag_sparse_nsym_precon_ilu_solve (f11dbc) Example Program Results");
  printf("\n\n");
  /* Skip heading in data file*/
  scanf("%*[^\n]");
  scanf("%ld%*[^\n]", &n);
  scanf("%ld%*[^\n]", &nnz);
  la = 2 * nnz;
  if (
      !(a = NAG_ALLOC((la), double)) ||
      !(x = NAG_ALLOC((n), double)) ||
      !(y = NAG_ALLOC((n), double)) ||
      !(icol = NAG_ALLOC((la), Integer)) ||
      !(idiag = NAG_ALLOC((n), Integer)) ||
      !(ipivp = NAG_ALLOC((n), Integer)) ||
      !(ipivq = NAG_ALLOC((n), Integer)) ||
      !(irow = NAG_ALLOC((la), Integer)) ||
      !(istr = NAG_ALLOC((n + 1), Integer))
      )
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }
  /* Read the non-zero elements of the matrix a*/
  for (i = 0; i < nnz; i++)
    scanf("%lf%ld%ld%*[^\n] ", &a[i], &irow[i], &icol[i]);
  /* Read the vector y*/
  for (i = 0; i < n; i++) scanf("%lf", &y[i]);

  /* Calculate LU factorization*/
  lfill = -1;
  dtol = 0.0;
  pstrat = Nag_SparseNsym_CompletePiv;
  milu = Nag_SparseNsym_UnModFact;
  /* nag_sparse_nsym_fac (f11dac).
   * Incomplete LU factorization (nonsymmetric)
   */
  nag_sparse_nsym_fac(n, nnz, &a, &la, &irow, &icol, lfill, dtol, pstrat,
                      milu, ipivp, ipivq, istr, idiag, &nnzc, &npivm, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_sparse_nsym_fac (f11dac)\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Check value of npivm*/
  if (npivm > 0) {
    printf("Factorization is not complete \n");
    goto END;
  }
  /* Solve P L D U x = y*/
  check = Nag_SparseNsym_Check;
  trans = Nag_NoTrans;
  /* nag_sparse_nsym_precon_ilu_solve (f11dbc)
   * Solution of linear system involving incomplete LU preconditioning matrix
   */
  nag_sparse_nsym_precon_ilu_solve(trans, n, a, la, irow, icol, ipivp, ipivq,
                                   istr, idiag, check, y, x, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_sparse_nsym_precon_ilu_solve (f11dbc)\n%s\n",
             fail.message);
      exit_status = 2;
      goto END;
    }
  /* Output results*/
  printf(" Solution of linear system \n");
  for (i = 0; i < n; i++) printf("%16.4e\n", x[i]);
END:
  NAG_FREE(a);
  NAG_FREE(x);
  NAG_FREE(y);
  NAG_FREE(icol);
  NAG_FREE(idiag);
  NAG_FREE(ipivp);
  NAG_FREE(ipivq);
  NAG_FREE(irow);
  NAG_FREE(istr);
  return exit_status;
}
```

10.2 Program Data

```
nag_sparse_nsym_precon_ilu_solve (f11dbc) Example Program Data
  4                   : n
 11                   : nnz
  1.    1    2
  1.    1    3
 -1.    2    1
  2.    2    3
  2.    2    4
  3.    3    1
 -2.    3    4
  1.    4    1
 -2.    4    2
  1.    4    3
  1.    4    4         : a[i], irow[i], icol[i], i=0,...,nnz-1
  5.0  13.0 -5.0  4.0  : y[i], i=0,...,n-1
```

10.3 Program Results

```
nag_sparse_nsym_precon_ilu_solve (f11dbc) Example Program Results

 Solution of linear system
      1.0000e+00
      2.0000e+00
      3.0000e+00
      4.0000e+00
```